Active modal control of hull radiated noise

Xia Pan, Yan Tso and Ross Juniper
Maritime Platforms Division, Defence Science and Technology Organisation, P0 Box 4331, Melbourne, Victoria 3001, Australia

ABSTRACT

A theoretical analysis of the active modal control of radiated pressure from a finite cylindrical pressure hull is presented. The control action is implemented through a Tee-sectioned circumferential stiffener driven by a pair of PZT stack actuators. The actuators are located under the flange of the stiffener and are driven out of phase to produce a control moment. This paper examines the effects of control actions, both structurally and acoustically, for a control moment applied around the circumference of the hull. The model considered is a water-loaded finite stiffened cylindrical shell with rigid ends caps. One end of the shell is excited by an axial force while the other end is free. Control action is achieved by using the PZT actuators and stiffener to minimize the structural response and radiated pressure. It was found that the control system was capable of reducing the radiated pressure by approximately two-thirds for the first three axial modes.

INTRODUCTION

The work described in this paper is concerned with the active control of the structural response and sound pressure radiation of a finite cylindrical shell subjected to an axial excitation.

Recently, the radiated pressure of a finite cylindrical shell in axisymmetric vibration has been investigated by Tso and Jenkins (Tso and Jenkins 2003). In their study, they simulated the response of a submarine hull due to propeller excitations as a water-loaded finite cylinder subjected to an axial force. Their model is developed for low frequency applications such as the blade tonal noise. The active control of vibration transmission in a cylindrical shell has been studied by Pan and Hansen (Pan and Hansen 1996, Pan and Hansen 1997) using circumferential arrays of vibration control actuators and sensors. Young (Young 1995) studied the active control of vibration of an air duct using an angled stiffener and point forces. Tso and Kessissoglou (Tso et al. 2003) carried out an analysis of the active control of the first two structural modes of a cylindrical shell using an axial force applied at the opposite end of a primary excitation source. However, the amplitude of the axial force required was about the same order as the primary excitation, making this method impractical for real maritime structures.

The work outlined in this paper is based on the sound pressure radiation model developed by Tso and Jenkins (Tso and Jenkins 2003) coupled with a novel active control technique where a control moment is applied to minimize the structural response and radiated pressure. The control moment is applied by using a Tee-sectioned stiffener combined with a pair of PZT stack actuators driven out of phase as shown in Figure 1. The control strategy used is the feedforward active control and, assumed that an ideal feedforward controller is available. Using this control strategy, the combination of the stiffener and the actuators are capable of developing a control moment of sufficient amplitude to enable the implementation of an effective control action.

ACTIVE CONTROL OF ACCELERATION

There are two fundamental approaches considered in this paper for developing control strategies for the active control of radiated pressure from a cylindrical shell, namely, acceleration control and radiated pressure control. This section describes the former approach while the latter approach is considered in the next section.

As a first approximation, the control action due to the stiffener and the stack actuators is replaced by circumferential line moment acting around a bulkhead as shown in Figure 2. The inclusion of the bulkhead demonstrates how the method of analysis may be applied to shells with structural discontinuities. A simplified model of the pressure hull may then be considered as a structural junction with two cylindrical shells and a circular plate.
If the pressure hull is excited by a sinusoidal axial force of amplitude $F$ located at $x = 0$, the flexural displacement $w_c(x)$ (see Tso and Jenkins 2003) at any location $x$ may be expressed as:

$$w_c(x) = Fw_{c-f}(x)$$

(1)

where $w_{c-f}$ is the flexural displacement per unit axial force.

Similarly, if a line moment of amplitude $M$ is applied at $x = x_i$, the flexural displacement due to this moment is:

$$w_c(x) = Mw_{c-m}(x)$$

(2)

where $w_{c-m}$ is the flexural displacement per unit line moment.

The total flexural displacement at $x$ due to the primary and control excitations together is then:

$$w_c(x) = Fw_{c-f}(x) + Mw_{c-m}(x)$$

(3)

The optimal moment which minimizes the flexural displacement at $x = x_e$ is obtained from Equation (3) by setting $w_c(x_e)$ to be zero: i.e.,

$$M = -F \frac{w_{c-f}(x_e)}{w_{c-m}(x_e)}$$

(4)

Similarly, the optimal moment which minimizes the axial displacement at $x = x_e$ is

$$M = -F \frac{u_{c-f}(x_e)}{u_{c-m}(x_e)}$$

(5)

where $u_{c-f}$ is the axial displacement per unit primary force and $u_{c-m}$ is the axial displacement per unit control line moment.

**ACTIVE CONTROL OF SOUND RADIATION**

The total sound radiation of a pressure hull may be considered as the sum of the pressure due to the end plates and the radial motion of the cylinder (Tso and Jenkins 2003). The pressure due to the radial motion of the primary and control excitation together is:

$$p_{rad} = \int_0^{2\pi} \left[ p_c(R, \theta) + p_{c-f}(R, \theta) \right] d\theta$$

(6)

where $p_c$ is the pressure due to a unit primary force excitation and $p_{c-f}$ is the pressure due to a unit control moment excitation. Similarly, the pressure due to the end plates can be shown as:

$$p_e(R, \theta) = \int_0^{2\pi} \left[ p_{e-f}(R, \theta) + p_{e-m}(R, \theta) \right] d\theta$$

(7)

where $p_{e-f}$ is the axial pressure due to a unit primary force excitation and $p_{e-m}$ is the axial pressure due to a unit control moment excitation.

The total sound radiation then becomes:

$$p_{rad} = \int_0^{2\pi} \left[ p_c(R, \theta) + p_{c-f}(R, \theta) \right] d\theta$$

$$= \int_0^{2\pi} \left[ p_{e-f}(R, \theta) + p_{e-m}(R, \theta) \right] d\theta$$

(8)

The optimal control moment to minimize the total radiated pressure is obtained by determining the derivatives of Equation (8) with respect to the control moment and setting the result to zero. The optimal control moment may then be expressed as:

$$M = -F \frac{\int_0^{2\pi} \left[ p_{c-f}(R, \theta) + p_{c-m}(R, \theta) \right] d\theta}{\int_0^{2\pi} \left[ p_{c-f}(R, \theta) + p_{c-m}(R, \theta) \right]^2 d\theta}$$

(9)

**NUMERICAL RESULTS**

The numerical results presented in this section were based on a steel pressure hull of 7 m diameter, 60 m length and a shell thickness of 25 mm. A primary excitation force of 1 N amplitude is applied at $x = 0$ m and the control moment at $x = 20$ m. An error sensor is placed at $x = 40$ m for the case of acceleration control while the sound pressure level at a distance of 1000 m is used as the error signal for sound radiation control. Additional results are presented in the Section (Effect of stiffener and control actuator location) for other control source locations.

It is assumed that the pressure hull is in axisymmetric vibration. Therefore, the only circumferential mode is the breath mode ($n=0$). For the purpose of this study, the results presented here are mainly for the first three axial modes.

Figure 3 shows the axial displacement at both ends of the pressure hull as a function of frequency. In order to obtain realistic amplitudes near the resonant frequency of the hull, a structural loss factor of 0.02 is used in the calculations. It can be seen that the first three axial modes are approximately 12, 24 and 35 Hz.
Minimization of acceleration

The cost function to be minimized in this control strategy is acceleration at the two end plates. Figure 4 shows the controlled and uncontrolled mode shapes of the first three axial hull modes. The phase relationship between the ends of the pressure hull can clearly be observed. The results demonstrate that the axial displacement at the ends of the hull is reduced significantly for the first three axial modes.

![Figure 4](image)

**Figure 4.** Axial displacement with the control moment (line moment) using axial displacement as the cost function: (a) at first axial mode; (b) at second axial mode; (c) at third axial mode. —, uncontrolled; - - -, controlled.

However, in order to minimize the axial motion of the end plates, the control actuators have to induce a significant radial motion on the cylinder. Figure 5 shows the controlled and uncontrolled radial displacement for the first three hull modes. With the application of control actions, large radial displacements at the actuator location can be observed for the first two modes (see Figure 5(a) and (b)), and to a lesser extent for the third mode (see Figure 5(c)).

![Figure 5](image)

**Figure 5.** Radial displacement with the control moment (line moment) using axial displacement as the cost function: (a) at first axial mode; (b) at second axial mode; (c) at third axial mode. —, uncontrolled; - - -, controlled.

The effect of the controlled radial displacement (presented in Figure 5) on the total radiated pressure is shown in Figure 6. It can be seen that, for the first two axial modes (Figure 6(a) and (b)), the total radiated pressure with acceleration control is very much higher than the uncontrolled case due to the large radial displacements. For the third axial mode (Figure 6(c)), the total radiated pressure with acceleration control is reduced, as the small increase in radial displacement is more than compensated by the elimination of axial displacement.
The results presented in this section suggest that a different cost function is warranted to account for all the modes.

Minimization of radiated pressure

The cost function to be minimized in this control strategy is the total radiated pressure. This method of control may be implemented by an array of accelerometers to measure the radial motion of the shell in order to determine the component of radiated pressure due to the radial motion (refer to the acceleration measurement system in Young (1995)). The component of radiated pressure due to axial motion may be determined by measuring the acceleration of the end plates.

Figure 7(a) shows the controlled and uncontrolled total radiated pressure at the first axial mode by minimizing the radiated pressure at 90° from the cylinder axis, where the control action is more effective in this orientation. It can be observed that approximately two-thirds of the total pressure has been reduced.

Figures 7(b) and (c) show the results for the second and third modes respectively. The results were obtained by minimizing the sum of the total radiated pressure from 0° to 180°.
Again, a significant reduction of radiation pressure can be observed.

By comparing the results between Figure 7 and Figure 6, it can be seen that the radiated pressure is a better-cost function in terms of reducing radiated pressure than the axial acceleration for this control configuration.

Refer to Figure 7, the line moment location and the ratio of the amplitude of control (the line) moment to the primary force at each axial mode (see Equation (9) where the total radiated pressure are used as the cost function), are recorded in Table 1.

**Table 1.** Line moment location and line moment/force ratio

<table>
<thead>
<tr>
<th>Axial mode</th>
<th>Location of moment (m)</th>
<th>Moment/Force ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>First mode</td>
<td>20</td>
<td>0.15</td>
</tr>
<tr>
<td>Second mode</td>
<td>20</td>
<td>0.16</td>
</tr>
<tr>
<td>Third mode</td>
<td>20</td>
<td>0.008</td>
</tr>
</tbody>
</table>

Table 1 shows that the amplitude of the control moment is much lower than the primary force for the first three axial modes. To put these figures into perspective, a typical PZT stack can generate a pushing force of 30 kN. By placing the stacks under a 200 mm flange (with a moment arm of 100 mm), it translates to a point moment of 3 kNm. If the PZT stacks are spaced at 500 mm apart around the circumference of the hull, it will give an equivalent line moment of 6 kNm/m. Referring back to Table 1, a control moment of 6 kNm/m is capable of controlling a primary force of 40 kN for the first axial mode which is sufficient for practical maritime applications.

**Effect of stiffener and control actuator location**

In practice, it may not be feasible to locate the control actuators at a specific position along the cylindrical shell, say at L/3. Another concern is that this arbitrary location may not be optimum for the attenuation of noise radiation. This section explores the effect of locating the control actuator at other positions and their effects on noise radiation.

In order to implement an effective control of the total radiated pressure, calculations were conducted with actuator locations at 1m increments along the length of the cylindrical shell. It was found that the control actuator should be located close to the primary source for optimum attenuation. This enables an effective control of the motion at the other end without causing an excessive radial motion.

Table 2 shows the locations of the control actuators and the amplitude ratio between the control moment and primary force where the control moment is close to the excitation source. Again, the amplitudes of the control moments are much lower than the primary force.

**Table 2.** Line moment location and line moment/force ratio with the control moment close to the excitation source

<table>
<thead>
<tr>
<th>Axial mode</th>
<th>Location of moment (m)</th>
<th>Moment/Force ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>First mode</td>
<td>0.25</td>
<td>0.13</td>
</tr>
<tr>
<td>Second mode</td>
<td>0.25</td>
<td>0.12</td>
</tr>
<tr>
<td>Third mode</td>
<td>0.25</td>
<td>0.007</td>
</tr>
</tbody>
</table>

Figure 8 shows the total radiated pressure of the first three modes that corresponds to the control actuator locations as shown in Table 2. The radiation pattern of the second mode (Figure 8(b)) differs considerably from Figure 7(b) due to the difference in location of the actuator. Also, a larger attenuation is achieved with the control moment located close to the excitation source. Both the first and third modes (Figure 8(a) and (c)) show similar reduction in radiated pressure compared with Figure 7(a) and (c). It seems that the second mode is more sensitive to the control moment location for this control configuration.

**Comparison between line moment control and point moment control**

The implementation of the control system requires a series of point moments to be applied around the circumference of the hull. To investigate the effect of replacing a line moment with a series of point moments, calculations were performed for the total radiated pressure of the first three modes with the actuator locations shown in Table 2, but in this case the system is controlled by three evenly spaced equivalent point moments. Figure 9 presents the results of these calculations. It can be seen that the results are very similar to those of line moment control, so that in practical terms point moments can be used without reducing the effectiveness.

**CONCLUSIONS**

The radiated pressure of a cylindrical shell subjected to an axial excitation may be reduced by approximately two-thirds using an active control moment. The amplitude of the control moment is small compared with the excitation force and may be implemented by a series of PZT stack actuators.

For a finite cylinder, the control of axial motion only may lead to a higher overall radiated pressure due to the excessive radial motion. This finding indicates that the phasing between the radial and axial motions is a significant factor in the application of active control to reduce the radiated pressure.
Figure 8. Total radiated pressure with the control moment (line moment) close to the excitation source using pressure as the cost function:
(a) at first axial mode; (b) at second axial mode; (c) at third axial mode. —, uncontrolled; - - -, controlled.

Figure 9. Total radiated pressure with the control moment (three point moments) close to the excitation source using pressure as the cost function: (a) at first axial mode; (b) at second axial mode; (c) at third axial mode. —, uncontrolled; - - -, controlled.

REFERENCES


**APPENDIX A**

**Superscripts**

* complex conjugate

**Subscripts**

p plate only

c cylindrical shell only

e end plate only

f primary force only

m control moment only

c-f cylindrical shell response due to unit primary force

c-m cylindrical shell response due to unit control moment

p-f plate-stiffener response due to unit primary force

p-m plate-stiffener response due to unit control moment