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How should acoustics adapt to meet future demands?

Acoustic cloaking of an elastic cylinder in a plane waveguide by active means

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ABSTRACT

A plane two-dimensional acoustic waveguide with an elastic infinite cylinder as a scattering object is considered. Based on the solution of the scattering problem obtained by the author previously, acoustic pressure distribution and energy streamlines are calculated. It is shown that, with some specific configurations of acoustic sources on the surface of the cylinder, the scattered field will contain evanescent waveguide modes only, thus being absent at large distances from the cylinder and making the cylinder acoustically invisible. It is also shown that such sources consume zero power on average without taking into consideration viscous losses. It is shown that, for a specific frequency and a specific modal composition of the incident field, there is infinitely large variety of possible source configurations leading to acoustic invisibility. Some issues related to acoustic invisibility in viscous fluids and free space are discussed.

INTRODUCTION

During recent years, some suggestions are made in literature about possible means to prevent a rigid object from reflecting acoustic waves, thus making the object acoustically invisible.

One way to achieve the invisibility of a rigid object is called “acoustic cloaking”. Cummer & Schurig (2007) suggested a cloaking shell consisting of a material with anisotropic density and scalar bulk modulus. The density of such a material is a tensor with different values corresponding to different directions in space. The authors showed that a layer of such a material placed on the surface of a two-dimensional cylindrical rigid shell will “smoothly and reflectionlessly direct compressional acoustic waves around” the shell.

Torrent & Sanchez-Deheza (2008) proposed that a cloaking shell can be built of a multilayered structure consisting of two layers and made up of two different acoustic isotropic metamaterials. These metamaterials consist of “sonic crystals”, i.e. periodic arrays of cylindrical scatterers. Numerical experiments conducted by the authors showed that a shell consisting of 200 double layers of metamaterials can efficiently cloak a rigid cylinder.

Another concept of achieving acoustic invisibility refers to “smart obstacles” (Fatone *et al* 2004, Fatone *et al* 2006, Zirilli 2007), which represent scattering objects that, when hit by an acoustic wave, “will react to pursue a preassigned goal”. There are at least three types of such goals (Zirilli 2007): to be undetectable, to appear with a shape and boundary impedance different from the actual shape and impedance (masking), and to appear in a location in space different from its actual location (ghost obstacle). The above-mentioned publications show that these goals can be achieved by circulating a suitable “pressure current” on the surface of a smart obstacle. This “pressure current” physically is simply additional acoustic sources on the surface of the obstacle.

In this paper, a theoretical possibility to achieve invisibility of an elastic cylinder in a waveguide is demonstrated. The necessary distribution of acoustic sources is calculated. It is shown that, for any given modal composition of the incident acoustic wave, there are an infinite number of the acoustic source configurations leading to invisibility.

The present work differs from the above-mentioned works by two aspects. First, this work is about cloaking an *elastic*, rather than rigid, cylinder. Second, the cylinder is located in a plane waveguide rather than in a free space. This consideration has been made possible by previous works of the present author and his co-authors, who developed a novel method of solving scattering problems in two-dimensional plane waveguides (Belov, Gorsky, Zinoviev & Khilko 1994, Belov, Gorsky, Zalezsky & Zinoviev 1998, Zinoviev & Belov 1998, Zinoviev 1998b, Zinoviev 2000). The method is based on a new representation of the waveguide Green’s function and allows taking into account all waveguide modes without difficulty.

More recently, this method has been applied also to a three-dimensional plane fluid layer (Zinoviev 2005). The Green’s function for the layer has been developed and its properties have been investigated.

It may also be noted that the results published in this paper have been obtained before the above-mentioned works on “acoustic cloaking” and “smart obstacles” and have been presented at several seminars. The present article is the first publication of these results in open literature.

STATEMENT OF THE PROBLEM

The configuration of the waveguide is shown in Figure 1. A planar two-dimensional waveguide of depth, D, with pressure release conditions on both upper and lower boundaries is filled with a compressible perfect liquid of density, ρ, and sound speed, c. The waveguide contains a homogeneous cylinder with radius, R_0. The elastic material of the cylinder is described by density, ρ_c, and Lame coefficients, λ, μ.

The incident (external) wave $P_e(x,y)$ of frequency, f, is emitted by a source located at $X = -\infty$. The total pressure field in the waveguide, $P(x,y)$, can be written as a sum of the incident wave, $P_e(x,y)$, and the scattered wave, $P_s(x,y)$:

$$P(x, y) = P_e(x,y) + P_s(x,y). \qquad (1)$$

The scattered acoustic field is separated into the reflected and transmitted fields, P^- and P^+ in the areas before and behind the cylinder respectively. Harmonic temporal dependence $\exp(-i\omega t)$ is assumed.

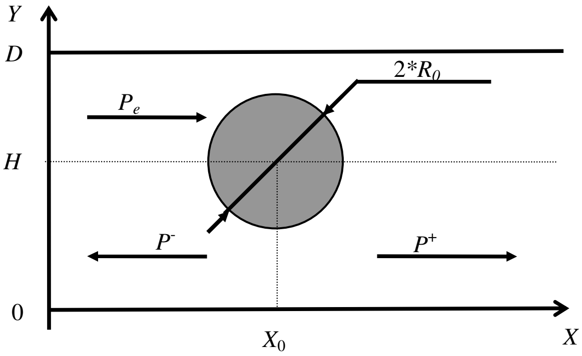

Figure 1. Cross-section of the waveguide under consideration

In this analysis, all variables having dimension of length are normalised by D/π, which simplifies the representation of the cut-on frequencies of the waveguide normal modes: they are equal to integer numbers on the non-dimensional wavenumber, $k=2fD/c$, scale. The non-dimensional wavenumber equals the number of acoustic half-wavelengths that fit across the waveguide. Using this normalisation, the incident, reflected and transmitted waves can be written as sums of waveguide normal modes as follows:

$$P_e(x,y) = \sum_{n=1}^{\infty} A_n \sin(ny)\exp(ig_n x), \quad -\infty < x < \infty, \qquad (2)$$

$$P^-(x,y) = \sum_{n=1}^{\infty} B_n^- \sin(ny)\exp(-ig_n x), -\infty < x < X_0 - R_0, \qquad (3)$$

$$P^+(x,y) = \sum_{n=1}^{\infty} B_n^+ \sin(ny)\exp(ig_n x), \quad X_0 + R_0 < x < \infty, \qquad (4)$$

where $g_n=(k^2-n^2)^{1/2}$ are longitudinal wavenumbers. The wavenumbers $k_n = 1,2,..$ correspond to the cut-on frequencies of the modes, below which the modes do not propagate, but exponentially decay as x increases.

Elastic waves inside the cylinder are described by the scalar potential, $F(r, \varphi)$, and the z-component of the vector potential, $\Phi(r, \varphi)$. The displacement vector $\Delta\mathbf{r}$ in the elastic material of the cylinder is expressed as

$$\Delta\mathbf{r}(r,\varphi) = \nabla F(r,\varphi) + \nabla\times\Phi(r,\varphi), \qquad (5)$$

where r and φ are polar coordinates with the origin in the centre of the cylinder, and z is the axial coordinate. The first and second terms in Eq. (5) describe the longitudinal and transversal waves in the cylinder respectively.

The variables P, F, and Φ satisfy Helmholtz equations in their respective regions:

$$(\Delta + k^2)P(x, y) = 0 \qquad (6)$$

in the region $r > R_0$, $0 < y < \pi$, $-\infty < x < \infty$,

$$(\Delta + k_l^2)F(x, y) = 0, \qquad (7)$$

$$(\Delta + k_t^2)\Phi(x, y) = 0, \qquad (8)$$

in the region $0 \le r \le R_0$, $0 \le \varphi \le 2\pi$, where Δ is the Laplace operator. The wavenumbers k_l and k_t are determined by

$$k_l = kc/c_l, \qquad (9)$$

$$k_t = kc/c_t. \qquad (10)$$

Velocities of the longitudinal and transversal waves c_l and c_t in the elastic material of the cylinder are expressed as

$$c_l = [(\lambda + 2\mu)/\rho_c]^{1/2}, \qquad (11)$$

$$c_t = [\mu/\rho_c]^{1/2}. \qquad (12)$$

It is assumed that both waveguide boundaries are pressure release, so that

$$P(x,0) = 0, \quad P(x,\pi) = 0. \qquad (13)$$

Conditions on the elastic cylinder boundary are considered to be as follows (Zinoviev & Belov 1998, Zinoviev 2000).
The absence of a tangential component of the stress tensor (the liquid is inviscid):

$$\mu\left[\frac{\pi}{D}\right]^2\left[2\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial F}{\partial\varphi}\right) - k_t^2\Phi - 2\frac{\partial^2\Phi}{\partial r^2}\right]_{r=R_0} = 0. \qquad (14)$$

The continuity of the normal component of the stress tensor:

$$2\mu\left[\frac{\pi}{D}\right]^2\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\Phi}{\partial\varphi}\right) - \frac{\lambda}{2\mu}k_l^2 F + \frac{\partial^2 F}{\partial r^2}\right]_{r=R_0} = -P|_{r=R_0} \qquad (15)$$

The continuity of the normal component of the displacement vector.

$$\left[\frac{\pi}{D}\right]^2\left[\frac{\partial F}{\partial r} + \frac{1}{r}\frac{\partial\Phi}{\partial\varphi}\right]_{r=R_0} = \left[\frac{1}{\rho\,(kc)^2}\frac{\partial P}{\partial r}\right]_{r=R_0}. \qquad (16)$$

The scattered field is represented as a field of a layer of monopole sources on the surface of the cylinder:

$$P_s(x,y) = \int_L G(x,y;x_0,y_0)\mu^0(x_0,y_0)dl_0 \qquad (17)$$

Here $\mu^0(x_0,y_0)$ is an unknown source distribution on the surface of the cylinder, (x_0,y_0) are the coordinates of the current integration point over the line L which is a circle of radius R_0 centred at the point y = H, $x = X_0$, (x,y) are the coordinates of an observation point in the liquid, and the Green's function $G(x,y;x_0,y_0)$ is the field radiated by a point source in the waveguide without the cylinder.

The main question which will be answered in this analysis is whether there is a distribution of sources $\mu^0(x_0, y_0)$ that would lead to the scattered field P_s vanishing at large distances from the cylinder in both x-negative and x-positive directions.

ZERO-RADIATION ACOUSTIC SOURCES

Representation of the acoustic sources and the Green's function

In this paper, the acoustic sources existing on the surface of the cylinder are represented as a Fourier series:

$$\mu(\varphi) = \sum_{m=-\infty}^{\infty} \mu_m \exp(im\varphi), \qquad (18)$$
$$\mu_m = \frac{1}{2\pi}\int_0^{2\pi} \mu(\varphi)\exp(im\varphi)d\varphi.$$

The Green's function of the waveguide, $G(x, y; x_0, y_0)$, traditionally is written in the following form:

$$G(x, y; x_0, y_0) = \frac{i}{\pi}\sum_{n=1}^{\infty}\frac{1}{g_n}\sin ny_0 \sin ny \exp\left(ig_n|x-x_0|\right). \qquad (19)$$

System of linear equations for the Fourier coefficients of the sources

Substitution of Eqs. (18) and (19) into Eq. (17) for the scattered field leads to the following link between the Fourier coefficients, μ_m, and the modal coefficients of the transmitted and reflected waves, $B_n^{\pm}$:

$$B_n^{\pm} = \frac{2i}{g_n}e^{\mp ig_n X_0}\sum_{m=-\infty}^{\infty}\mu_m i^m J_m(kR_0)\sin\left[nH \pm m\left(\alpha_n \pm \frac{\pi}{2}\right)\right], \qquad (20)$$

where $J_m(kR_0)$ is the Bessel function of the first kind of order m, and $\alpha_n = \arccos(n/k)$.

In the waveguide, for any given wavenumber k only a finite number of waveguide modes, with the orders n satisfying the condition $n < k$, are propagating to infinite distance from their source. Therefore, to calculate a zero-radiation distribution of acoustic sources on the surface of the cylinder, it is sufficient to require the modal coefficients of only the propagating modes be equal to zero. As a result, the condition of zero radiation in the far field can be written as the following finite system of linear equations for Fourier coefficients, μ_m, of the acoustic source distribution:

$$\sum_{m=-\infty}^{\infty}\mu_m i^m J_m(kR_0)\sin\left[nH \pm m\left(\alpha_n \pm \frac{\pi}{2}\right)\right] = 0, \qquad (21)$$

where $n = 1, 2, ..., N$, and N is the integer part of the wavenumber k. The total number of equations in the above system is $2N$.

The system of equations (21) contains a *finite* number of equations, but an *infinite* number of Fourier coefficients in the distribution of acoustic sources, μ_m. Therefore, one can conclude that, for each wavenumber k, there is an *infinite* number of sets of the coefficients μ_m that will lead to the disappearance of the scattered far field.

Passive scattering of sound by the cylinder

The problem of the scattering of the incident wave by the cylinder under consideration has been solved previously by the author (Zinoviev, 2000). In this solution, a modified form of the waveguide Green's function (Eq. (19)) has been proposed and utilised. By efficiently taking into account all the waveguide modes, this Green's function leads to a quickly converging solution to the scattering problem by means of the integral equation method.

The solution of the passive scattering problem has shown that the Fourier series of the acoustic sources induced on the surface of the cylinder by the incident wave contain only a finite number of significant non-zero terms and can be represented as follows.

$$\mu^0(\varphi) = \sum_{m=-M}^{M}\mu_m^0 \exp(im\varphi), \qquad (22)$$

where M is a positive integer number which depends on the ratios between the wavenumbers of acoustic waves in the fluid, elastic waves in the cylinder, and the diameter of the cylinder. The Fourier coefficients with orders $|m| > M$ can be negelected as they quickly tend to zero with increasing m.

Distribution of acoustic sources required for the cloaking of the scattering cylinder

Eqs (21) and (22) show the possibility to achieve acoustic invisibility of the scattering cylinder by adding external acoustic sources on the surface of the cylinder. The external source distribution, $\mu^e(\varphi)$, needs to contain only a finite number of terms, $N_e \geq 2N$ to ensure that the modal coefficient of both the transmitted and reflected waves vanish. The system of $2N$ linear equations to determine $\mu^e(\varphi)$ can be represented as a system of N pairs of equations as follows:

$$\begin{cases} \sum_{l=1}^{2N}\mu_{m_l}^e i^{m_l} J_{m_l}(kR_0)\sin\left[nH + m_l(\alpha_n + \pi/2)\right] = \\ -\sum_{l=2N+1}^{N_e}\mu_{m_l}^e i^{m_l} J_{m_l}(kR_0)\sin\left[nH + m_l(\alpha_n + \pi/2)\right] \\ -\sum_{m=-M}^{M}\mu_m^0 i^m J_m(kR_0)\sin\left[nH + m(\alpha_n + \pi/2)\right] \\ \sum_{l=1}^{2N}\mu_{m_l}^e i^{m_l} J_{m_l}(kR_0)\sin\left[nH - m_l(\alpha_n - \pi/2)\right] = \\ -\sum_{l=2N+1}^{N_e}\mu_{m_l}^e i^{m_l} J_{m_l}(kR_0)\sin\left[nH - m_l(\alpha_n - \pi/2)\right] \\ -\sum_{m=-M}^{M}\mu_m^0 i^m J_m(kR_0)\sin\left[nH - m(\alpha_n - \pi/2)\right] \end{cases} \qquad (23)$$

where n=1,2,…2N.

In Eq. (23), the left-hand side contains 2N unknown Fourier coefficients of the external source distribution. The first term in the right-hand side represents the part of the external sources which is known. The Fourier coefficients in this term can be assigned specific values to make the acoustic cloaking of the cylinder more efficient or easier to realise. The second term in the right-hand part is determined by the distribution of sources due to the passive scattering.

Eq. (23) shows the following characteristics of the proposed method of achieving acoustic invisibility. First, there are an infinite number of finite sets of Fourier coefficients of the external acoustic sources, which would lead to the invisibility at any given wavenumber. Second, the number of the unknown Fourier coefficients in each of such sets equals twice the number of propagating waveguide modes. Third, the orders of these unknown Fourier coefficients can be chosen arbitrarily. Fourth, a number of the Fourier coefficients of the external sources can be pre-set before calculating the distribution of sources required to achieve the invisibility. Fifth, the distribution of sources due to the passive scattering must be known before calculating the zero-radiation acoustic sources.

It may be noted that, whereas scattering by the cylinder is frequency dependent, the proposed method is valid for any frequency, including the resonance frequencies of the elastic vibrations of the cylinder. At such resonances, the distribution of acoustic sources due to passive scattering will be significantly different from the distributions of sources at frequencies between the resonances. However, this will lead only to a change in the right-hand part of Eq. (23), and, therefore, in the *shape* of its solution, but will not affect the *existence* of its solution.

In further analysis, the proposed method will be applied to the waveguide under consideration and the achieved acoustic invisibility will be demonstrated by graphical representation of the instantaneous acoustic pressure and energy streamlines.

NOTION OF ACOUSTIC ENERGY STREAMLINES

Definition

The energy streamlines, $y(x)$, are defined as the lines to which the acoustic power density vector is tangential at any point. They can be defined by the following equation:

$$\frac{dy}{dx} = \frac{q_y}{q_x} \tag{24}$$

where q_x, q_y are the components of the time-averaged power density vector, $\mathbf{q}$, which is defined by the well-known expression:

$$\mathbf{q}(x, y) = \frac{1}{2}\mathrm{Re}\left[P(x, y)\mathbf{V}^*(x, y)\right] \tag{25}$$

Here $P(x,y)$ is the amplitude of the total pressure in the liquid, and $\mathbf{V}^*(x,y)$ is the complex conjugate of the amplitude of the fluid velocity vector.

Method of calculations

Acoustic energy streamlines have been used before to investigate acoustic scattering (Skelton & Waterhouse 1986, Hickling, Burrows, Ball & Petrovic 1991). The present author calculated the energy streamline for passive scattering in the two-dimensional waveguide (Zinoviev & Belov 1998). In the current paper, the energy streamlines are calculated for both passive scattering and active cloaking in the same way as in the above-mentioned publication. However, for clarity purposes, major steps in calculating the energy streamlines are included here.

The energy streamlines in the waveguide are drawn as equally spaced contours of the acoustic stream function, $\Psi(x,y)$, satisfying the equations

$$\frac{\partial \Psi(x, y)}{\partial x} = -q_y; \quad \frac{\partial \Psi(x, y)}{\partial y} = q_x, \tag{26}$$

which is equivalent to the definition (24). The stream function $\Psi(x,y)$ is calculated as follows:

$$\Psi(x, y) = \int_0^y q_x(x, y_0)dy_0. \tag{27}$$

The x-component, q_x, of the power flow density vector, $\mathbf{q}$, is found as

$$q_x(x, y) = \frac{1}{2}\mathrm{Re}\left[P(x, y)V_x^*(x, y)\right] \tag{28}$$

where $P(x,y)$ is determined by the formulas (1), (2), and (17), and the x-component, $V_x(x,y)$, of the velocity vector, is found as

$$V_x(x, y) = -\frac{i}{\rho\omega}\frac{\partial P(x, y)}{\partial x} \tag{29}$$

The derivative $\partial P(x,y)/\partial x$ is determined by differentiating Eqs. (1), (2), and (17):

$$\frac{\partial P(x, y)}{\partial x} = \frac{\partial P_e(x, y)}{\partial x} + \frac{\partial P_s(x, y)}{\partial x}, \tag{30}$$

$$\frac{\partial P_e(x.y)}{\partial x} = i\sum_{n=1}^{\infty} g_n A_n \sin(ny)\exp(ig_n x), \tag{31}$$

$$\frac{\partial P_s(x, y)}{\partial x} = \int_L \frac{\partial G(x, y; x_0, y_0)}{\partial x}\mu^0(x_0, y_0)dl_0. \tag{32}$$

In the numerical experiments described below, the Green's function in Eqs. (17) and (32) has been used in its modified form derived in Zinoviev (2000) and related publications.

NUMERICAL EXPERIMENTS

Parameters

The following parameters have been used in the numerical experiments described in this paper:
Parameters of the fluid: $\rho = 1000$ kg/m^3, $c = 1493$ m/s.
Parameters of the cylinder: $R_0 = D/20$; $H = D/2$ $\rho_c = 7700$ kg/m^3, $\lambda = 1.11*10^{10}$ Pa, $\mu = 8*10^{10}$ Pa.
Parameters of the incident wave: $k = 4.6$, $A_1 = 1$, $A_n = 0$, $n > 1$.

Results

The results of the numerical experiments are presented in Figure 2. Figures 2a and 2b show the absolute value of the instantaneous acoustic pressure. Instantaneous minima and maxima of the pressure are not distinguishable in these figures and both are shown in dark grey colour, whereas white colour corresponds to zero pressure fluctuations from the value at equilibrium. Figures 2c and 2d show the energy streamlines calculated as described above. The incident wave is coming from the left. The scattering cylinder is shown as the circle in the centre of the figures. For clarity, x- and y-axes are normalised on the waveguide depth, D.

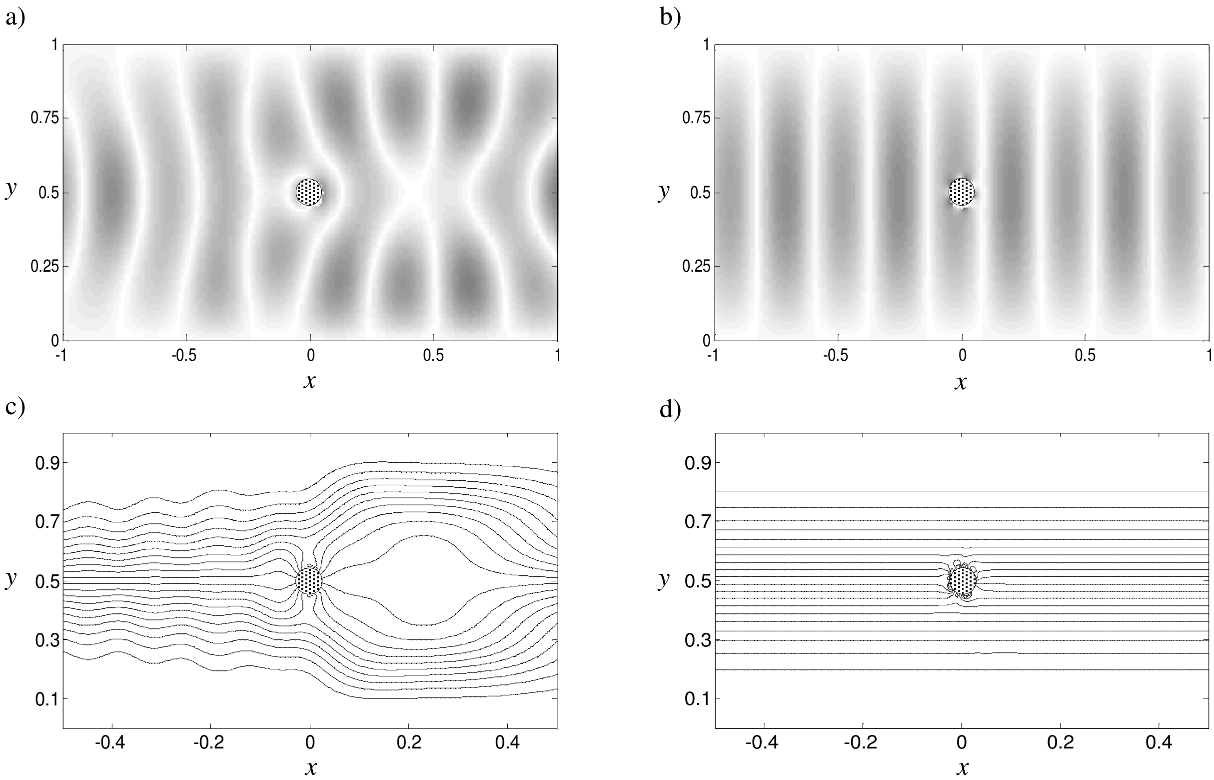

Figure 2. Structure of the acoustic field in the waveguide. The incident wave is coming from the left. $k = 4.6$; a), b) – instantaneous absolute value of the acoustic pressure field; c), d) – energy streamlines; a), c) – passive scattering; b), d) – active cloaking. The coordinates are normalised by the waveguide depth.

Figures 2a and 2c show the results for the passive scattering of the incident wave by the cylinder. These results were obtained using the method developed by the present author (Zinoviev 2000).

Figures 2b and 2d correspond to the cloaking of the cylinder due to additional acoustical sources placed on its surface. The configuration of these sources is calculated using the method described above in this paper. Due to the value of the non-dimensional wavenumber ($k = 4.6$) the number of the propagating modes $N = 4$. Therefore, it is necessary to determine 8 Fourier coefficients in the external acoustic source distribution.

The following configuration of the external sources has been chosen out of infinite number of possible zero-radiation configurations. It is assumed that other coefficients in the external source distribution are such that the series in the right-hand part in Eq. (23) vanish with the exception of one term, which value is arbitrary, as it affects only the *amplitude* of the generated acoustic field, but not the existence of cloaking itself. In the calculations, the value of this term is chosen in such a way that the amplitude of the acoustic field produced by the external sources is comparable with the amplitude of the incident wave.

Discussion

The main conclusion from the analysis of Figure 2 is that the acoustic cloaking by active means is achievable. On the one hand, Figures 2a and 2c demonstrate that the scattering of the incident wave by the cylinder is significant. These figures clearly show the area of shadow behind the cylinder, as well as the distortion of the incident wave in front of the cylinder due to the reflected wave of significant amplitude. On the other hand, Figures 2b and 2d correspond to the case where the additional acoustic sources of the required configuration are placed on the surface of the cylinder. These figures show that the acoustic field generated by the sources on the cylinder exists only near the cylinder, whereas the acoustic far field is not changed as compared with the incident wave. Therefore, it can be concluded that the cylinder is not acoustically visible in far field in both backward and forward directions.

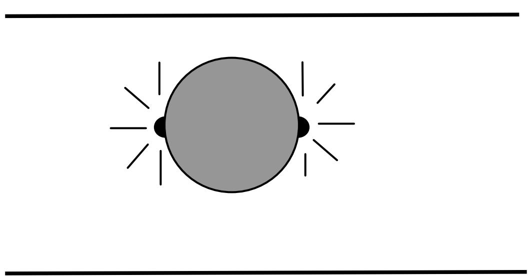

Figure 3. Suppressing the lowest propagating mode by two monopole sources on the opposite sides of the cylinder.

The external acoustic sources that lead to acoustic invisibility of the cylinder have an important property. In an inviscid fluid, they do not consume any power at all. In Figures 2c and 2d, the streamlines are calculated in such a way that they correspond to equal parts of the total energy flow. As the numbers of the energy streamlines at the left and right edges of the figures are equal, one can conclude that the acoustic sources on the cylinder do not radiate any power. Instead, the acoustic energy travels back and forth between the cylinder and the surrounding fluid during different intervals within the period of the acoustic wave.

The distribution of the external acoustic sources on the cylinder, which is calculated in the numerical experiments, is such that only 8 Fourier coefficients are non-zero. These coefficients have the orders m = -4,-3,-2,-1,1,2,3,4. It may be noted that multiplying all these coefficients by the same number will result only in changing the amplitude of the generated near field. At the same time, after the multiplication the modal coefficients of propagating modes will remain zero and, therefore, the cylinder will remain invisible at sufficiently large distances.

The numerical experiments also show that it is easier to achieve acoustic invisibility at low frequencies. The higher the frequency is, the more waveguide modes can propagate, and the more Fourier coefficients in the distribution of the external sources need to be determined.

ACTIVE ACOUSTIC CLOAKING IN OTHER SITUATIONS

Viscous fluids

In the analysis above, the fluid in the waveguide is considered to be inviscid. However, many real fluids can be considered as slightly viscous, so that their viscosity affects only the propagation of sound, not its generation. In such a fluid, the acoustic cloaking can be achieved even easier than in an inviscid fluid. As higher waveguide modes decay quicker with increasing distance from the sources, at large distances only the lowest waveguide mode will be detectable. Therefore, it appears to be sufficient to suppress only the lowest mode in the transmitted and reflected waves that, as shown above, requires the presence of only two Fourier coefficients in the distribution of the external sources.

The orders of these two coefficients, m_1 and m_2, in general can be arbitrary, but it is convenient to consider them to be $m_{1,2} = \pm 1$. In this case, the external sources contain only the dipole component, and this distribution can be achieved with some degree of approximation by placing two monopole sources on opposite sides of the cylinder, as shown in Figure 3.

The source distribution produced by the two monopole sources will be only approximately dipole, as the radiated sound will induce multipole sources on the surface of the cylinder. However, the two sources may be sufficient to significantly reduce the amplitude of the lowest propagating mode, if not to cancel it entirely. The definite answer to this question requires further investigation.

Free space

In free space without boundaries, there is no separation of the acoustic field into propagating and evanescent modes. At the same time, the acoustic field can be represented as a series of multipoles. It is known that, the lower the multipole order is, the more efficient the radiation becomes. Therefore, it may be possible to achieve partial, although significant, suppression of the acoustic radiation in the far field by a small number of monopole sources placed on the surface of the scattering body in a way similar to that described above for viscous fluids. The target for cancelling may be, for example, the monopole and dipole radiation components.

Vibrating object

Active acoustic cloaking can be also used to cancel the radiation of a vibrating object. The acoustic invisibility of this object can be achieved in a way analogous to the way described above by placing external acoustic sources on the surface of the object. The acoustic sources induced on the surface of the object by its internal vibrations need to be known to be able to calculate the correct amplitudes and phases of the external sources. As in the scattering problem described above, only lowest modes need to be cancelled to make the object acoustically invisible.

CONCLUSIONS

In this paper, a problem of making an elastic object acoustically invisible is considered. By the example of an elastic cylinder in a two-dimensional fluid layer, it is shown that acoustic invisibility is achievable by placing additional acoustic sources on the surface of the cylinder.

As only the amplitudes of a finite number of the waveguide modes need to be equal to zero to achieve acoustic invisibility, there are an infinite variety of possible configurations of the active acoustic sources, which will lead to such invisibility.

The acoustical invisibility is demonstrated by graphical representation of the instantaneous amplitude of the acoustic pressure and energy streamlines. The figures are shown for both passive scattering and active acoustic cloaking. In the latter case, the structure of the acoustic field far from the cylinder is unchanged as compared with the incident wave, thus proving acoustic invisibility. The figures also prove that the external acoustic sources do not consume any power if viscous losses are neglected.

The general possibility of the use of this method for achieving acoustic invisibility is considered in some other situations that differ from the problem under consideration in this paper. It is argued that, in a viscous fluid, acoustic invisibility is easier to achieve, as normally only the lowest waveguide mode is present at large distances of the source. Therefore, to achieve the invisibility at large distances, it is sufficient to consider only two Fourier coefficients in the distribution of the external source, and such a distribution can be realised approximately by placing two monopole sources on the surface of the cylinder. The same method can be used also for the acoustic cloaking of a vibrating elastic object.

Also, some considerations are given to the possibility of the use of a similar method for the acoustic cloaking of an object in a free space.

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