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How should acoustics adapt to meet future demands?

New Approaches for Understanding the Mechanisms of Brake Squeal

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ABSTRACT

Brake squeal has become an increasing concern for the automotive industry because of associated warranty costs and the requirement for the continued reduction of interior vehicle noise. Low and high frequency noises in car brakes, often referred to as brake squeal, are known to be a result of nonlinearity, unstable behaviour and bifurcations leading to limit cycle behaviour. By using the data from a separate experimental study designed to determine the influence of the geometric parameters of brake pads (such as the number and location of slots) on brake squeal noise, we examine two new approaches for providing improved understanding of the brake squeal phenomenon: statistical and nonlinear dynamics analyses. Results of the statistical analysis indicate that the performances of certain pad designs correlate with their levels of nonlinearity. The nonlinear time series analysis reveals that, in the experimental data, not only are limit cycle behaviours present but also a route to chaotic solutions can be observed.

INTRODUCTION

The development of a brake system involves optimising many aspects of its performance, including its propensity for brake noise, in order to meet customers' specifications. Brake noise is often perceived by most customers as an indication of a poorly functioning product. In addition, brake noise is an annoyance for both the occupants of the vehicle and those in close proximity to it; it can also become an environmental issue. Further, excessive wear, due to high vibrations and micro cracks caused by travelling waves in the lining material, are an issue for maintenance costs and, more importantly, for security in daily traffic. Therefore, it is understandable that a lot of interest exists in, and money is spent on, resolving these issues. Akay (2002) states that up to one billion US dollars per annum were spent in North America on Noise, Vibration and Harshness (NVH) in the early 2000s and, according to Abendroth and Wernitz (2000), as cited in Kinkaid et al (2003), up to 50% of the engineering budgets of manufacturers of brake pad materials have been spent on NVH problems. In a J.D. Power survey published in 2002, 60% of warranty claims concerning the brake corner were due to brake squeal (Yang el al 1997). In Vadari et al (2001) indicated that a single Los Angeles city traffic test (a special test procedure in which the brake is applied very often in a short time) takes up to 6 weeks of development time, including 300-500 man hours.

Since the early 20th century, researchers have been investigating problems related to brake noise (Akay 2002). However, although other areas of vehicle NVH have produced significant improvements, the friction levels of brake materials have been increasing due to higher vehicle performance and increased total weight. Further, the move away from asbestos has increased the difficulty of producing brake systems with adequate NVH performances.

There have been a number of reviews of brake squeal: (North 1976, Yang and Gibson 1997) and, more recently, Papinniemi et al (2002) or Kinkaid et al (2003). In 1976, North described a number of analytical models, with a limited number of degrees-of-freedom (dof), for analysing the role of geometry and friction in brake squeal. In 1997, Yang and Gibson produced a survey of research into brake squeal conducted since the 1950s which covered the theoretical, experimental and numerical approaches. They noted that the limitations of large multi-dof finite element models were due to the difficulty of modelling the friction interface. Overall they considered experimental methods had been more productive in addressing brake noise concerns.

More recently, Kinkaid et al (2003) provided a comprehensive review of disc brake squeal research, covering a range of topics: its history, theories, experimental studies, determination of parameters (such as contact conditions), analytical and numerical modelling as well as practices undertaken to suppress brake squeal. A monograph by Chen et al. (2006) provides an excellent treatment of all aspects of disc brake squeal.

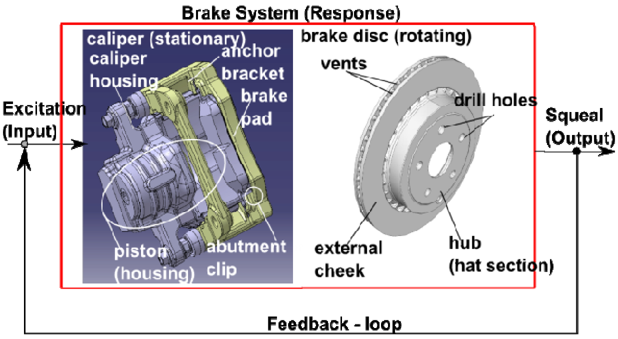


Figure 1: A brake system showing calliper housing, anchor bracket, piston, brake pad and rotor

Figure 1 displays the complete assembly of a modern brake system showing the essential components: calliper housing, anchor bracket, piston, brake pad and rotor. Brake noise can be classified into many categories according to its frequency range (Akay 2002). Disc brake squeal, the subject of this paper, generally refers to brake noise at frequencies greater than 1 kHz. From the analytical perspective, brake squeal can be regarded as a self-excited vibration of the brake assembly, with the friction interface providing the energy source (input) and the flat surfaces of the brake rotor radiating the bulk of the sound (output), as depicted schematically in Figure 1. As is widely acknowledged, the system enters an unstable vibration mode and settles into a limit cycle vibration as the energy provided by the friction interface is balanced by the dissipation caused by sound radiation, friction damping, and other system nonlinearities. Low frequency squeal, generally considered to be between 1 kHz and 5 kHz where the first tangential in-plane rotor mode normally occurs, is influenced by the dynamic behaviour of the brake calliper, its mounting and, often, the steering knuckle and other suspension components. On the other hand, high frequency squeal (greater than 5 kHz) is usually dominated by the dynamic properties of the brake rotor and the brake pads.

The primary nature of brake squeal is transient and often cannot be repeated under apparently similar conditions. Small changes in speed, brake line pressure, temperature, contact conditions, material properties, geometry and/or environmental conditions can produce significantly different results. This has often been attributed to the highly nonlinear character of the contact interface. Unfortunately, the contact pressure distribution of this area is difficult to measure, let alone to model. Therefore, as pointed out by Oberst and Lai (2008a), contemporary approaches in industry are based more on trial and error and experiments which are inclined to focus on the limitation of damage a-posteriori.

Numerical tools and guidelines for accurately predicting squeal propensity in a brake system's configuration are currently not available. Although some of the physical active mechanisms causing these self-excited vibrations are known, they cannot always be generalised. Since the problem is very complex due to geometry, highly nonlinear behaviour in the contact zone and complicated material laws (composites in layer structure) as well as often unknown material formulations, an overall solution is not in sight.

Therefore, the objective of this paper is to explore the potential of non-linear dynamics instead of traditional linear methods in analysing brake squeal phenomena. Two new methods for the analysis of brake squeal are introduced: non-linear statistical analysis and chaos theory. In the statistical analysis (Oberst et al 2008b), joint recurrence plots are used to investigate the nonlinear relationship between the friction coefficient and the peak sound pressure level. In the second study, a nonlinear time series analysis is performed which focuses on finding whether chaotic regimes exist in experimental squeal data (Oberst et al 2008c). Brake squeal data obtained from a full brake system in noise dynamometer tests (Moore et al 2008) are used to illustrate the potential of these methods in understanding mechanisms for brake squeal.

STATISTICAL ANALYSIS

The basis for the statistical analysis was established by an extensive design of experiment (DoE) study at PBR in Melbourne (now BOSCH Chassis Systems Asia Pacific Ltd.). A more detailed discussion of this research and the test-rig set up is given by Moore et al (2008). The effect of different pad configurations on brake squeal noise was analysed by Oberst et al (2008b). Here, only part of the DoE-study is presented to illustrate the potential of statistical analysis.

The data used for this investigation were the time-averaged friction coefficient and the peak sound pressure level of each stop. The experimental test matrix consisted of 1669 stops, separated into a warm section (stop number 1-992) and a cold section (stop number 993-1669). A *stop* is the application of the brake in the noise dynamometer. It should be noted that the brake disc is either slowed down to a certain speed or that a certain speed is maintained (as if driving down-hill). The dynamometer simulates the procedure of braking according to specified environmental conditions. These conditions are varied over the course of the dynamometer test. The altered parameters are, for instance, humidity, temperature, rotational velocity of the disc and brake line pressure. Basically, two different major temperature regimes (sub matrices), warm and cold were considered in which rotor temperature was between 50-300 [ºC] (warm section) and 0-60 [ºC] (cold section). The question for the statistical analysis was whether it is possible to correlate the relative performance of a pad, with the time-averaged friction coefficient. The performance of a pad is measured according to a noise index based on the peak sound pressure level (as described by Moore et al (2008)). Four pad modifications were considered, and were ranked according to their performance (best to worst) in terms of the noise index in the following order (with the symbolic representation of each configuration):

1. VEF39 (single central vertical slot) → I
2. VEF43 (single diagonal slot) → \
3. VEF37 (baseline model, no slot) → o
4. VER41 (double vertical slots) → II

The statistical analysis, as described in Oberst et al (2008b), was performed first using descriptive statistics and examining histogram distributions of peak sound pressure levels and the time-averaged friction coefficient over 1669 stops. Time-averaged friction coefficient and peak sound pressure levels were used for the analysis, instead of an instantaneous friction coefficient and sound pressure levels, in order to reduce the data set to a manageable size that is practical and useful for industry. This is then followed by using novel techniques in nonlinear theory, namely, joint-recurrence plots. Joint recurrence plots work by comparing two or more trajectories in different phase spaces so that systems' solutions and their general behaviour can be analysed, as proposed by Marwan et al (2007), Marwan (2003), Romano (2004), and Thiel (2004).

After determining an appropriate embedding(dimension and delay) and reconstructing a copy of the system's phase space, joint-recurrence plots of the time-averaged friction coefficient and peak sound pressure levels were investigated (Figure 2). The embedding parameters were determined by using the first minimum of the averaged cross-mutual information for the delay. The minimum of the false nearest neighbour algorithm determined the embedding dimension. This is necessary since the measurements consist only of a one dimensional array, whereas the system's phase space can have more components. In the case of a one degree of freedom dry frictional oscillator, these dimensions are the displacement, the velocity and slip. Taken's embedding theorems guarantee that the reconstructed phase space gives an appropriate description of the system's dynamics (Kantz and Schreiber 2004). With this theorem it is possible to build a copy of the solution's phase space from a single measurement.

A joint recurrence plot is a qualitative picture of the system's recurrent states. Recurrence in dynamical systems measures the similarity of the system's states. A ε-environment applied to a specified norm, measures successively the distance from each point in phase space to all other points. Without going into detail, a norm is a mapping function to measure distances (like Euclidean distance) and a ε-environment is a point's neighbourhood, which should go in the limit to zero. This distance function is the argument of an indicator or Heaviside function, which is either 0 (points far away from each other) or 1 (points in the neighbourhood).A 'one' gives a black dot in the recurrence plot, a zero remains white. The plot is usually symmetrical, depending on the norm applied. Here the maximum norm was used, such that the lower diagonal matrix is the mirrored image of the upper diagonal matrix. The line in the middle of Figure 2 is called the line of identity (LOI).

Clearly, if one follows this LOI in Figure 2, two major regimes can be distinguished: The warm section of the test-matrix from 0-992 sample points and the cold section after that (in Figure 2 marked with a dashed square). Comparing the four different joint-recurrence plots with each other revealed that only differences between the single vertically/diagonally and the baseline/double slotted pad could be distinguished. Usually a visual inspection as performed in a joint-recurrence analysis can give valuable hints and could render further analysis steps obsolete. Unfortunately in this case, the differences in performance of the four different pad configurations cannot be ascertained. Since visual inspection only did not provide a complete picture of the underlying nonlinear correlations, a joint-recurrence quantification analysis was undertaken

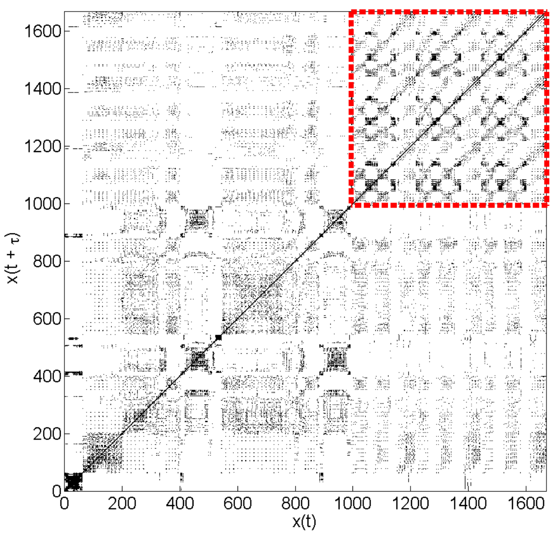


Figure 2: Joint recurrence plots. The cold section of the test matrix is marked with a (red) dashed line in the upper right corner.

In Figure 3, comparisons between a classical linear measure, the *cross-correlation* (xcorr), and the nonlinear measure, the recurrence rate (RR), are shown. The recurrence rate corresponds to the percentage of total recurrence points in the recurrence plot. It is equivalent to the correlation sum (Marwan 2007). The major peaks are captured by both the linear and the nonlinear complexity measures. However, in comparison with the xcorr, the RR exhibits dissimilar behaviour. It clearly shows more distinguishable curves and differences between the in the pad modifications. The baseline configuration peaks in the second half of the cold matrix section in Figure 3 b) whereas, in the first half (apart from the beginning), the double slotted pad and the baseline model dominate.

The nonlinear dependency changes with time. Therefore, it is necessary to know in which part of the test matrix the most severe squealing events take place. In Figure 4, each recorded squeal received an index according to its position in the test matrix and, thereafter, this vector was ranked and formed the x-axis. The y-axis shows the position of the whole test matrix. Only the cold matrix was investigated since it contained more squealing incidents than the warm matrix.

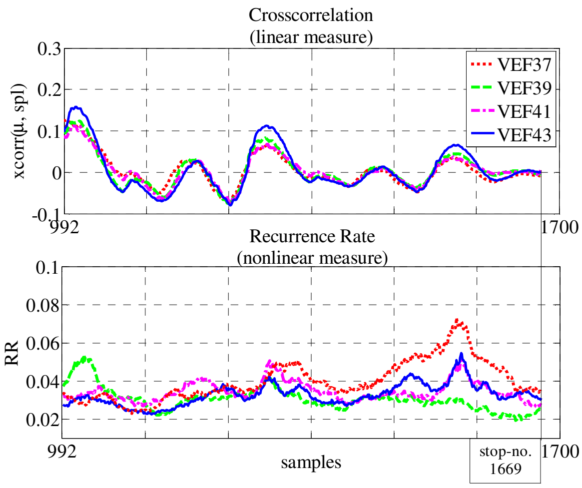


Figure 3: a) Cross-correlation and b) Recurrence Rate for four different pad modifications

For the highest 50 squealing events, the pad configurations VEF37 and VEF41 show that some major squeals are in the test matrix segment [1150-1250]. Together with the knowledge that VEF41 and VEF37 pads perform the worst and, by examining the recurrence rate (RR) in this test matrix segment in Figure 3 b), it can be seen that the nonlinear linkage is relatively high. This is also valid for the best-performing configuration VEF39 (peaking in the beginning of Figure 3 b) and the baseline model (no slot) VEF37 (at very high "stop number" high nonlinear relationship). VEF39 indicates that the high sound pressure level peaks at around stop 1000 and is due to high nonlinearity (see Figure 3 b). The same is true of VEF37 for the test matrix points in the interval [1400-1600].

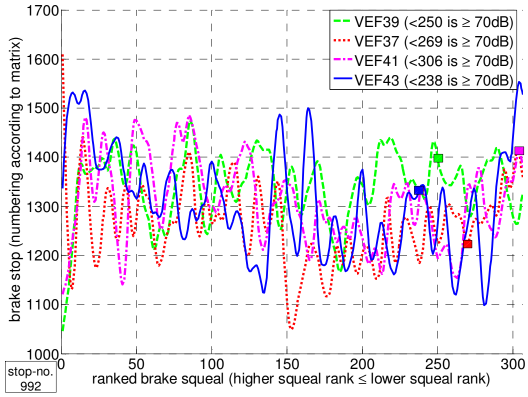


Figure 4: Distribution of squealing events

In Figure 5, several nonlinear complexity measures (namely, recurrence rate, average line length, determinism, laminarity and entropy) were investigated based on joint-recurrence plots, as described by Marwan et al (2007).

The average line length is a measure based on the diagonal line lengths in a recurrence plot. These diagonal lines display similar behaviour and correspond to parallel running trajectories in phase space. The laminarity measures the system states, where not much change is observed. Further, determinism is a measure of predictability and corresponds to the percentage of diagonal lines in the recurrence plot. The entropy is a measure based on the Shannon entropy of the probability distribution of the diagonal line lengths. All these measures were calculated for each brake pad configuration and normalised by the maximum value to highlight the ranking of nonlinearity for the various pad modifications.

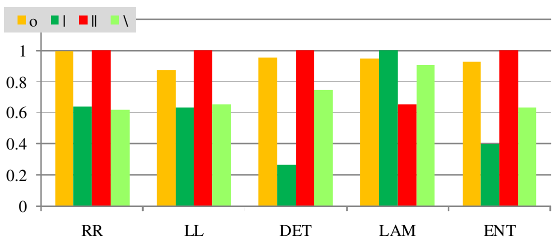


Figure 5: Non-linear complexity measures, from left to right: Recurrence Rate (RR), Average Line Length (LL), Determinism (DET), Laminarity (LAM), and Entropy (ENT).

For the four test pad configurations, the ranking of nonlinearity is from top to bottom for ||→ o→|←→\. The double arrow at the end of this sequence indicates that sometimes the ranks interchange. LAM is a measure of linearity, that is, the lower this value, the lower the linearity. Comparison with the noise performance of the four test pad configurations suggests that the higher the non-linearity, the worse the noise performance of the pad.

Nonlinear Time Series Analysis

After having applied nonlinear correlation analysis techniques, the microphone signal itself was investigated using recurrence plots and quantification measures as well as classical analyses. A recurrence plot is similar to a joint-recurrence plot, only that the trajectory of one phase space is investigated rather than that of two (possibly) different physical systems. A thorough explanation of nonlinear time series analyses can be found in Kantz and Schreiber (2004) and for recurrence plots in Marwan et al (2007). Here, the purpose of using nonlinear time series analyses is to examine whether nonlinear dynamics (including the dynamics of chaos) can shed some light on the development of brake squeal. Firstly, the techniques were applied to a single degree of freedom friction oscillator in order to highlight the dynamics involved in chaos. A more detailed explanation of this benchmarking process using the analytical model is provided in Oberst et al (2008c). By investigating the variation of the sound pressure signal over time of especially severe squealing events, consistent behaviour could be observed and it was possible to separate parts of the signal into three major zones, here accordingly named zone 1, zone 2 and zone 3. Two cases (case A and B) were chosen as representatives of a relatively often encountered route to chaos (case A) and of a strange attractor (case B). A detailed description of these routes can be found in Oberst et al (2008c). In Figure 6, the power spectra of zones 1 to 3 of case A are given. Clearly, the power spectrum becomes broader and is lifting up. In Figure 6a) squeal is developing with one dominant frequency and its harmonic. The spectrum is lifting up, which can be seen in b) whereas in c) more frequencies are involved. As soon as more than one frequency is involved, the former stable limit cycle switches to a torus in phase space and displays a quasi-periodic time series. Then, if more frequencies get involved, the torus can become chaotic, which was the case for Figure 6c). The development of a chaotic torus-like structure is depicted in Figure 7 from a vortex-structure (a), which corresponds to the transition from a stable point into a limit cycle, over a limit cycle (b) into a torus-structure (c).

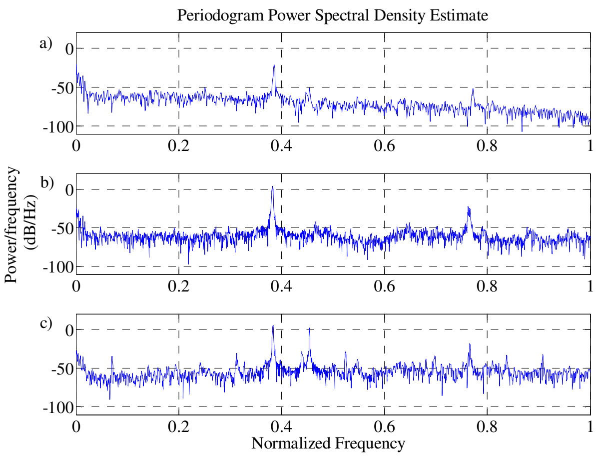
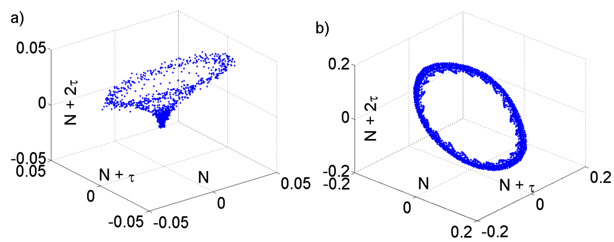


Figure 6: Power spectral density for a representative times series segment in case A: a) vortex regime, b) limit cycle regime and c) a chaotic regime.

This route to chaos is described in detail as a classical one in Schuster and Just (2005) and will not be discussed further in relation to nonlinear time series analyses. It is the so-called Ruelle-Takens-Newhouse route to chaos.



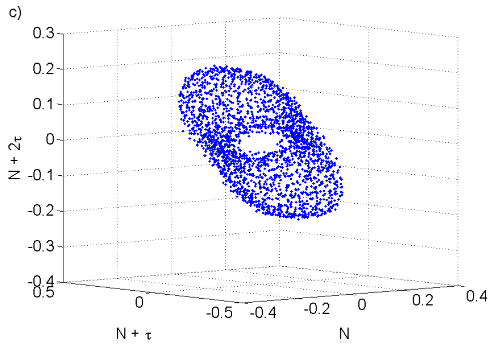


Figure 7: The development to chaos over a a) vortex regime, b) limit cycle and, in the end, to an c) unstable torus.

As another crucial point of evidence, complexity measures (namely, correlation dimension D_2, Kolmogorov-Sinai entropy estimates $K_1$and K_2, correlation entropy h_2 and the maximal Lyapunov exponent L_{max}) are calculated for the sound pressure signal recorded during squealing events and listed in Table 1. Quasi-stationary segments were extracted from the three zones described before (see Table 1, column 3). This was done with the help of distance and recurrence plots (not depicted here) and is described in more details in Kantz and Schreiber (2004).$D_2$is a lower bound estimate for the fractal dimension of an attractor in phase space. The entropy estimates K_2>K_1 and h_2 give an approximate value of the sum of positive Lyapunov exponents. If these values are positive, then the system has diverging trajectories and is chaotic. One positive Lyapunov exponent is enough, to have a chaotic condition, since it describes the maximal exponentially diverging behaviour of the attractor's trajectories. For a stable point L_{max} should be negative, a limit cycle gives a L_{max} of around zero and a chaotic attractor has at least one positive Lyapunov exponent.

Table 1: Dynamic invariants: correlation dimension D_2; entropies K_2, K_1; correlation entropy h_2 and maximum Lyapunov exponent L_{max}.

Case	Zone	Segment, (*10^3)	D_2	K_2	K_1	h_2	L_{max}
A	1	28 – 32	0.1	0.014	0.0298	0.00	-0.070
	2	32– 36	1.0	0.003	0.0099	0.012	0.006
	3	41.5 – 43.9	2.6	0.478	0.3790	0.174	0.147
B	3	333– 337	1.9	0.420	0.3125	0.150	0.175

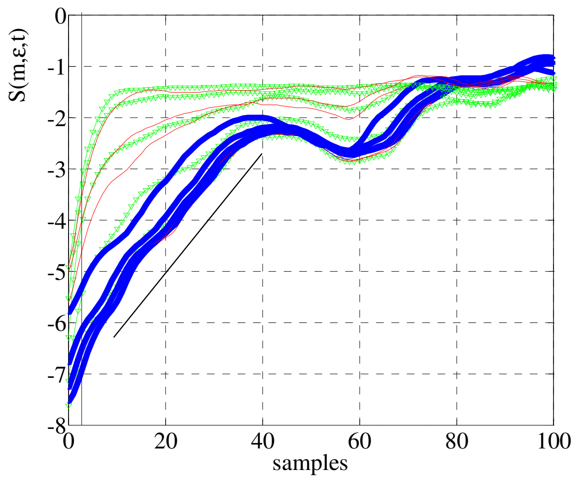


Figure 8: Maximal Lyapunov exponent estimate for a range of embedding delays and three different embedding dimensions (m={2=green, 3= red ,4=blue}).

In Figure 8, an example of a calculation of the maximal Lyapunov exponent for zone 3 in case A, using the algorithm of Kantz (2004), is presented. *S(m,ε,t)* is the so-called stretching factor and samples represent sampling points and is therefore equivalent to time. The correct embedding dimension is obtained when the slope for different ε is similar. Apart from parameters characterizing the embedding, the initial neighbourhood size ε important, which is necessary to calculate the closed returns of the trajectory. The smaller this environment, the larger is a possible linear range of *S*, if there is one. Hence Figure 8 indicates that the embedding dimension *m* should be 4as indicated by the family of blue lines and the slope is the Lyapunov exponent. Interestingly, a similar regime could be simulated with a single-dof friction-induced oscillator (Popp 1990). This attractor was reported by Awrejewicz (1986) who called it a *slip-slip-attractor*. The rendered version of this computational slip-slip-attractor is illustrated in Figure 9.

The fractal dimension of the analytical friction-induced model is estimated using the correlation dimension of D_2=2.2. Experimental data are always contaminated by noise. This artificially blows up the correlation dimension of zone 3 in case A to D_2=2.6 (Table 1). Nevertheless, this does not differ very much from the dimension estimated of the analytical model, which is D_2=2.2.

In Figure 9 and 10 a), a comparison of the attractors for the analytical model and the attractor for zone 3 in case A of the brake-squeal data is provided. Both were rendered to show their similarity. The attractor of the single dof oscillator is represented by only using 30,000 data points. Otherwise, it would appear more compact and fill out more space.

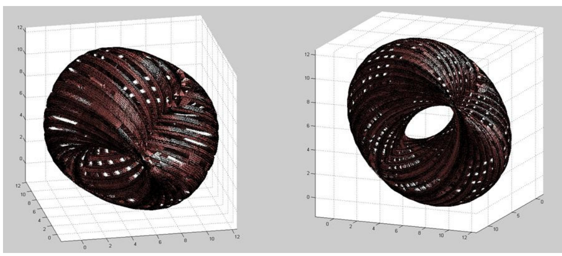


Figure 9: Two different views of a rendered attractor of friction-induced oscillator in a slip-slip-attractor regime.

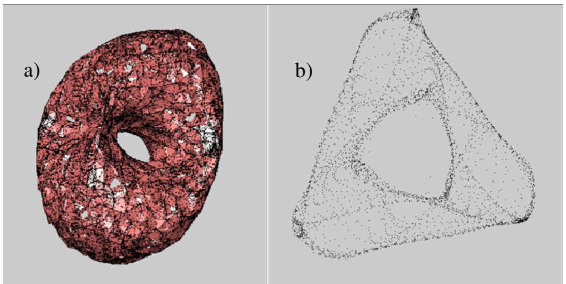


Figure 10: a) rendered slip-slip-attractor obtained from microphone measurements (case A), b) chaotic attractor (case B)

In case B, only the chaotic regime (zone 3) is considered here, which became active right after the limit cycle regime. In Figure 11, the power spectrum for case B is much richer in terms of frequencies and more broadband in nature, suggesting an attractor as shown in Figure 10(b). The dynamic invariants as shown in Table 1 clearly indicate that chaos is present. Although the attractor for case B was found very often in this sound file, it was not possible to reconstruct it with a simple single-dof oscillator.

Finally, with the help of the novel measures based on the theory of recurrence plots (Eckmann et al (1987)), a description of the route to chaos found in case A is possible. Details of this route can be found in Oberst et al (2008c) and are illustrated in Table 2.

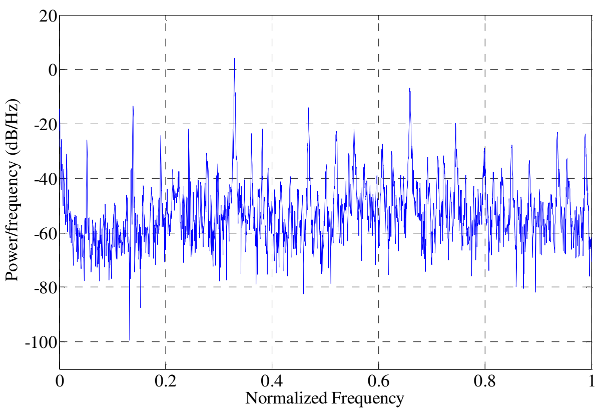


Figure 11: Power spectral density estimate of attractor case B.

Table 2: General behaviour of a brake system exhibiting chaos quantified by recurrence rate (RR), laminarity (LAM), determinism (DET), average diagonal line length (LL) and entropy of diagonal lines (ENT).

Zone	RR	LAM	DET	LL	DIV	L-ENT	Amplitude of squeal	Frequencies
1	High	High	High	High	Low	High	Low/no	No/very few
1→2	-	--(c)	+(c)	+(c)	=	= - (c)	+/yes	+
2→3	-	-+	-	--	++	-	++/yes	++
3→1	++	++	+	++	--	+	--/no	--

It was discovered that, at each stop where chaos developed, the route to chaos could be divided into basically 3 zones. In table 2, the base measures for zone 1 and transitions between zones are given qualitatively. For each transition, the relative qualitative change to the preceding zone is described: '-, =, and +' indicate the decreasing, equal and increasing values of the measure of interest relative to the previous zone, respectively; '- -(c)' means that, after a strong decrease, the value stays basically constant whereas '=-' shows a tendency to decrease slightly. The first zone consists of a vortex regime (see Figure 7), the second a limit cycle regime and the third a chaotic regime. In summary, chaos can be described as relatively high in divergence, low in average LL value, relatively low (in relation to determinism) in RR value and decreasing but slightly recovered in DET and LAM. In addition, the entropy is relatively low, more frequencies are involved. Still, squeal was always clearly audible.

DISCUSSION AND CONCLUSION

By applying new methods of nonlinear time series analysis to experimental brake squeal data, new insights into brake squeal mechanisms were gained. The methods used are already widely applied to nonlinear systems and, in the case of recurrence plots, are more often found in the medical, earth, chemical or biological sciences. The application of these mathematical tools is fruitful and shows that, in the case of the statistical correlation analysis, it is possible to correlate the performance of a brake system with system-specific nonlinearity favoured by geometric modifications: the more non-linearity is involved, the poorer is the performance. This is interesting as the system changes are minimal and only local, and only overall global data (such as time-averaged friction coefficient and occurrence of peak sound pressure level) were used. The measurability of nonlinear correlations confirms the significance and the very high sensitivity of pad geometry. In addition, it can be pointed out that, during the whole analysis, basically two groups could be distinguished in terms of their noise performance. They behaved similarly, according to the measures, and could be observed mostly by inspection in semblance analysis and joint-recurrence plots (both qualitative studies are not further discussed in this paper). The two lesser-performing pad configurations were clearly distinguishable from the two better-performing ones. Within each group, it is difficult to determine which one is the better modification.

In a further step, the dynamics of the system werea nalysed using nonlinear time series analysis and recurrence quantification measures. The results showed close agreement with the theory presented in a benchmark model and with experimental data. As described in Oberst et al (2008c), a possible route to instability could be observed. It is clear that the system dynamics does not settle into a limit cycle regime only but that strange attractors, belonging to chaotic regimes, are formed. Meanwhile, squeal was not attenuated and was still clearly perceivable, even in zone 3 of case B.

In addition, even if a brake system can be seen as complex, the behaviour of some of its regimes indicates that there are underlying mechanisms which are related to very simple mechanical systems. One example was provided of a single-dof friction oscillator illustrating that the mechanism responsible for instability is part of a set, or family, of mechanisms responsible for very severe brake squeal. This is the first time that a friction oscillator, well-known in literature, displays very similar dynamic behaviour to a fully tested brake system. While previous studies using simplified laboratory apparati such as pin-on-disc set-ups revealed chaotic behaviour, it was uncertain, whether a real brake system would display similar features.

In the course of this study, it was discovered that squeal is, of course, also possible when chaotic behaviour is not established (or if the process is so fast that it could not be observed). Therefore, even though chaotic regimes are not the only source of higher squeal propensity, it is observed that, in most of the cases where chaos is present, the peak sound pressure amplitude in the test matrix is very high as well.

This study of nonlinear dynamics indicates that nonlinearity and transient behaviour should be incorporated and considered in numerical simulations in order to bridge the gap between theory and practice.

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REFERENCES

H. Abendroth and B. Wernitz, "The integrated test concept: Dyno-vehicle, performance-noise," SAE Technical Paper Series, 2000-01-2774 (2000).

A. Akay, "Acoustics of friction," Journal of the Acoustical Society of America, **111**(4), 1525–1548 (2002).

J. Awrejcewicz, Chaos in simple mechanical systems with friction, Journal of Sound and Vibration, **1**, 178–180, (1986).

L. Baillet, S.D. Errico, and B. Laulagnet, "Understanding the occurrence of squealing noise using the temporal finite element method," J Sound and Vibration, **292**, 443-460, (2006).

F. Chen, C.A. Tan and R.L. Quaglia, *Disc brake squeal: mechanism, analysis, evaluation and reduction/prevention* (SAE International, Warrendale, PA, 2006).

F. Chen, J. Chern, and J. Swayze, "Modal coupling and its effect on brake squeal," SAE Technical Paper No. 2002-01-0922, (2002).

J.-P. Eckmann, S. Oliffson Kamphorst, D. Ruelle, Recurrence plots of dynamical systems, EurophysicsLetters, **4**(9), 973–977, November,(1987).

A. Felske, G. Hoppe, and H. Matthai, "Oscillation in Squealing Brakes - Analysis of Vibration Modes by Holographic Interferometry," SAE Paper, No. 780333(1978).

Fieldhouse, J. D., and Newcomb, T. P., 'Double Pulsed Holography used to Investigate Noisy Brakes', *Optics and Lasers in Engineering*, **25**, pp 455-494, 1996.

R.A.C. Fosberry and Z. Holubecki, "An investigation of the cause and nature of brake squeal," MIRA report 1955/2 (1955).

D. Guan and J. Huang, "The method of feed-in energy on disc brake squeal," J. Sound & Vibration, **261**, 297-307,(2003).

N. Hoffmann and L. Gaul, "Effects of damping on mode-coupling instability in friction induced oscillations," ZAMM,Z. Angew. Math. Mech**.**, **83**(8), 524-534 (2003).

N.M. Kinkaid, O.M. O'Reilly, and P. Papadopoulos, "Automotive disc brake squeal," Journal of Sound and Vibration, **267**(1),105-166(2003).

N.M. Kinkaid, O.M. O'Reilly, and P. Papadopoulos, "On the transient dynamics of a multi-degree-of-freedom friction oscillator: a new mechanism for disc brake noise," J Sound and Vibration, **287**, 901-917 (2005).

H. Kantz, A robust method to estimate the maximal Lyapunov exponent of a time series. Phys. Lett. A, **185**(1), 77–87, January, (1994).

H. Kantz and T. Schreiber, Nonlinear Time Series Analysis, (Cambridge University Press, 2004)

H. Lee and R. Singh, "Determination of sound radiation from a simplified disk-brake rotor by a semi-analytical method," Noise Control Eng J, **52**(5), 225-239 (2004).

N. Marwan, EncounterswithNeighbours.PhD thesis, Mathematisch-NaturwissenschaftlicheFakultät, InstitutfürPhysik, (2003).

N. Marwan, M. Carmen Romano, M. Thiel, and J. Kurths. Recurrence plots for the analysis of complex systems. Physics Reports, **438**, 237–329 (2007).

F. Massi, O. Giannini and L. Bailet, "Brake squeal as dynamic instability: an experimental investigation,", Journal of the Acoustical Society of America, **120**(3), 1388-1398 (2006).

F. Massi, L. Bailet, O. Giannini and A. Sestieri, "Brake squeal: linear and nonlinear numerical approaches," Mechanical Systems and Signal Processing, **21**(6), 2374-2393 (2007).

H.R. Mills, "Brake squeak," Research Report Nos. 9000 B (1938) and 9162 B, The Institution of Automobile Engineers (1939).

S. Moore, J.C.S. Lai, S. Oberst, A. Papinniemi, S. Hamdi and D. Stanef, "Determining the effect of pad lining geometry on brake noise performance," Internoise '08, Shanghai, China, 26-29 Oct (2008).

H. Murakami, N. Tsunada and T. Kitamura, "A study concerned with a mechanism of disc brake squeal," SAE Transactions, **93**(5), 604-616 (1984).

W.V. Nack, "Brake squeal analysis by finite elements," Int J of Vehicle Design, **23**(3/4), 263-275,(2000).

M.R. North, "Disc brake squeal," Proc. Instn Mech. Engrs, Part C: J. Mechanical Engineering Science, C38/76, 169-176 (1976).

S. Oberst and J.C.S. Lai, "A critical review of brake squeal and its treatment in practice," Internoise '08, Shanghai, China, 26-29 Oct (2008a).

S. Oberst, J.C.S. Lai, S. Moore, A. Papinniemi, S. Hamdi and D. Stanef, "Statistical analysis of brake squeal noise," Internoise '08, Shanghai, China, 26-29 Oct (2008b).

S. Oberst, J.C.S. Lai, S. Moore, A. Papinniemi, S. Hamdi and D. Stanef, "Chaos in brake squeal noise," Internoise '08, Shanghai, China, 26-29 Oct (2008c).

H. Ouyang, J.E. Mottershead and W. Li, "A moving-load model for disc-brake stability analysis," ASME Journal of Vibration and Acoustics, **125**(1), 53-58 (2003).

H. Ouyang, W. Nack, Y. Yuan and F. Chen, "Numerical analysis of automotive disc brake squeal: a review," International Journal of Vehicle Noise and Vibration, **1**, 207-231 (2005).

A. Papinniemi, J.C.S. Lai, J.Y. Zhao, and L. Loader, "Brake squeal: a literature review," Applied Acoustics, **63**(4), 391-400 (2002).

A. Papinniemi, J.C.S. Lai, and J. Zhao, "Numerical prediction of brake squeal propensity," Internoise 2006, Honolulu, Hawaii, USA, 3-6 December (2006), 10 pp.

A. Papinniemi, D. Stanef, S. Moore and Said Hamdi, "Brake Noise in Practice", ICSV 14, Cairns, Australia, 9-12 July (2007).

A. Papinniemi, J.C.S. Lai and J. Zhao, "Disc brake squeal: progress and challenges," ICSV14, Cairns, Australia, 9-12 July (2007), 8 pp.

http://www.polytec.com/int/_files/LN_AM_INFO_0104_E_Squealing_Brakes.pdf

K. Popp and P. Stelter, Stick-slip vibrations and chaos. Philosophical Transactions: Physical Sciences and Engineering, **332**(1624), 89–105, July (1990).

S.K. Rhee, P.H.S. Tsang, and Y.S. Wang, "Friction-induced noise and vibration of disc brakes," Wear, **133**, 39-45,(1989).

M. C. Romano Blasco, Synchronization Analysis by Means of Recurrences in Phase Space, PhD thesis, Mathematisch-Naturwissenschaftliche Fakultät, Universität Potsdam, Institut für Physik, (2004).

H.G. Schuster and W. Just, Deterministic Chaos, (Wiley-VCH, 2005)

R.T. Spurr, "A theory of brake squeal," IMechE Auto Div Proc, 1: 30-40,(1961).

M. Thiel, Exploiting Naturally Occuring Analogues. PhD thesis, Universität Potsdam, Mathematisch-NaturwissenschaftlicheFakultät, InstitutfürPhysik, (2004).

V. Vadari, M. Albright, D. Edgar, "Anintroduction to brakenoise engineering", Sound and Vibration, July, (2001)

S. Yang and R.F. Gibson, "Brake vibration and noise reviews, comments and proposals", International Journal of Materials and Product Testing,**12**, 496-513 (1997).

M.M. Yang, A.-H. Afaneh and P. Blaschke, "A study of disc brake high frequency squeals and disc in-plane/out-of-plane modes," SAE Technical Paper Series, 2003-01-1621 (2003).