The role of theoretical models in shaping railway noise policy and mitigation strategies

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ABSTRACT

The main source of noise from railway operations is associated with the wheel/rail contact. It is now widely established that railway rolling noise is caused by the 'roughness' of the wheel and rail running surfaces; this roughness induces vibration of both wheel and track and the vibration radiates sound. A review is presented of the theoretical modelling that has led to this conclusion, following which the implications of this for noise control as well as for legislation are discussed. Legal limits for the noise emission from road vehicles and aircraft have been in force for many years. By contrast the noise emission from rail vehicles has only recently been subject to legal limits, introduced in Europe through the Technical Specifications for Interoperability. This has been facilitated by the theoretical knowledge embodied in models of rolling noise. Finally, the scope for using 'virtual testing' based on calculations to partially replace costly field tests is discussed.

INTRODUCTION

The main source of noise from railway operations is usually rolling noise associated with the wheel/rail interaction. It is now widely established that railway rolling noise is caused by the 'roughness' of the wheel and rail running surfaces; this roughness induces vibration of both wheel and track and the vibration radiates sound (Thompson, 2008).

Theoretical modelling of this phenomenon commenced in the 1970s, in particular through work by Remington in the USA. His basic model was developed and extended in Europe in the 1980s culminating in the TWINS model produced for the European railways in 1991 as part of the work of ERRI C163.

The most important conclusion from this work was the recognition that rolling noise is caused by the vibration of *both* the wheel and the track, excited by the combined wheel and rail surface roughness. This has considerable implications in terms of noise mitigation at source as treatments are required to deal with radiation from both the wheel and track. The model has subsequently been used widely to develop various low noise designs. For example, new designs of wheel are routinely analysed using TWINS, rail and wheel dampers have been developed and shielding options have been assessed.

Moreover, a number of aspects of the theoretical model have now become an integral part of noise legislation in Europe. Noise limits for individual road vehicles were introduced by the EU in the early 1970s and limits for aircraft were also introduced. However, for railway vehicles this was delayed by the need to account for the complex interaction of wheel and track. Since 2002 noise regulations for new vehicles have been introduced in Europe through the medium of Technical Specifications for Interoperability or TSI (European Commission, 2006, 2008). The pass-by noise test in the TSIs is based largely on ISO 3095 (International Organization for Standardization, 2005). It includes a limit on the allowable rail roughness at the test site to ensure that the roughness is mainly dependent on that of the wheels. Within the TSI test there is also a requirement that the decay rate of the track should exceed a certain threshold, so that the track component of noise should be kept within reasonable bounds.

The testing of new vehicles is an expensive operation. For the future there are moves to try to replace at least part of the noise testing process with calculations ('virtual certification'). While it will not be possible to remove the need for testing completely, theoretical models will allow some of the process to be simplified, for example by making more use of static tests. There is also a need to extend the process to include other sources, particularly aerodynamic noise, traction noise and fan noise.

This paper gives a brief review of the theoretical modelling, following which the implications of this for noise control as well as for legislation are discussed. Finally, the scope for using 'virtual testing' based on calculations to partially replace the costly field tests is discussed.

THEORETICAL MODELS FOR ROLLING NOISE

The noise due to the wheel/rail interaction is usually the main contribution to the noise from railway operations. This can be divided into rolling noise, a broad-band noise occurring on straight and gently curved track; curve squeal, a tonal noise occurring in sharp curves; and impact noise caused by discontinuities in the wheel/rail surface such as wheel flats, rail joints, switches or crossings. Rolling noise is much more widespread than curve squeal or impact noise and has therefore received much greater interest. It has a typical frequency range of 100 to 5000 Hz.

In the 1970s and 1980s there was still considerable discussion about whether the wheels or the rails were the main source of noise. In some situations it appeared to be the wheels and in other situations the track that produced the noise. This was resolved by a combination of field measurements of vibration and noise together with theoretical models of sound radiation, leading to the conclusion that both of them formed significant sources of noise (Thompson, 1988). This conclusion is somewhat unfortunate as it means that effective noise control requires measures to deal with both sources in order to obtain significant reductions.

A model for rolling noise was first developed by Remington (Remington, 1975; Remington, 1987). The models used today such as TWINS (Track-Wheel Interaction Noise Software) are based extensively on this early work. Figure 1 shows the framework for the TWINS model (Thompson et al 1996a).

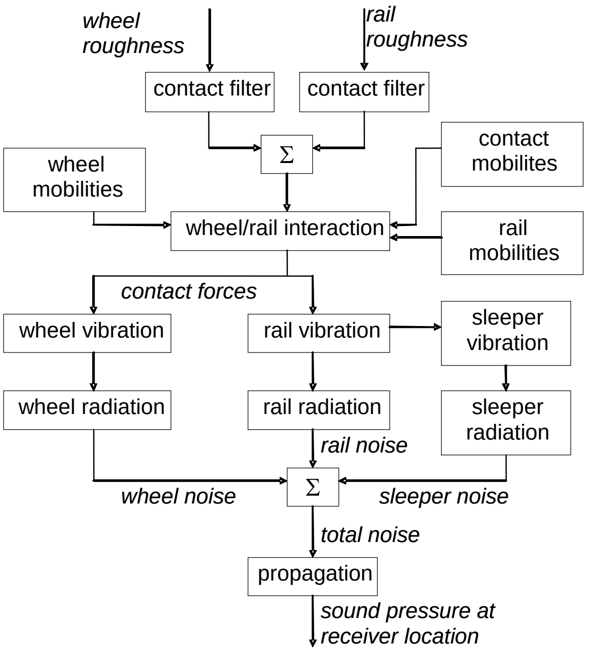

Figure 1. TWINS model for rolling noise generation.

The model is based on the excitation of vibration by surface roughness as indicated in Figure 2. The term 'roughness' in this context refers to wavelengths between about 5 and 500 mm, rather than micro-roughness used in other fields, but the term has become widely used in the railway noise field. A roughness of wavelength λ traversed at a speed v excites vibration at a frequency $f = v / \lambda$. From this the wavelengths of importance can be derived, as listed in Table 1 for some example speeds.

Table 1. Examples of roughness wavelengths (in mm) for various frequencies and train speeds.

Speed (km/h):	40	80	160	320
50 Hz	230	450	900	1800
100 Hz	110	230	450	900
250 Hz	45	90	180	360
500 Hz	23	45	90	180
1000 Hz	11	23	45	90
2500 Hz	4.5	9.0	18	36
5000 Hz	2.3	4.5	9.0	18

Wavelengths that are short compared with the contact patch length (typically 10-15 mm) are attenuated in their excitation of the system, the so-called contact filter effect (Remington, 1976; Thompson, 1996c).

The interaction between wheel and rail relies on their dynamic properties, expressed in Figure 1 as mobility. The wheel, rail and contact spring vertical mobilities are shown in Figure 3. The component with the larger mobility responds at the amplitude of the roughness – for example between about 100 and 1000 Hz the rail vibration is driven by the roughness.

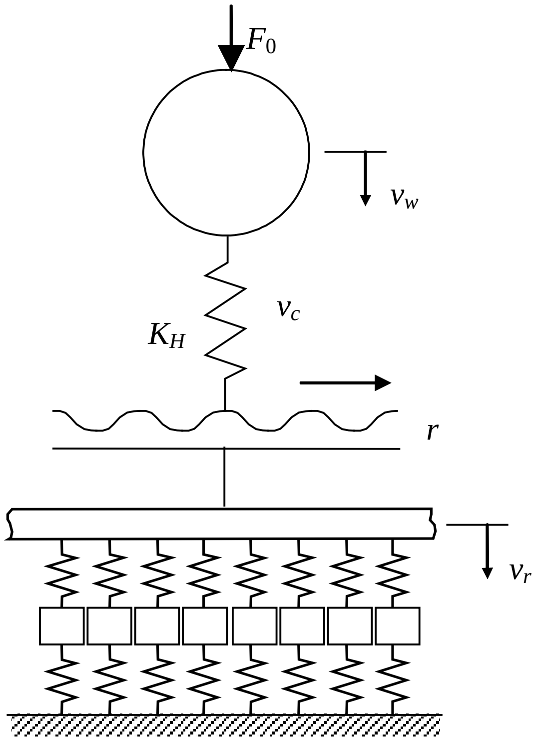

Figure 2. Schematic diagram of the wheel/rail system excited by a roughness r.

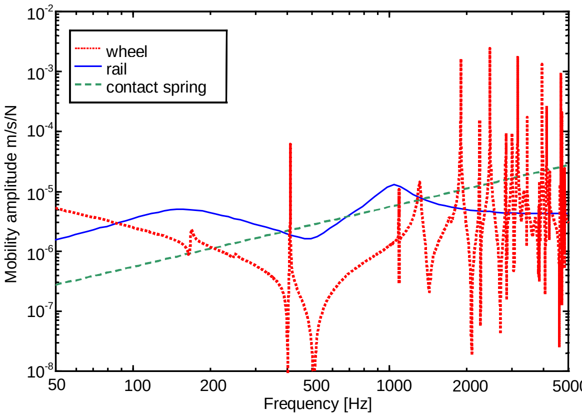

Figure 3. Typical wheel, rail and contact spring mobilities.

A typical prediction is shown in Figure 4. At low frequencies the sleepers produce most of the noise as they have a large area and are well coupled to the rails. The frequency range where the rails become decoupled from the sleepers depends on the rail pad stiffness. In this frequency range waves propagate along the rail relatively unhindered. In the middle of the frequency range, therefore, the rails radiate most of the noise where they respond with the amplitude of the roughness and then propagate waves along the track. At higher frequencies the rail vibration at the contact is attenuated by the contact spring, which has a higher mobility. Here, it is the wheels that produce the largest component of noise as a series of modes of vibration occur with large radial motion and axial motion of the web. These modes can be seen clearly as sharp peaks at high frequencies in the mobilities plotted in Figure 3. The corresponding modeshapes are shown in Figure 5. The balance between the overall noise produced by wheel and track depends on a number of parameters including the roughness spectrum and the train speed.

The situation is made more complicated by the fact that it is the combined wheel and rail roughness that excites the system, see Figure 1. This combined roughness affects both wheel and track noise. It is therefore possible to have a situation where a high *wheel* roughness level causes a large radiation from the *track* or vice versa. It is unclear in such a situation whether this noise should be attributed to the vehicle or the track.

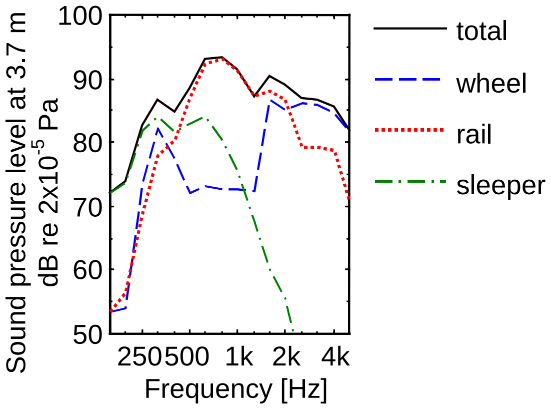

Figure 4. Components of rolling noise for a freight wheel at 100 km/h on track with medium stiffness pads (Thompson et al 1996b).

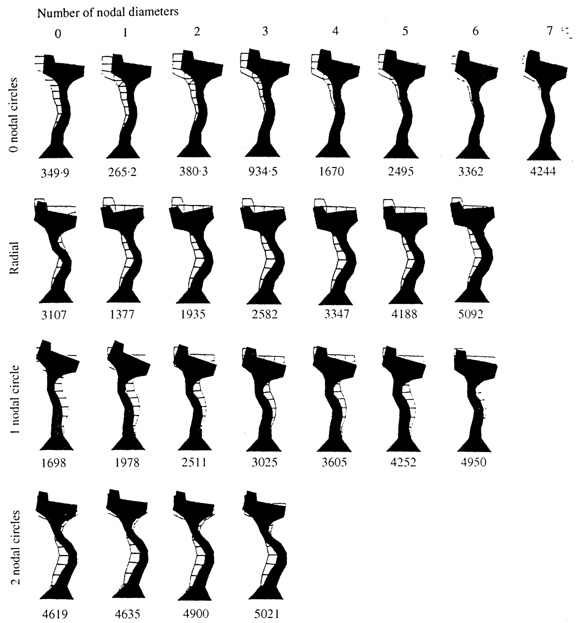

Figure 5. Modeshapes of a UIC 920 mm standard freight wheel shown in cross-section with natural frequencies in Hz (Thompson 2008).

Extensive validation tests were carried out by ERRI C163 (Thompson et al 1996b) which showed that the model gave adequate results. The overall sound level could be predicted to within about ±2 dB while the result in individual one-third octave bands could be obtained to within about ±5 dB. The validation was later extended to include a range of novel low noise designs (Jones & Thompson 2003) with a similar conclusion. The remaining uncertainty, although large, has been shown to be consistent with the level of uncertainty in the roughness input.

CONTROLLING NOISE: THE IMPORTANT PARAMETERS

The overview of the model shown in Figure 1 also provides an indication of which parameters could be used to control noise.

The TWINS model has been used in a number of research projects to help design low noise wheels and tracks (Thompson & Gautier, 2006). Of particular importance were the Silent Freight and Silent Track projects, supported by the European Commission (Hemsworth et al, 2000).

Surface roughness

The importance of surface roughness has been recognised since Remington's work in the 1970s. Visibly corrugated rails have long been known to produce high levels of noise. In the late 1970s it was observed that wheels with disc brakes were up to 10 dB quieter than wheels with cast-iron block brakes and this was attributed to the presence of smoother wheel surfaces on disc braked wheels (Hemsworth, 1979).

In Europe, freight vehicles have long been fitted with cast-iron brake blocks to facilitate interoperability as a single type of braking is required throughout a train. Consequently freight trains tend to dominate the noise exposure from railway lines, especially where these trains run at night and a night-time penalty has to be included.

A move from cast-iron brake blocks for freight vehicles towards composite brake blocks (K and LL) has formed a major initiative of the European railways in recent years (Hübner, 2001; de Vos et al 2006). The basic principle has been demonstrated clearly although there have been some problems with implementation and 'homologation' of particular products.

Figure 6 shows the development of noise from vehicles with two types of brake blocks. The cast-iron brake blocks can be seen to lead to quite rapid development of roughness reaching a stable level after about 15,000 km whereas the LL blocks lead to a much slower development over about 50,000 km before a stable level is reached.

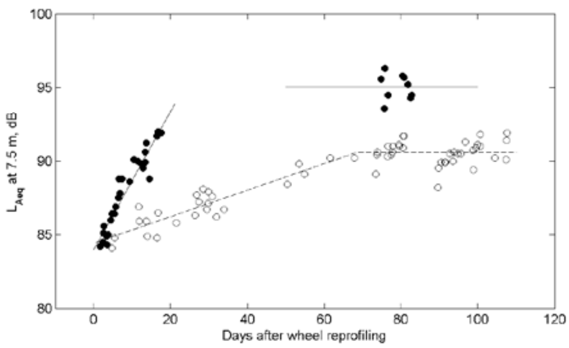

Figure 6. Development of noise level after reprofiling of wheels. Noise measurements at 7.5 m from the track, train speed 120-135 km/h. ●, —, cast-iron brake blocks; ○, – – –, composite (LL) brake blocks; on average these vehicles travel 700 km per day (Thompson, 2008).

Track decay rates

The decay rate determines the length of rail that vibrates and radiates sound for each wheel/rail contact. The track decay rate was also identified by Remington as an important parameter but only more recently has it been acknowledged more widely. The sound power from the rail is inversely proportional to the decay rate Δ in dB/m so that the sound power level is related to $10.\log_{10}(\Delta)$.

The rail pad stiffness has a critical effect on the decay rate. Figures 7 and 8 show the track acceleration and rail decay rate for three different pad stiffness. The rail pad stiffness affects the peak in the frequency response between 300 and 900 Hz in Figure 7 – at this frequency the rail bounces on the pad stiffness. Above this frequency waves in the rail propagate freely leading to a reduction in the decay rate and an increase in the noise radiated by the rail.

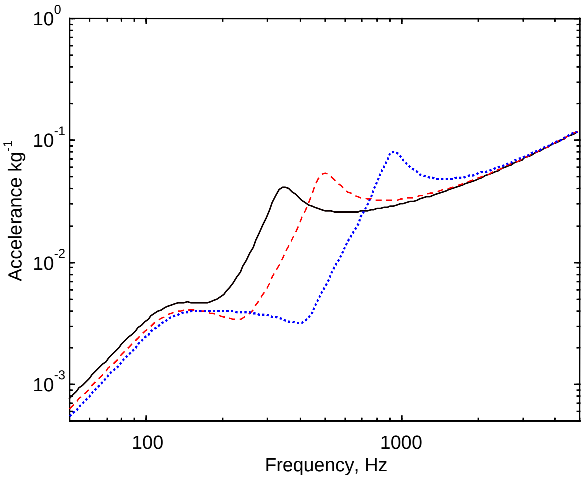

Figure 7. Predicted effect of rail pad stiffness on track vertical accelerances, —— 140, – – 300 and ······ 1000 MN/m.

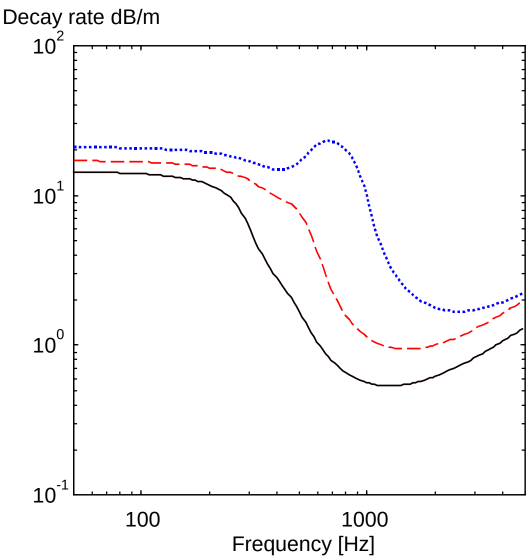

Figure 8. Predicted effect of rail pad stiffness on track vertical decay rates. —— 140, – – 300 and ······ 1000 MN/m.

Stiff pads cause a greater decay rate and hence reduce the rail component of noise; but they also lead to an increase in the noise radiated by the sleepers by coupling the rail and sleeper over a greater frequency range. The dependence of noise on pad stiffness is plotted in Figure 9. The optimum value of stiffness has been found to lie at the hard end of the range of possible values (Vincent et al, 1996).

In order to increase the track decay rate a number of different rail dampers have been designed in recent years, see e.g. (Thompson and Gautier, 2006). These also allow soft rail pads to be used without an increase in noise radiation. Figure 10 shows one example of a rail damper attached to the rail web. Typical results (actually from an earlier prototype) are shown in Figure 11.

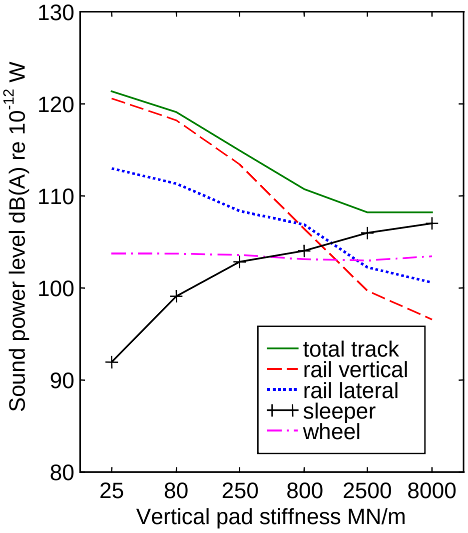

Figure 9. Example of predicted sound power due to one wheel and the associated track vibration versus high frequency rail pad stiffness. Calculations using TWINS for a standard 920 mm freight wheel at 100 km/h, with a typical tread-braked roughness.

Figure 10. Rail dampers produced by Tata Steel.

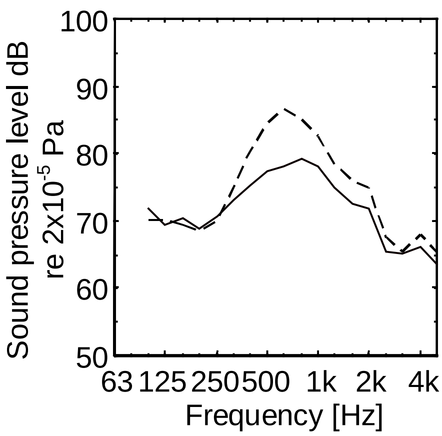

Figure 11. Measured reduction in overall noise due to rail damper tested with a low-noise wheel design. - - - reference track, — track with damper (Thompson et al 2007).

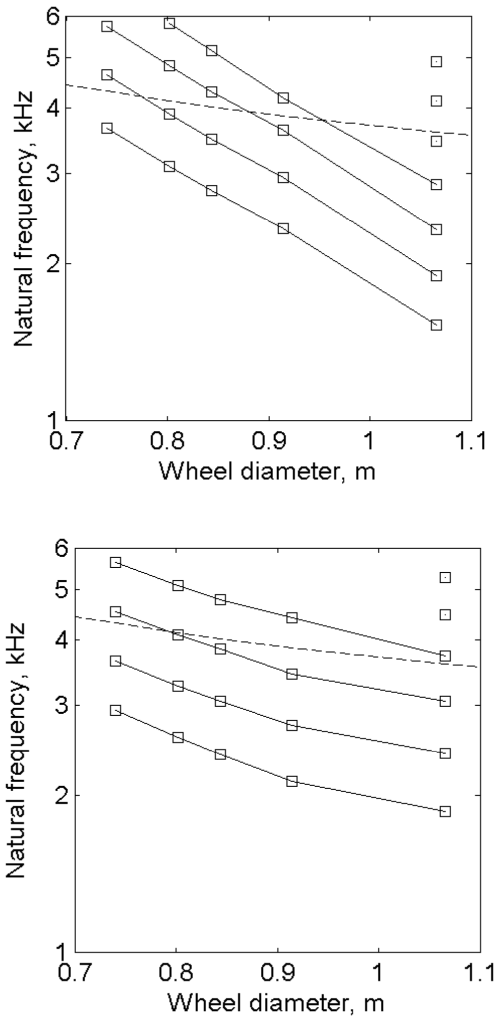

Figure 12. Effect of wheel diameter on natural frequencies of straight webbed wheels. Upper: 1-nodal-circle axial modes; lower: radial modes. ☐ natural frequencies of modes with $n \geq 2$; – – – 'cut-off' frequency of contact filter (Thompson, 2008).

Wheel design

TWINS is now routinely used in assessing new designs of wheel. Where it is necessary to continue using tread brakes the wheel web must be curved, as in Figure 5. A straight web is now known to produce lower noise levels due to a reduction in the coupling between the radial excitation (due to roughness) and the axial response which radiates most of the noise.

It has also been shown that wheel diameter can have a significant effect. Figure 12 shows that the natural frequencies of one-nodal-circle and radial modes increase with reducing diameter. The contact filter is also affected as the contact patch becomes shorter in the rolling direction, but this effect is much smaller. Consequently small wheels have fewer modes in the frequency range of excitation. They also have reduced radiating surface area but this effect is less significant. However, it should be pointed out that very small wheels can lead to an increase in the noise radiated by the track due to the change in the contact filter.

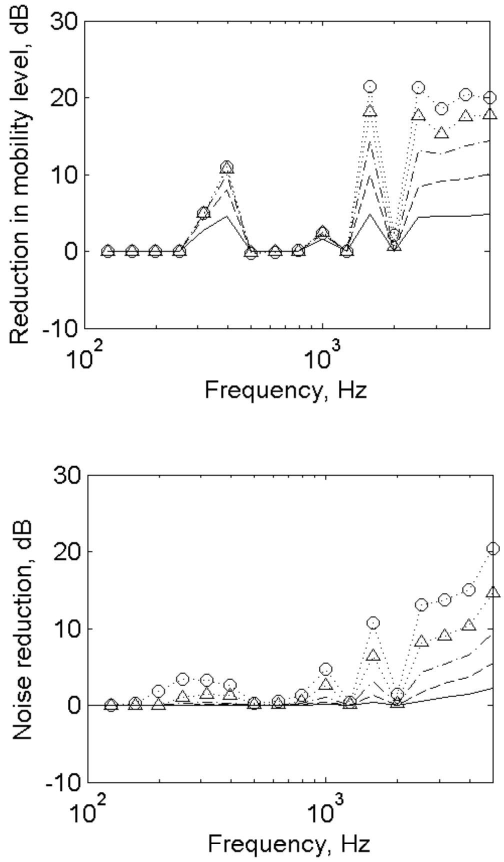

Figure 13. Effect of increasing the damping of the wheel relative to an initial situation with wheel damping ratios of 10^{-4} in the modes with $n \geq 2$. (a) Reduction in radial mobility level, (b) reduction in noise radiated by the wheel. Damping ratios increased to: —, 3×10^{-4}; – – –, 10^{-3}; – · – ·, 3×10^{-3}; Δ, 10^{-2}; o, 3×10^{-2} (Thompson, 2008).

The wheel is a lightly damped structure so adding a damping treatment should be a promising noise control measure. However, the wheel/rail contact introduces significant apparent damping to the wheel. It is therefore essential that this level of damping is exceeded if a damping treatment is to be effective. To illustrate this, Figure 13 shows the change in average wheel mobility in one-third octave bands and the corresponding change in noise radiated by the wheel. It is clear that the first factor of 10 increase in damping has only a very small effect on the noise radiation. Increases in damping above this level start to have a greater effect. Studying only the free response of the wheel would lead to misleading conclusions as far as rolling noise performance is concerned.

A project called Stardamp is currently underway under the Franco-German collaboration scheme Deufrako. The purpose of this project is to propose standardised methods for assessing wheel and rail damping treatments to avoid such misleading conclusions.

Radiation and shielding

A final area where noise reduction can be sought is by modifying the sound radiation (see Figure 1). It is difficult to affect the radiation ratio of either the wheel or the rail in the frequency range of interest. However, it is possible to introduce local shielding, either on the wheel or rail itself, through shrouds around the bogie or barriers close to the rail.

MEASUREMENTS AND REGULATIONS

As mentioned in the introduction, limit values on individual vehicles have only been introduced for rail vehicles in recent years. Prior to that, it was considered too complicated to separate the contribution of the vehicle and the track. It is clearly of no value to introduce limits for rail vehicles if the noise is dominated by the contribution from the track.

Using theoretical modelling it has been possible to identify the respective contributions of vehicle and track radiation and of wheel and rail roughness and from this to specify parameters that can be used to control the influence of the track. In this context the concepts of rail roughness and track decay rate have taken on a very important role.

ISO3095 and the TSIs

The measurement standard ISO3095 (International Organization for Standardization, 2005) is used, amongst other things, to describe the pass-by noise of a rail vehicle. In the 2005 revision it was recognised that the contribution of the track had to be limited as much as possible – the previous version had stated merely that the rail running surface should be in good condition and free from visible corrugation. This has led to a limit in terms of the rail roughness spectrum.

Meanwhile in Europe the Technical Specifications for Interoperability (European Commission, 2008, 2009) have been introduced as a legal framework which had the motivation to remove barriers to cross-border operation. These have been used as a mechanism to introduce legal limits for the noise emitted by new rolling stock. The pass-by test is based on ISO3095 but with a modified rail roughness limit.

Roughness limits

In both ISO3095 and the TSIs a limiting rail roughness spectrum is defined, slightly different in the two cases, which the rail roughness should not exceed. The purpose of such a limit is to restrict the contribution of the track to the pass-by noise in terms of its excitation of the complete wheel/rail system. There are no controls on the wheel roughness apart from a requirement to have run 3000 km; it is supposed that it is typical of the type of vehicle under test (but see Figure 6). Figure 14 compares these rail roughness limits.

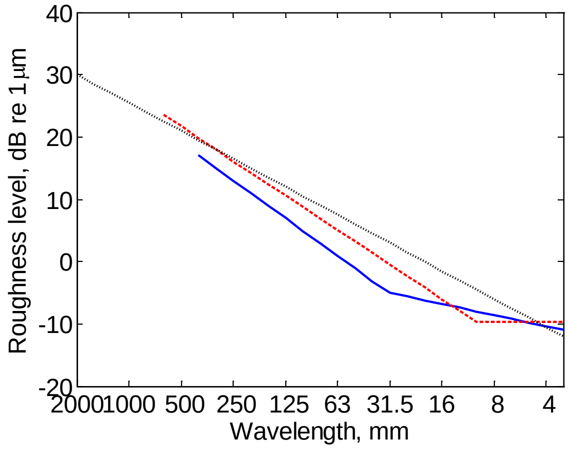

Figure 14. Roughness limits (upper bounds): —, TSI limit for rail roughness applying during vehicle type testing; – – –, rail roughness limit in ISO 3095:2005; ······ limit on combined wheel and rail roughness in Crossrail Parliamentary undertaking.

The limit in the original TSI was different for high speed and conventional rail. In the latest revision this was modified to a common limit (known as TSI+) following work in the Noemie project (Fodiman and Staiger, 2006).

It must be emphasised that these roughness limits are only intended for the purposes of defining a test site for pass-by measurements. They do not form limits for normal operation. Such limits do not normally apply. Nevertheless, Parliamentary approval for the Crossrail project in London includes an undertaking to maintain the total (combined wheel/rail) roughness spectrum below a certain threshold for the purposes of ground-borne noise mitigation (Crossrail, 2008). This is shown for comparison in Figure 14. For groundborne noise the frequency range of interest is 20 to 250 Hz. At 80 km/h this corresponds to wavelengths between 1200 and 90 mm.

In the TSIs a minimum wavelength range of 3-100 mm is specified. The upper limit is not sufficient for the lower part of the frequency range (see Table 1) but is introduced due to limitations of 1 m straight edge based instruments. A new standard EN 15610 (European Committee for Standardization, 2009) has been introduced that covers the measurement and analysis of rail roughness.

The situation is made more complicated by the fact that it is the combined wheel and rail roughness that excites the system, see Figure 1. This combined roughness affects both wheel and track noise. It is therefore possible to have a situation where a high *wheel* roughness level causes a large radiation from the *track* or vice versa. It is unclear in such a situation whether this noise should be attributed to the vehicle or the track.

Decay rate limits

In addition, within the TSIs (though not in ISO3095) there are requirements on the dynamic properties of the track which attempt to limit the acoustic radiation from the track. This is currently enforced through limits on the track decay rate. The current limits on track decay rates are shown in Figure 15; the track decay rate should exceed these values. Comparison with Figure 8 shows that a stiff pad will allow the limit for vertical decay rate to be met; this is also the case for the lateral decay rate.

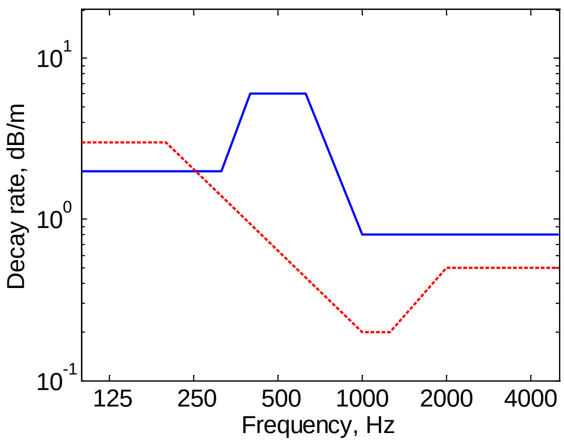

Figure 15. Track decay rate limits (lower bounds). —, vertical; – – –, lateral.

Initially the TSI-HS was based on a requirement for a certain rail pad stiffness (Fodiman and Staiger, 2006) since a stiff pad was known to minimise the contribution of the track radiation (see Figure 9). However, this was replaced with the above limit on the track decay rate as this is more directly linked to noise radiation. It would also be possible to use suitable rail dampers to achieve even higher decay rates than these limits.

The measurement method used to determine the track decay rates is also important. A European standard EN15461 (European Committee for Standardization, 2008) has been developed based on the work of (Jones et al, 2006). This uses frequency response measurements at a series of distances along the track. The decay rate Δ is then determined using

$$\Delta = \frac{4.343|A(0)|^2}{\sum_{z_i=0}^{z_{max}} |A(z_i)|^2 \, dz_i} \qquad (1)$$

where $A(0)$ is the acceleration (or mobility) at the drive point and z_i are measurement positions spaced along the track. The result corresponds to an overall equivalent decay rate that is valid for use in noise estimates. However, it relies particularly on the accuracy of the drive point frequency response function.

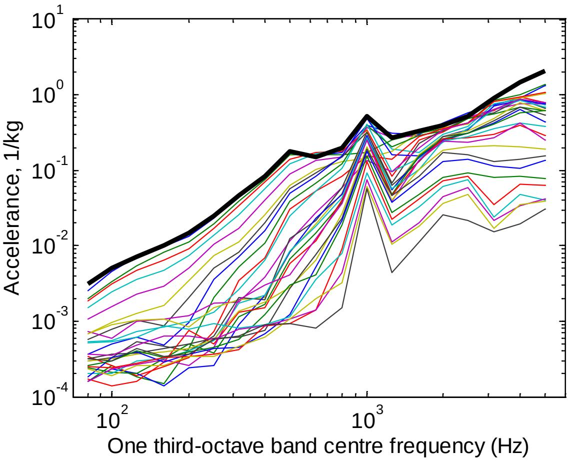

Figure 16. Examples of measured accelerations at various distances from excitation point. The thick line is the point acceleration.

Figure 16 shows examples of measured accelerations converted to one-third octave bands (these measurements were supplied as part of the German/French Stardamp project). These are all from a single site and are transfer measurements to various distances according to EN15461 (European Committee for Standardization, 2008). It can be seen that at low frequencies the measurement data is affected by measurement noise. Figure 17 shows the same data in some example frequency bands along with the decay curve obtained from equation (1). At 125 Hz it can be seen that a clear decay is obtained using data close to the drive point but the points further away should be neglected. A threshold of -15 dB has been introduced below which data points are ignored. At 800 Hz a clear double slope can be seen. This is believed to be caused by lateral waves with a low decay rate appearing in a vertical measurement but at a much lower amplitude. It should therefore not be included in the decay rate as the lateral waves are determined separately. At 5000 Hz the decay curve appears much steeper than the data points. This is caused by a much larger value of acceleration at the drive point than at adjacent points. The result is nevertheless correct according to the calculation procedure and gives an equivalent decay rate that is valid for use in noise estimates.

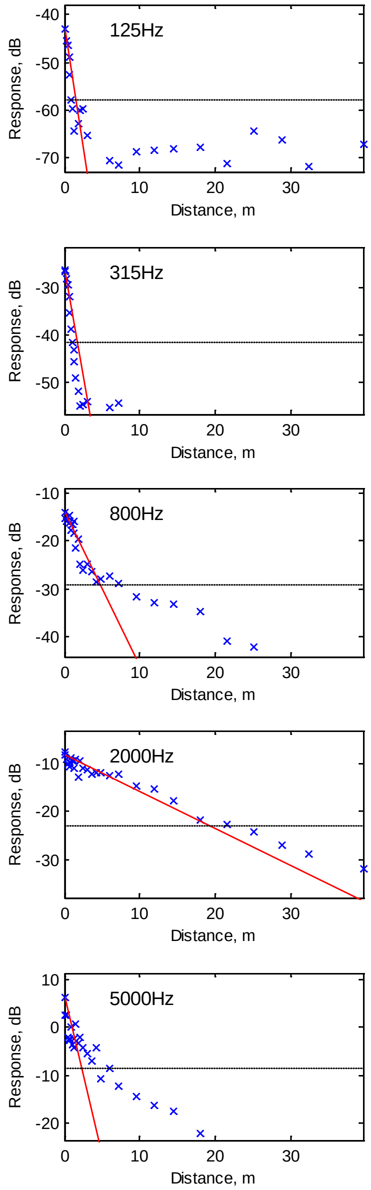

Figure 17. Examples of decay curves obtained from equation (1) and measured data.

TOWARDS VIRTUAL TESTING

Every new vehicle type is required to undergo type approval testing including the acoustic tests. This is expensive and difficult logistically – there are only a few test sites which are approved for use in carrying out such acoustic tests. Moreover, the rail roughness and decay rates must be controlled regularly. In other fields, such as crashworthiness and vehicle dynamics, work is underway to allow certification on the basis of calculations, either partially or in full, in place of testing. For acoustic tests there is probably too much uncertainty to allow a completely virtual approval process. For example, how would the wheel roughness of a new vehicle type be identified?

Nevertheless there is scope for a partial use of 'virtual testing' to replace some of the field testing. Example scenarios include the following:

1. A train type is certified comprising a three car multiple unit with two powered bogies. A new design variant with four cars and three powered bogies could be certi-

fied on the basis of calculations in combination with the original measurements.

2. A diesel train type is certified on the basis of one type of diesel engine. A new design variant with a different diesel engine could be certified on the basis of test bench measurements on the engines in combination with a model for the overall train noise.
3. A train type is certified on the basis of one wheel design. A new wheel design is to be introduced that is claimed to produce less noise. This could be certified on the basis of calculations, possibly in combination with laboratory tests on the new (and old) wheel designs.
4. A test site does not comply with the strict requirements of the TSI, for example the roughness exceeds the limit in some wavelength bands. Can the measured results nevertheless be used to establish whether the vehicle is compliant?
5. The pantograph is replaced by a new design. Will this have an influence on the pass-by noise?

To achieve this, a framework is needed to ensure that any models that are used can be verified. This includes models for the source terms, such as TWINS, but also overall models for the whole train that include the various sources and the propagation paths to a wayside receiver. As well as rolling noise, provision is required for models of other sources such as engine noise, electrical equipment, fans and aerodynamic noise. This approach forms the basis of the project Acoutrain which is expected to be launched in the Autumn of 2011. This is supported financially by the EU under its Framework 7 Programme and will be led by UNIFE with technical coordination by SNCF and Bombardier.

CONCLUSIONS

Theoretical models for rolling noise are now well established and have been validated by several experimental campaigns. Although some uncertainty remains in the predicted noise levels this is associated particularly with uncertainty in the roughness inputs. The models can therefore be used with confidence in the design of quieter wheels and tracks, for example rail dampers or new wheel designs. The models have also formed the backdrop to new regulations for vehicle noise such as the TSI pass-by tests. For the future the models also have the potential to allow partial replacement of testing by calculations in various ‘virtual testing’ scenarios. But it should not be expected that testing can be eliminated altogether.

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