Gearbox Fault Simulation using Finite Element Model Reduction Technique

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ABSTRACT

Lumped Parameter Models (LPMs) are widely used to simulate the dynamics of mechanical systems such as shafts and gears in a gearbox. In addition to simulating the complex behaviour between the various components, LPMs can also be used to simulate vibration signals in the presence of gear and bearing faults. However, the limited representation of the modes of the gearbox casing and the internals (shafts and gears) in the LPM model results in poor spectral matching over a wide frequency range. Finite element analysis (FEA) provides an alternative approach to simulate the dynamic behaviour of the structure. However, the use of FEA results in a large number of degrees-of-freedom (DOF), requires large computing resources and presents additional difficulties to simulate the entire system's response in the presence of nonlinearities and gear and bearing faults. The limitations of LPM and FE models are addressed in this paper by using finite element model reduction techniques. FE model reduction based on Craig-Bampton component mode synthesis (CMS) is used to extract greatly reduced mass and stiffness matrices of the gearbox components (internals and casing) based on the frequency range of interest. The extracted mass and stiffness matrices are imported and assembled into the dynamic model of the gearbox previously developed using Simulink®. The model has the capability of simulating time-varying stiffness nonlinearities and geometric faults for both gears and bearings. The reduced matrices comprising both physical and modal coordinates enable inclusion of component flexibility in the dynamic model up to a specified frequency defined by the number of modes included. The dynamic model was solved interactively to obtain simulated vibration signals in the presence of extended inner and outer race bearing faults and compared with the measured signals.

INTRODUCTION

Faults in machine components can be detected and diagnosed using the traditional Machine Condition Monitoring (MCM) techniques that rely mainly on continuous monitoring and analysis of vibration signals, but gathering enough data to train fault recognition algorithms, in particular for catastrophic faults, is not viable. The other alternative is to simulate these faults and capture the vibration signals by creating realistic dynamic models which can be solved using high speed computers. Using fault simulation the location and dimensions of faults can be easily varied and a large amount of data can be gathered without the need for costly and time consuming experiments, or real life failures, which otherwise would be required to gather enough data. The simulation data can for example be used to train neural networks to automate the diagnostic and prognostic processes.

The gearbox under investigation is typical of commonly used machinery in the engineering industry. Due to complex dynamic interaction among the individual components such as gears, shafts, bearings and the casing, the main challenge is to apply a suitable diagnostic technique that is capable of clearly differentiating between the bearing and gear faults (Randall, 2004b, Randall, 2004a). Furthermore it should clearly identify the characteristics of bearing faults (localised and extended faults) and location of the faults (inner race, outer race, rolling elements etc).

Fault simulation models can be developed using a Lumped Parameter Model (LPM) or Finite Element Model (FEM) or by a combination of the two models. In the LPM model, the gearbox masses are lumped at certain locations such as gears, shafts, bearings etc. The resulting simulation model has a limited number of degrees-of-freedom (DOF), which facilitates studying the behaviour of gears and bearings in the presence of nonlinearities and geometrical faults (Sawalhi and Randall, 2008a, Sawalhi and Randall, 2008b). The main limitation of LPM models is their inability to include the casing flexibility, which is an important consideration in light weight structures such as in aircraft applications and cannot be ignored in the simulation models. Hence, LPMs are found to exhibit poor spectral matching over a wide frequency range.

FEM is well suited to predicting the dynamic behaviour of continuous elastic systems such as a gearbox casing, where the mass is distributed over the structure. FE models in general have a large number of DOFs, which makes the simulation of the whole system's response in the presence of nonlinearities and to gear and bearing faults much more complicated. This in turn limits the validity of the simulated results and restricts their later usage in the diagnostics and prognostics of gears and bearings.

The limitations of both the LPM and FEM models are addressed in this paper by using finite element model reduction techniques. Unlike stress analyses, which require large number of DOFs in the FE model, the dynamic behaviour can be predicted with reasonable accuracy using a reduced or much smaller number of DOFs. Hence there is a growing trend to use FE model reduction methods to create accurate low order dynamic models before calculating eigenfrequencies and eigenmodes (Qu, 2004).

An improved dynamic simulation model is developed where a finite element model reduction technique based on Craig-

Bampton component mode synthesis (CMS) (Craig and Bampton, 1968), is used to extract greatly reduced mass and stiffness matrices of the gearbox components including both the internals (shafts, and gears) and the casing. The extracted mass and stiffness matrices are imported into the dynamic model of the gearbox previously developed using Simulink®. This paper presents updated results of the earlier simulation of an extended inner race fault (Deshpande et al., 2011a) and also describes results of simulation of a bearing extended outer race fault. The main aim is to present a working dynamic model based on a full FEM reduced model rather than the combined LPM and reduced FEM model used earlier, to represent the effects of extended bearing faults, in the sense of giving a better match of overall spectra than the LPM model.

GEARBOX TEST RIG

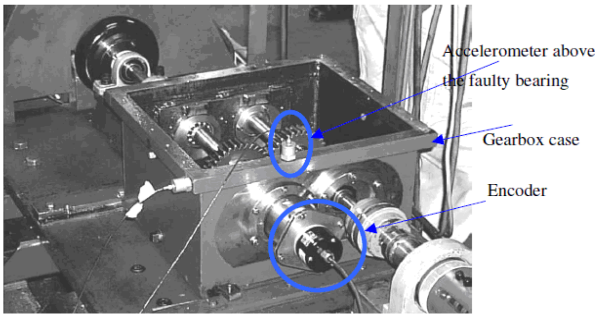


Figure 1: Spur Gear Test Rig (Sawalhi and Randall, 2008a, Sweeney and Randall, 1996)

Figure 1 shows the UNSW gearbox test rig used for the current investigations. The test rig was built originally to investigate the effect of gear faults on transmission error (Sweeney, 1994).

The gearbox consists of a pair of spur gears with 1:1 ratio and 32 teeth on each gear which are driven primarily by a 3-phase electric motor, but with circulating power via a hydraulic pump/motor set. The two shafts are each supported by two ball bearings. The flywheels are used to reduce the

Figure 2: Bearing under test

Figure 3: Extended inner race fault (Sawalhi and Randall, 2008b)

Fig 4: Extended outer race fault (Sawalhi and Randall, 2008b)

fluctuations of the input and output shaft speeds and couplings to attenuate the shaft torsional vibration.

The bearings under test (Fig 2) were double row self-aligning (Koyo 1205) with a contact angle of 0^{o}, a ball diameter of 7.12 mm and a pitch diameter of 38.5 mm.

An extended fault was inserted in the inner and outer race of the bearing by grinding one eighth of the circumference as shown in Fig. 3 and 4. In the simulation model an extended rough profile was programmed into the bearing's S-function to simulate the vibrations generated from the test rig as a result of the extended inner and outer race faults (Sawalhi and Randall, 2008b). This has yet to be optimised. The effect of the oil film was not taken into account in the simulation model.

Both the experimental tests and the simulations were carried out in the presence of bearing extended inner and outer race faults under a 50 Nm load at 10 Hz shaft frequency. An accelerometer was positioned on top of the gearbox casing above the defective bearing to record the vibration signals. The signals were sampled at 48 kHz. A shaft encoder mounted on the free end of the output shaft provided a once per rev tacho signal. The torque for each case was measured at the input shaft. The approximate ball pass frequency of the inner race (BPFI) was 71.1 Hz and that of outer race (BPFO) was 48.9 Hz.

LUMPED PARAMETER MODEL (LPM)

The dynamic model of the UNSW gearbox was previously developed using a 34 degrees-of-freedom (DOF) LPM model (Fig 5). The model takes into account time varying nonlinearity of both the gear and bearing stiffness including the Hertzian contact, and also random slippage in the bearings. The model is capable of simulating bearing faults (both localised and extended faults in the inner and outer race), in addition to spalls and cracks in the gears. The model includes gears, shafts and bearings in detail but only a basic model of the casing with 2 DOFs to represent one low frequency rigid body mode of the casing and one resonance at 15 kHz to represent a high frequency response excited by the bearing faults. Due to limited representation of the casing, poor spectral matching between the simulated and measured bearing faults was observed over a wide frequency range, though envelope analysis of the demodulated high frequency resonance gave good results for localised faults.

FINITE ELEMENT MODEL

The improved dynamic simulation model developed in this paper is based on the reduced mass and stiffness matrices of the gearbox components, which are obtained by using finite element model reduction techniques.

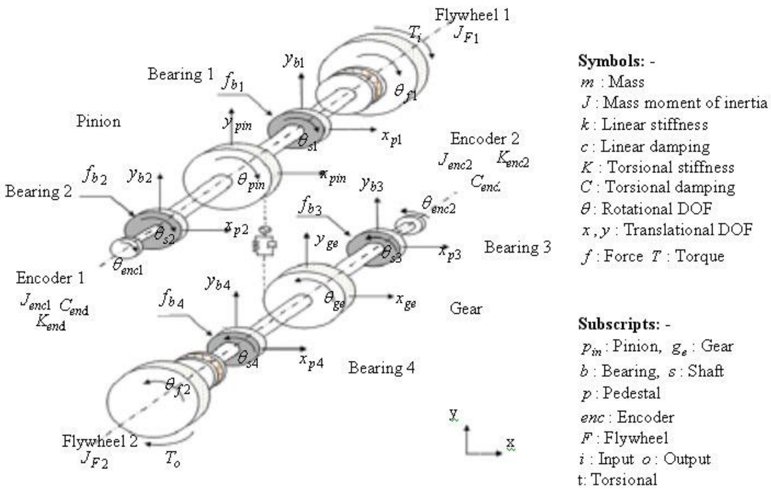


Figure 5: LPM 34 DOF model (Sawalhi and Randall, 2008a)

The FE model of the gearbox and the internals (shafts, gears and bearings) is shown in Fig 6. The casing was modelled with both shell and solid elements, whereas the shafts were modelled with beam elements and gears with shell elements. Rigid body elements were used to connect the gears to the shafts and the bearings to the casing. Flywheels and encoders were modelled using mass elements with appropriate inertial properties and connected to the shafts with torsional springs. The gearbox casing was mounted on rubber pads which were modelled as spring elements. The FE model was updated by tuning the stiffness values of these spring elements so as to correlate with the results of EMA (Experimental Modal Analysis) carried out in (Endo, 2005). Before reduction, the FE model of the casing had 104340 DOFs whereas each shaft-gear assembly (with flywheel and encoder) had 840 DOFs.

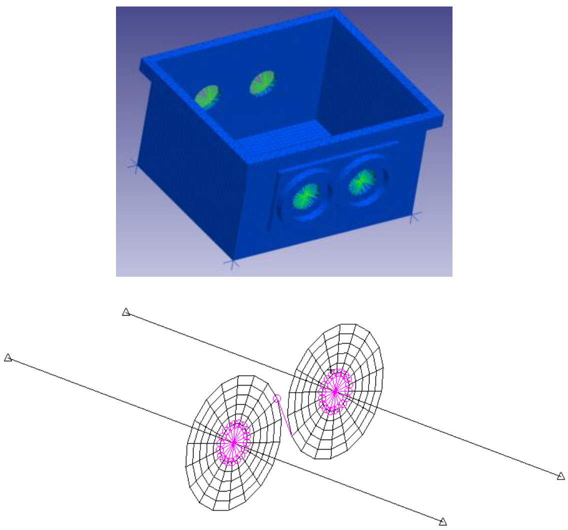


Figure 6: FE model – gearbox casing and internals

Master degrees of freedom (MDOFs) must be specified in order to create reduced FE models. Twenty four MDOFs were selected for the casing and eighteen for the gearbox internals. The MDOF set mainly included the degrees-of-freedom associated with the nodes corresponding to the bearing centre points, shaft gear connection points, flywheels etc. The MDOFs were selected so as to enable connecting the reduced model of the casing with the reduced model of the internals and capture the flexibility of components.

FE MODEL REDUCTION TECHNIQUES

Application of finite element technique for structural analyses normally requires fine mesh (nodes) to accurately predict the component stresses and strains. This results in FE models with very large, perhaps several hundred thousand DOFs which require large computing resources. Unlike static analysis, dynamic analysis is mainly concerned with evaluating the dynamic properties of the structure, namely natural frequencies, mode shapes and damping, which can be predicted using reduced FE models with far fewer DOFs (Young, 2000). Various model reduction techniques have been developed in the past decades such as Guyan, Dynamic, CMS, IRS, SEREP etc.

The static reduction method also known as Irons-Guyan method (Guyan, 1965, Irons, 1965) produces smaller size system matrices by eliminating the coordinates at which no external force is applied. The reduction method is exact for static problems; however, for dynamic problems large errors may be introduced due to the fact that the DOFs eliminated may experience inertia forces, which cause their dynamic displacements to differ from the static, the deviation increasing with frequency (Chen and Géradin, 1997). High frequency motion is better approximated using dynamic reduction. However, the transformation matrix T_{dyn} depends on the choice of an appropriate initial frequency ω, which is not a trivial task. Guyan reduction is a special case of dynamic reduction, when $\omega = 0$.

Component Mode Synthesis (Craig-Bampton Method)

Dynamic condensation, as an efficient method for model reduction, was proposed in 1965 (Qu, 2004). One such method known as the Component Mode Synthesis (CMS) technique (Bayoumi, 2005, Carmignani et al., 2009, Sellgren, 2003) consists of dividing the complex structure into smaller *substructures* (or *superelements*) and recovering afterwards the dynamic behaviour of the original structure by assembling the *superelements* and considering the equilibrium of nodes at the interfaces between the various components. The dynamic analyses of large structures are often carried out using *superelements* based on the *substructuring* principle.

The Craig-Bampton (CB) method (Craig and Bampton, 1968) is a dynamic reduction method used to reduce the size of the finite element models. In this method, the motion of the whole structure is represented as a combination of boundary points (master degrees of freedom) and modes of the structure assuming that the master degrees of freedom are held fixed. Unlike Guyan reduction (Guyan, 1965), which only accounts for the stiffness matrix, Craig-Bampton accounts for both the mass and stiffness. Furthermore, it enables defining the frequency range of interest by identifying the modes of interest and including these as a part of the transformation matrix. The decomposition of the model into both physical DOFs (master DOFs) and modal coordinates allows the flexibility of connecting the finite elements to other substructures, while maintaining a reasonably good result within a required frequency range. The lower order CB modes differ from those of the connected structure, but the frequencies of the higher order modes are much less dependent on the boundary conditions. The algorithmic scheme of CMS is divided into three categories namely, fixed-interface, free-interface, and the residual-flexible free interface. In this paper only the fixed-interface CMS method is considered.

The CMS algorithm does have certain disadvantages and the static term (Guyan) constitutes the source of the largest amount of information loss. The error can be compensated to some extent by increasing the number of CB modes (Koutsovasilis and Beitelschmidt, 2010).

REDUCED DYNAMIC MODEL

Initial investigations on implementing model reduction techniques involved combining the reduced FE model of the casing with the LPM of the internals. The reduced mass and stiffness matrices for the casing were imported into the LPM model developed earlier using Matlab/Simulink®. The equations of motion were redefined at the bearing / casing connection points and the combined model (LPM + reduced FEA) was solved to obtain the vibration signals in the presence of bearing faults.

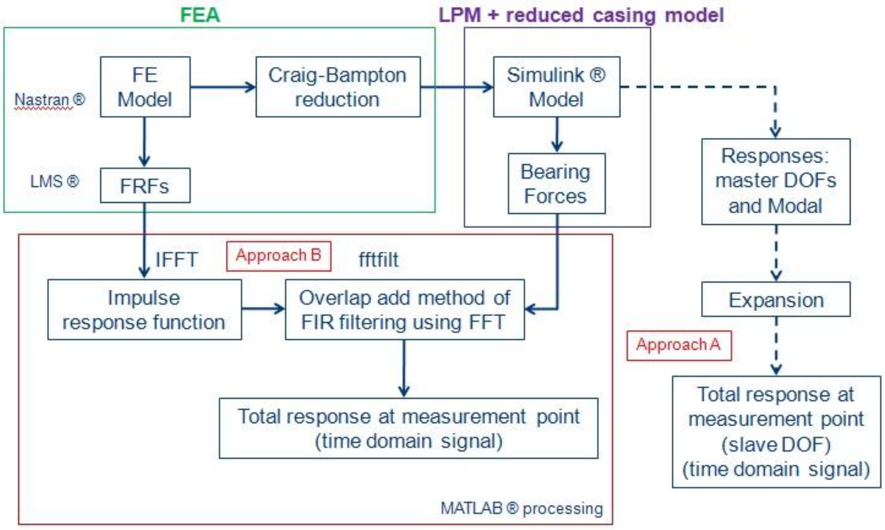


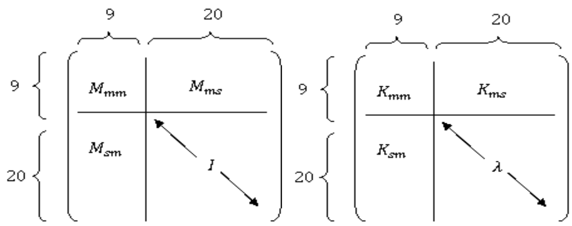
Figure 7: Evaluation of total response

In (Deshpande et al., 2010) a simple dynamic reduction of the casing was used whereas in (Deshpande et al., 2011b, Sawalhi et al., 2011) the Craig-Bampton based reduction technique was used to reduce the casing model. The resulting model (combined LPM + reduced FEM) had 146 DOFs. In (Sawalhi et al., 2011) the total response of the 146 DOF model was evaluated by expanding both the physical and modal responses back to a slave DOF of interest which represented a virtual sensor corresponding to an accelerometer location in experimental testing (Approach A, Fig 7). However, these results were valid only up to 4 kHz (100 modes). In (Deshpande et al., 2011b) a more interactive approach was used for the same 146 DOF model to obtain a higher valid frequency range by convolving the reduced model forces with the impulse responses of the gearbox (Approach B, Fig 7).

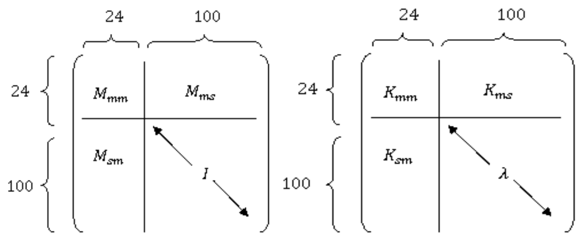
The main limitation of the earlier models was that the gear/bearing interaction was not properly modelled. The models failed to give adequate results in the mid-frequency range where there is interaction between the forces at the gearmesh and the dynamic properties of the casing. Hence, an improved dynamic model of the gearbox was created by applying Craig-Bampton based model reduction to both the gearbox internals (shafts, gears and bearings) and the casing.

The improved model has a total of 182 DOFs with 42 physical (masters) and 140 modal coordinates. The mass and stiffness matrices for the two shafts (with gears) and that of the casing were assembled in Matlab® (Fig 8).

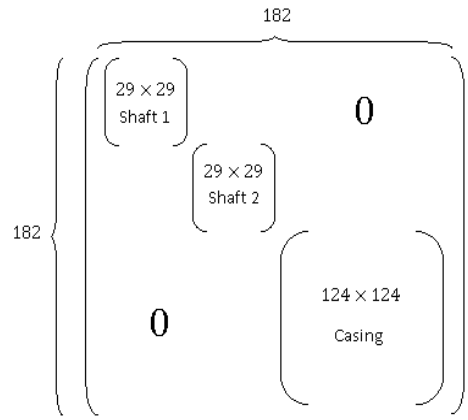
The total response now includes contributions due to flexibility and dynamics of both the internals and the casing. The simulation model contains more DOFs for the internals as compared with the LPM model, in addition to a large number of casing modes. The model was used to simulate an extended inner race fault and the results, while similar to those of the 146 DOF model, indicate that it is a valid way to model more complex systems (Deshpande et al., 2011a).



[M] Reduced and [K] Reduced (Shaft 1 and 2)



[M] Reduced and [K] Reduced (Casing)



[M] Reduced and [K] Reduced (Shaft 1 + Shaft 2+ Casing)

Figure 8: CMS Reduction: Full gearbox (Internals + Casing)

In this paper, updated results for the bearing extended inner race fault using the 182 DOF model are presented. The model has also been used to simulate an extended outer race fault. The model is run in Simulink® environment using Approach B discussed earlier (Fig 7) to give a total response at a virtual sensor location, corresponding to the actual accelerometer position. In both new simulations, the sampling frequency was increased by a factor of four (then downsampled after lowpass filtering) to solve an aliasing problem. Interestingly, this did not greatly increase computation time because of quicker convergence.

RESULTS AND DISCUSSION

The dynamic models developed earlier (discussed in the previous section) were based on LPM only or LPM of internals combined with the reduced FE model of casing. An updated model (182 DOF) used in this paper was created by FE model reduction of both the internals and the casing. The LPM and combined models were used to simulate localised and extended faults in the bearing inner race whereas the updated model developed in this paper was used to simulate extended faults in both the inner and outer race of bearing. The simulation results of all the models are compared and discussed in this section.

Extended inner race fault – PSD comparison

Figure 9 (a-d) show PSD comparisons for the good and the faulty bearing with the extended inner race fault in case of the UNSW measured data, LPM model, 146 DOF and 182 DOF models. The presence of extended inner race fault results in a dB increase (between the good and faulty bearing) in the high frequency region in the case of measured data as seen in Fig 9 (a) whereas in case of all three simulation models (Fig 9 b-d) the dB increase is mainly seen in the mid-frequency region. This can be attributed to the fault geometry used and implies that further tuning of the simulation models is required which can be achieved by increasing the abruptness of the fault exit.

Extended outer race fault – PSD comparison

Figure 10 (a-d) shows PSD comparisons for the UNSW measured data, LPM, 146 DOF and 182 DOF models. The presence of an extended outer race fault results in a dB increase in the high frequency region in case of all three simulation models (Fig 10 b-d), which is similar to the measured results as seen in Fig 10 a. Both the 146 and 182 DOF models show the effects of the much greater number of resonances in the high frequency region compared with the LPM. Although the results are not greatly different, the 182 DOF model can be considered to be more realistic due to inclusion of additional DOFs for both the casing and internals (more bending modes of the shafts for example, compared with the single mode in the LPM).

The increased simulated response in the case of the outer race fault can be attributed to the fact that the fault is in the load zone for the whole time, but only part of the time for the inner race fault. The actual fault geometry could be modified to give better correspondence with the measurements.

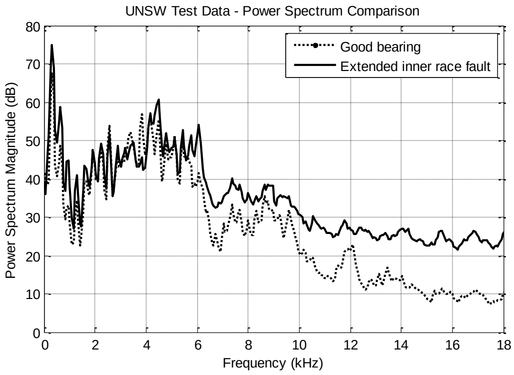
This initial study of various simulation models was limited to the PSD comparison of the good and the faulty bearing. As such only a general trend of increase in dB difference over a wide frequency range was mainly used to establish the existence of a fault rather than the quantitative comparisons of PSD differences for various models. However, further studies using spectral correlation (Sawalhi & Randall 2008b) need to be carried out to accurately predict the location of fault i.e. inner race or outer race fault.

The analysis time for the 34 DOF LPM model was about 15 minutes on a standard desktop PC (Intel Core 2 Duo, 3 GHz, 4 GB RAM) and approximately 3 hours for the 146 and 182 DOF models. It is expected that the simulation will be run for only a limited number of times to generate realistic vibration signals which can be subsequently used to automate the diagnostics process and to train neural networks.

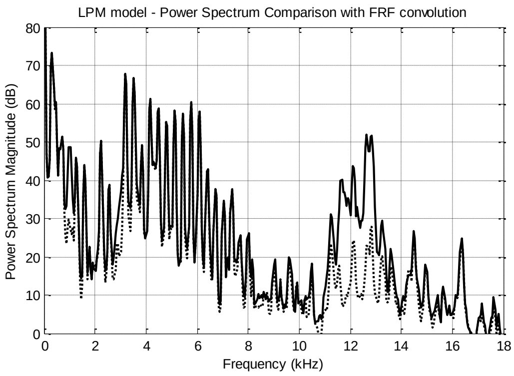
CONCLUSION

A finite element model reduction technique based on the Craig-Bampton component mode synthesis method was used to create a greatly reduced dynamic model of a gearbox. The full FEM reduced model was found to give similar results to the combined LPM and reduced FEM approach, but can be considered more realistic, and more easily expanded to more complex models. The model was able to simulate vibration signals in the presence of extended inner and outer race bearing faults. A previous problem with aliasing has been solved by increasing the sampling frequency for the time domain computation.

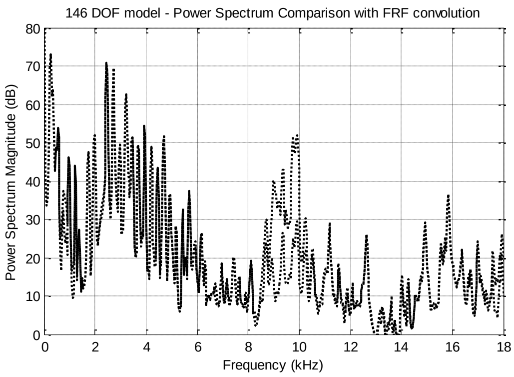
The full FEM reduced model is found to adequately represent the effects of extended rough inner and outer race faults and results in a better match of overall spectra with the experimental results. Since this is an ongoing work, the above findings will be further improved by making necessary adjustments, in particular to the fault geometries, and adding realistic noise to give a better match with measurements. The detailed interaction of the bearing faults with the dynamic response of the internals and casing will then be studied using spectral correlation, and is expected to give a better match with the measurements than the LPM model.



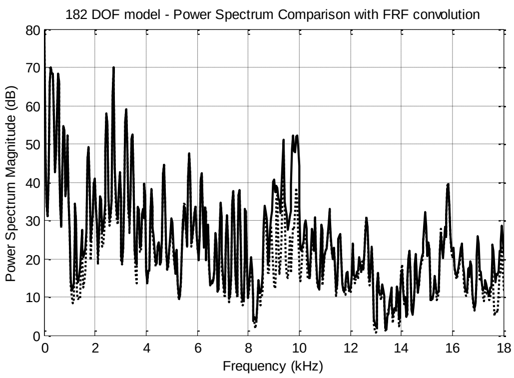
(a) UNSW Test Data



(b) LPM 34 DOF Model

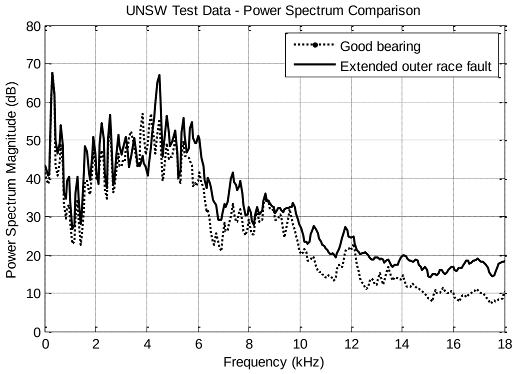


(c) 146 DOF Model

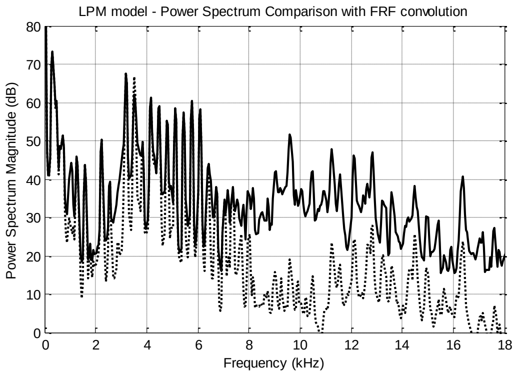


(d) 182 DOF Model

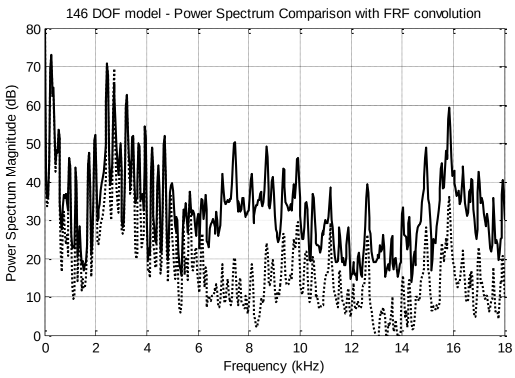
Figure 9: PSD comparison – Extended inner race fault



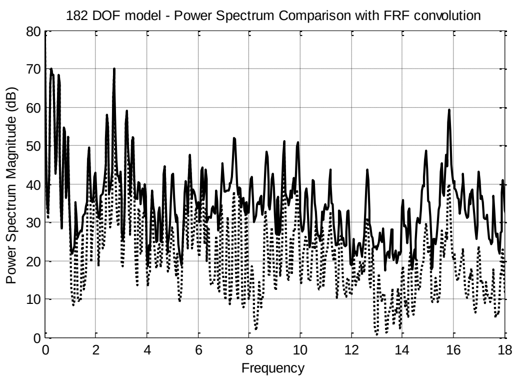
(a) UNSW Test Data



(b) LPM 34 DOF Model



(c) 146 DOF Model



(d) 182 DOF Model

Figure 10: PSD comparison – Extended outer race fault

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