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A Tool to Estimate the Sensitivity of Underwater Acoustic Transmission Loss to Environmental Uncertainty

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ABSTRACT

Inputs to mathematical models can be very uncertain because of incomplete knowledge. These input uncertainties propagate to output uncertainties, and an estimation of these output uncertainties, and of the importance of the individual inputs to these output uncertainties, would be desirable. These estimates can be calculated via Uncertainty Analysis (UA) and Sensitivity Analysis (SA) respectively. A software library, Algorithms for Stochastic Sensitivity Analysis (ASSA) has recently been written (Jansen 2005) to carry out these analyses. ASSA estimates the total variance (*VTOT*) for uncertainty, and for each input parameter, the "top marginal variance" (*TMV*) for its sensitivity - the reduction in variance that would occur if the input were perfectly known. We adapt the ASSA methodology to underwater acoustic propagation. Our tool uses efficient uncertainty models for sound speed profile (*SSP*) and bottom loss (*BL*) to compute an ensemble of *SSP*s and *BL* curves. A propagation model is then used to compute an ensemble of transmission losses (*TLs*) from which *VTOT* and the *TMV*s are used to estimate *TL* uncertainty and sensitivity respectively. Example results for a real ocean environment are presented.

INTRODUCTION

In many fields of science, engineering, sociology and finance, systems in the real world are represented by mathematical models. The inputs to these models can be very uncertain because of incomplete knowledge or spatial or temporal variability. As a result, there are corresponding uncertainties in the model output. Much work has been done recently (e.g. Hanson & Hermez 2005), in the study of output uncertainties. These studies yield the answer to two questions:

(i) what is the uncertainty in the output arising from the uncertainties in the inputs

and

(ii) how sensitive is the output to uncertainty in each individual input.

The answer to the first question is provided by *uncertainty analysis* (UA), and to the second question by *sensitivity analysis* (SA).

The main steps of UA and SA are

(1) generate a Monte-Carlo ensemble of uncertain inputs according to a specified multivariate distribution,

(2) propagate the ensemble of inputs through the mathematical model to yield an ensemble of outputs,

(3) estimate the uncertainty in the output from its ensemble and

(4) estimate the sensitivity of the output to uncertainty in each individual input.

A C-language software library, *Algorithms for Stochastic Sensitivity Analysis* (ASSA) (Jansen 2005) has recently been written to facilitate the implementation of these steps. We give more details of the algorithms of the ASSA library below.

An important factor in the prediction of sonar performance is the *transmission loss* (*TL*), a logarithmic measure of the reduction in sound intensity from the (underwater) transmitter to the receiver. *TL* in the underwater channel depends on channel parameters such as sound speed profile and bottom properties. These parameters can be very uncertain because of incomplete measurements and/or variability.

In this paper, we describe a tool which can be used to estimate *TL* uncertainty and sensitivity. It adapts the methodology of ASSA to underwater acoustic propagation. It uses a modified version of Dosso *et al*'s (2007) uncertainty model of SSP and bottom parameters. Dosso represents the SSP uncertainty by a single parameter. The bottom parameters can also be described by a single parameter: the slope of the bottom loss versus gradient curve (Jones et al. 2008). Following ASSA, the tool generates a Monte Carlo ensemble of environmental inputs, and uses a propagation model to yield an ensemble of output *TL*s. Because there are only two uncertain parameters, the number of *TL* evaluations – the main contribution to the computational load – is greatly reduced. The tool then estimates *TL* uncertainty and sensitivities using measures implemented in ASSA.

This paper first describes the algorithms provided in ASSA to perform the four steps of UA and SA. The adaptation of this methodology to develop an efficient tool for the *TL* problem is then discussed – how the ensemble of *SSP*s and bottom parameters is generated, how the ensemble of output *TL*s is computed, and how the *TL* uncertainty and sensitivities are estimated. The application of the tool to an example case is described, followed finally by some conclusions and proposals for future work.

THE ASSA LIBRARY

The ASSA library's algorithms fall into four groups corresponding to the four main steps of UA and SA –

- input generators;
- input-output models;
- uncertainty analysis; and
- sensitivity analysis.

Input Generators

The task of an input generator is to provide a set of multidimensional samples (one dimension per input) whose distributions and correlations are as specified by the user. ASSA provides two basic generators – uniform and multinormal. For other distributions, ASSA carries out two steps. Here, m is the number of inputs.

- Draw a set of m-dimensional samples uniformly from the m-dimensional hypercube, with the desired correlation matrix (which could be diagonal, i.e. uncorrelated inputs).
- Transform these samples to variables with the desired distribution – available distributions are uniform, triangular, normal, log-normal, beta and gamma.

Input-Output Models

Only two very simple models, suitable for software development, are available (Jansen 2005). The user is expected to integrate their own model into their analyses, which includes computation of the output ensemble, say y, from the input ensemble.

Uncertainty Analysis

Variance based methods are used for both uncertainty and sensitivity analysis. ASSA uses the total output variance due to all input factors

$$VTOT = \mathrm{Var}(y) \,.$$

Sensitivity Analysis

ASSA uses two measures of sensitivity to input parameters, the *top marginal variance (TMV)* and the *bottom marginal variance* (Jansen et al. 1994). The former is the most useful, and we review that here. The *TMV* due to a group of inputs S is the variance reduction that would occur if one knew S perfectly. It can be shown that the TMV due to S, denoted by TMV_S, is given by

$$TMV_S = \mathrm{Var}[\mathrm{E}(y|S)] \qquad (1)$$

where y is the model output (in our case, *TL*) and E denotes expectation. ASSA's measure of sensitivity is actually the ratio $TMV_S\,/\,VTOT$, the *relative TMV*.

ASSA provides two methods of computing TMV and BMV, a *regression-based* method, which fits a linear model to the output data, and a *regression-free* method, which makes no assumptions about the form of the model. We consider only the latter in this paper.

Computation of VTOT and TMV_S/VTOT

The direct evaluation of equation (1) is computationally expensive, since it requires the evaluation of an ensemble of expectations, one for each sample of S. ASSA uses Saltelli *et al*'s (2000) *resampling-based* method, which estimates TMV_S more efficiently. They use the following procedure:

Algorithm 1 (Jansen 2005)

1. Construct *two* independent ensembles of inputs X_1 and X_2. X_1 and X_2 can be represented as matrices with N rows and m columns, where N is the number of samples in each ensemble and m is the number of inputs.
2. Let the column indices of the inputs in S be $\{i_1, \ldots i_p\}$. Note that $p \le m$. Set columns $i_1, \ldots i_p$ of X_2 equal to columns $i_1, \ldots i_p$ respectively of X_1.
3. Using the chosen model, compute two output ensembles y_1 and y_2 from X_1 and X_2 respectively.
4. Compute the two sample variances v_1 and v_2 from y_1 and y_2 respectively. Estimate $VTOT = \mathrm{Var}(y)$ by the geometric mean of v_1 and v_2, that is $VTOT = \sqrt{v_1 v_2}$.
5. Let corr(y_1, y_2) be the correlation coefficient of y_1 and y_2, defined as
$$\mathrm{corr}(y_1, y_2) = \frac{\sum_j (y_1(j)-\bar{y}_1)(y_2(j)-\bar{y}_2)}{\sqrt{\sum_j (y_1(j)-\bar{y}_1)^2 (y_2(j)-\bar{y}_2)^2}}$$
where $\bar{y}_1$ and $\bar{y}_2$ are the means of y_1 and y_2 respectively. It can be shown (Saltelli et al. 2000) that corr(y_1, y_2) is an estimate of the relative *TMV*:
$$TMV_S/VTOT \cong \mathrm{corr}(y_1, y_2) \,. \qquad (2)$$

It is intuitively clear that if S is a very sensitive input group, y_1 and y_2 will be very much the same, and their correlation, and hence their relative *TMV*, estimated by equation (2), will be close to unity.

COMPUTATION OF TRANSMISSION LOSS (*TL*)

Transmission Loss (*TL*) is an important factor in sonar performance prediction. It is a measure of the reduction in sound intensity level (measured in decibels re 1 micropascal) from the (underwater) transmitter to the receiver. One measure of sonar performance is the range at which a sonar attains a given probability of detection (e.g. 0.5) for a specified probability of false alarm (e.g. 10^{-4}).

Let p_1 and p be the rms sound pressures at 1m from the transmitter and at the receiver respectively. Then *TL* at the receiver range is

$$TL = -20 \log_{10}(p\,/p_1) \,. \qquad (3)$$

There are a variety of methods to calculate *TL*, or equivalently, rms pressure at the receiver for unit rms pressure 1m from the transmitter. Two well-known classes of methods are *ray methods*, where rays are traced from the source to the receiver and added, and *modal methods*, where the sonar field in the acoustic waveguide is modelled as a set of modes propagating outward from the transmitter. A ray model is used in the sensitivity tool described here, but other models are easily incorporated.

GENERATION OF ENSEMBLE OF INPUTS

Transmission loss depends on the sound speed profile (*SSP*) and the bottom properties, and these parameters can be uncertain because of incomplete measurements and/or temporal or spatial variability.

SSP ensemble

For the ensemble of *SSP*s, we follow Dosso et al's (2007) uncertainty model, which represents the variability near the surface due to heating, cooling and wind mixing. The perturbations of the sound speeds are assumed to have Gaussian distributions, with standard deviations decaying exponentially with depth. Mathematically, this can be expressed as

$$\Delta c_z = \Delta c_0 \exp(-\gamma z) \quad (4)$$

where z is the depth, Δc_z is the perturbation at depth z, Δc_0 is the perturbation at the surface, and γ is the rate of exponential decay.

For a value of $\gamma = 0.15$, the perturbation at depth 30m is about 1% of that at the surface. Below 30m, the perturbations are assumed to be zero. This variability is shown schematically by the dashed curves in Figure 1. To summarize, the *SSP* ensemble is generated as follows:

Algorithm 2 (Dosso et el. 2007)
1. Generate a Gaussian ensemble of surface perturbations Δc_0
2. Generate an ensemble of *SSP* perturbations using (4).
3. For each sample of the ensemble, the *SSP* is given by

$$c_z = \tilde{c}_z + \Delta c_z \quad (5)$$

where $\tilde{c}_z$ is the unperturbed *SSP*.

Bottom parameter ensemble

Regarding the bottom parameters, it is well-known that the bottom can be characterized by the complex bottom sound pressure reflection coefficient (*BRC*) versus grazing angle curve. Urick (1969) and others have shown that for small grazing angles, typical of shallow-water transmission, the bottom loss in decibels (BL=-20 log10|BRC|) may be approximated as being proportional to the grazing angle β, that is

$$BL \cong F\beta \quad (6)$$

where F is the "bottom loss slope" in decibels per radian. Jones et al. (2008) have shown that for small β, the reflection phase angle (φ = arg(BRC)) can be approximated by a straight line from $-\pi$ at $\beta = 0$ to 0 at β_{6dB}, the angle at which BL = 6 dB (given by $\beta_{6dB} = 6/F$), and zero thereafter. This can be expressed by

$$\varphi \cong -\pi + \pi F\beta/6, \quad \beta \le 6/F, \quad (7a)$$
$$\cong 0 \quad \text{otherwise.} \quad (7b)$$

Hence the single parameter F specifies BL and φ for small β.

Jones et al. (2008) have further shown that for many bottom types, the *TL* computed from using (6) and (7) for *BL* and φ *for all grazing angles* is close to that computed using the exact curves for *BL* and φ. So for these bottom types, the parameter F specifies the bottom well enough to give good *TL* estimates.

If the distribution of F can be estimated and an ensemble of Fs generated in accordance with this distribution, equations (6) and (7) will provide an adequate Monte-Carlo sample of perturbed *BRC*s. In this work, the distribution of F is estimated by using Dosso et al's (2007) uncertainty model for 11 geoacoustic parameters such as compressional sound speeds, layer thicknesses, etc. The uncertainties in these parameters are modelled in (Dosso et al. 2007) by Gaussian distributions, with unrealistic samples (such as negative thicknesses) being excluded. These parameters are illustrated in Figure 1.

In this work, we generate a *very* large set of samples of these parameters (varying all parameters), assuming that the perturbations were independent. For each sample of the 11 parameters, the *BL* versus β curve is calculated using a multilayer reflection coefficient formula (Bartel, D.W., private communication 2007). The bottom loss slope F is then estimated by fitting a straight line to curve at small β, specifically the interval from 0° to 10°.

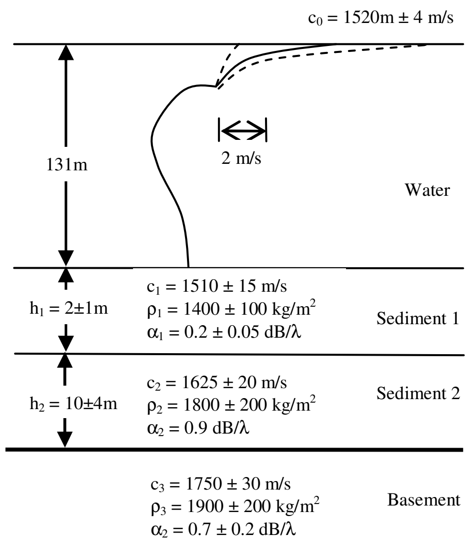

Figure 1. Sound speed profile, bottom parameters and associated uncertainties used by Dosso et al. 2007. Here, c, ρ, α and h denote compressional sound speed, density, attenuation coefficient and layer thickness respectively.

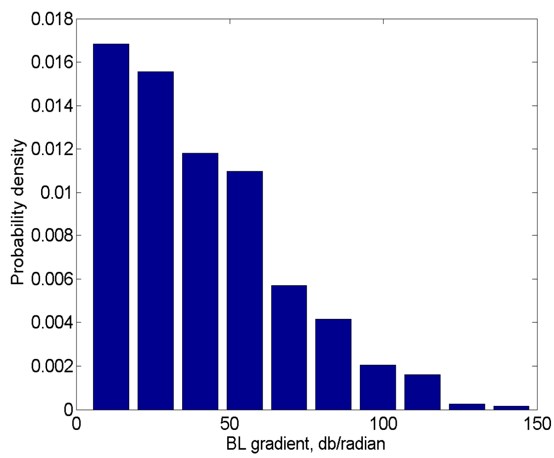

Figure 2. Probability density function of bottom loss gradients

Figure 2 shows the probability density function (pdf) of the Fs which are computed using the above procedure. Note that this pdf is highly skewed towards the lower bottom loss slopes.

Having estimated the distribution of the Fs, we can use this and the approximation of Jones et al. (2008) to generate the ensemble of BL/φ curves required for the uncertainty and sensitivity analyses. We observe that an ensemble size of 1000 is sufficient for our analyses – increasing it further does not change the sensitivity and uncertainty estimates significantly.

Generation of BL/phase angle(φ) ensemble – summary

Algorithm 3

1. Generate an ensemble of bottom loss slopes F with pdf as shown in Figure 2. Assume that the random variables F and Δc_0 (surface speed perturbation) are independent.
2. Compute the ensemble of BL/φ curves using (6) and (7).

We reiterate that only a two-parameter (F and Δc_0) ensemble of inputs is required, as opposed to a 12-parameter ensemble which would be required if the surface speed and all the 11 bottom parameters were varied independently. This greatly reduces the number of output evaluations (discussed below) and hence the overall computational load.

COMPUTATION OF ENSEMBLE OF OUTPUTS

As mentioned above, TL can be computed using a variety of models. Porter's (2008) *Acoustics Toolbox* contains a number of codes, including BELLHOP (ray model) and KRAKENC (modal model). Which code is best depends on a number of factors such as frequency, environment, speed of execution, etc. We choose BELLHOP because of its speed of execution and the fact that it gives accurate results at the chosen frequency of 1200 Hz.

BELLHOP does not compute TL directly, but a complex pressure $p_c=(p_r, p_i)$ whose magnitude and phase are given by

$$|p| = \sqrt{p_r^2 + p_i^2} \qquad (8)$$

and $\arg(p_c)$ respectively. Here arg is the complex argument. The pressure is normalized so the pressure magnitude 1m from the source is unity. Hence from (3) TL is given by

$$TL = -20 \log_{10}|p| \qquad (9)$$

For each pair of sample curves of (i) SSP and (ii) BL/φ, BELLHOP is run to yield p_c , then TL is computed using (8) and (9). Suppose there are N samples of each of the SSP and BL/φ curves. The steps in computing an output ensemble are as follows:

Algorithm 4

For *i_sam* = 1 **to** N {*i_sam* is sample count}
Write *i_sam*th BL/φ curve to a BELLHOP *BRC* file.
Launch BELLHOP. (The output p_c is written to a BELLHOP "shade" file).
Read p_c .
Compute TL using (8) and (9).
TL_ens(*i_sam*) = TL {*i_sam*th sample of TL ensemble}
end

UNCERTAINTY & SENSITIVITY TL ANALYSES

Following ASSA, we estimate $VTOT$ as the measure of TL uncertainty, and $TMV_{SSP}/VTOT$ and $TMV_{BL}/VTOT$ as the measures of TL sensitivity to uncertainties in SSP and BL respectively. These are estimated using Algorithm 1. Some details of the application of Algorithm 1 to TL are as follows:

Step 1. X_1 and X_2 are independent ensembles of the pairs $[\Delta c_0, F]$

Step 2. The set S is either Δc_0 or F . For sensitivity to Δc_0 , set the Δc_0's in X_2 equal to those in X_1. For sensitivity to F , set the F 's in X_2 equal to those in X_1.

Step 3. For each pair $[\Delta c_0, F]$ in the independent ensembles, compute the SSP using (4) and (5), and the BL curve using (6) and (7). This generates two ensembles of SSPs and BLs. Generate corresponding ensembles of TLs, say *TLens1* and *TLens2*, using BELLHOP. These correspond to y_1 and y_2 in Algorithm 1.

Step 4. Compute $v_1 = \mathrm{Var}[TLens1]$, $v_2 = \mathrm{Var}[TLens2]$ and $VTOT = \sqrt{v_1 v_2}$.

Step 5. If $S = \Delta c_0$ (in Step 2) , then $TMV_{SSP}/VTOT = \mathrm{corr}(y_1, y_2)$, otherwise $S = F$ and $TMV_{BL}/VTOT = \mathrm{corr}(y_1, y_2)$

EXAMPLE UNCERTAINTY/SENSITIVITY ANALYSES

We present analyses of the uncertainty and sensitivity of *incoherent TL* for the environment and associated parameter uncertainties for the Malta Plateau of the Mediterranean Sea (Dosso et al. 2007) (see Figure 1). The environment contains a 131m water layer over a seabed with two sediment layers and a semi-infinite basement. The SSP in the water column is strongly downward refracting in the top 50m and has a weak sound channel near mid-water depth. The standard deviation of the surface sound speed is 4m/s. The unperturbed values and standard deviations of the 11 geoacoustic parameters are as in Figure 1. As indicated above, these yield the probability density function in the bottom loss slope, F, that was illustrated in Figure 2.

The source and receiver depths are both 15m, and uncertainty and sensitivity analyses (computation of $VTOT$ and relative TMVs) are performed at ranges from 500m to 10km in steps of 500m.

The results are shown in Figures 3 and 4. The solid curve pf Figure 3 shows the mean ensemble incoherent TL versus range. Note the peaks at 4km and 8km. The two dashed curves in Figure 3 show plots of mean $TL \pm \sqrt{VTOT}$, i.e. mean TL ± one standard deviation. Recall that TL is expressed in decibels, so also is its standard deviation. These curves give an indication of the "spread" of TL, i.e. its uncertainty due to uncertainty in both SSP and BL. The standard deviations vary between 4 dB and 8.5 dB.

The solid and dashed curves in Figure 4 show the relative TMVs due to SSP and BL uncertainty respectively – we denote these by $TMV(SSP)$ and $TMV(BL)$. They are plotted as percentages, i.e. 100•TMV/VTOT. Note the peaks in $TMV(SSP)$ and troughs in TMV(BL) at 500m, 4km and 8km. Note also the troughs in $TMV(SSP)$ and peaks in $TMV(BL)$ at 2km and 6.5km.

Discussion of TL and TMV results

Figure 5 shows a range-depth plot of unperturbed TL. A slice through Figure 5 at depth 15m shows peaks at 4km and 8km, and these correspond to the peaks in the TL curves in Figure 3.

Figure 5 shows that TL changes rapidly with position at depth 15m and ranges 500m, 4km and 8km. So at these locations, the *TL* would be expected to be sensitive to changes in *SSP* which have the effect of "shifting" the *TL* pattern as illustrated in Figure 5.

Similarly, from Figure 5, *TL* at ranges 2km and 6.5km and a depth of 15m changes only slowly with position, hence *TL* would be expected to be relatively insensitive to *SSP* changes. Conversely *TL* would be expected to be relatively more sensitive to *BL* changes at these ranges.

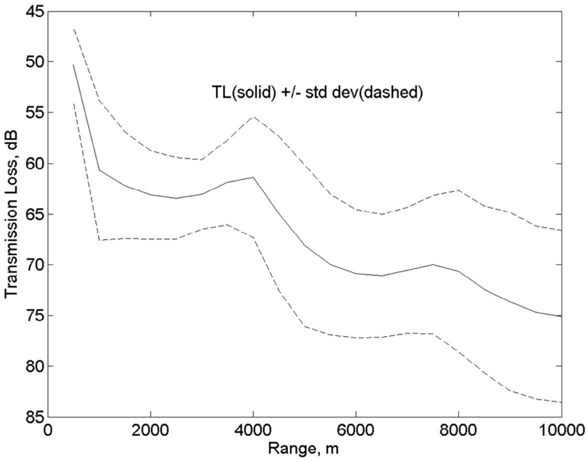

Figure 3. Transmission Loss ± one standard deviation

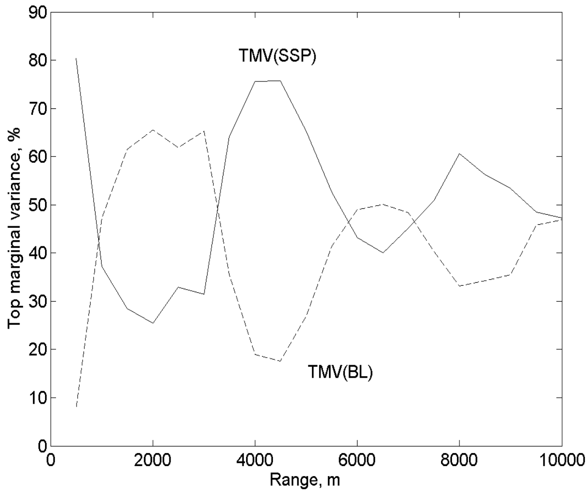

Figure 4. Top marginal variances for Dosso environment due to (i) sound speed profile uncertainty and (ii) bottom loss uncertainty

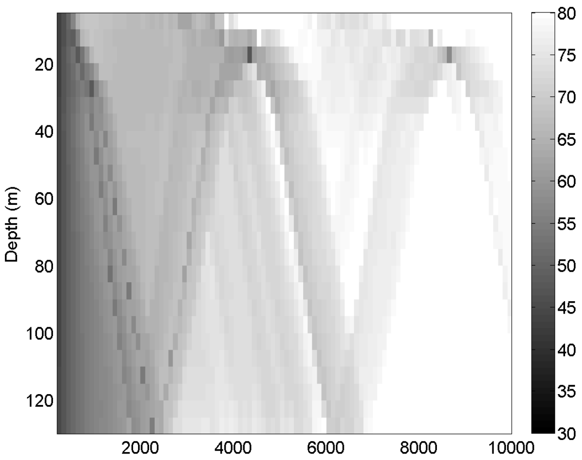

Figure 5. Unperturbed *TL* versus range and depth field for Dosso environment

CONCLUSIONS

A tool has been presented which yields uncertainty and sensitivity measures in *TL*. These are respectively, the total variance and the top marginal variances due to the input factors, following the ASSA methodology. The variability in the bottom parameters has been represented as variability in a single parameter, the bottom loss slope, following Jones et al's (2008) single parameter bottom representation. The variability in the SSP has also been represented by a single parameter, following (Dosso et al. 2007). The reduction in the uncertainty space to two dimensions greatly decreases the number of model evaluations required and hence speeds up the analyses.

The tool has been applied to the Dosso et al. (2007) environment, and the uncertainty and sensitivity results discussed. Further work is required in analysing uncertainty and sensitivity in a large number of representative environments. Further work will be carried out in generalising the tool to implement

(i) other uncertainty models,

(ii) other propagation models,

(iii) other uncertainty/sensitivity measures and

(iv) other output parameters, such detection range for a given probability of false alarm.

Another avenue of exploration is to ascertain whether bottom loss gradient distributions can be described by a small number of generic distributions, according to bottom type, but still produce realistic estimates of *TL* uncertainty and sensitivity.

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