Acoustic source beam control using a compact virtual loudspeaker array

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ABSTRACT
The directivity of an electroacoustic sound source can be controlled by dividing its surface into independently controlled elements, and there have been several approximately spherical loudspeaker arrays developed from this idea recently. This paper describes a virtual dodecahedral loudspeaker array, which is achieved using a sealed loudspeaker possessing a single driver (through successive measurements). We compare the measured spatial response of this loudspeaker to a computer modelled loudspeaker array in terms of the applicable spherical harmonics.

INTRODUCTION
There has been a growing interest in controlled directivity sound sources for acoustic measurement over the past decade, complementing the much more rapidly growing developments in high-order microphone (Noisternig et al. 2011). Such transducers use an array of discrete transmitters or receivers, often around a sphere (or a Platonic solid approximation of a sphere). The spatial control of these transducers can be generically encoded using a spherical harmonic series (Williams 1999), using techniques that are often referred to as higher-order Ambisonics (HOA). The theoretically possible HOA order increases by one as the number of transmitters or receivers is increased to the next squared integer: i.e., four elements can yield first-order, nine elements can yield second order, sixteen can yield third order, etc. As an alternative to spherical harmonics, the transducer can be controlled in terms of its acoustic radiation modes (ARMs), which have the advantage of preserving the degrees of freedom established by the number of transducer elements, and the disadvantage of a transducer-specific encoding, which may be frequency dependent (Pasqual et al. 2010; Zotter and Pasqual 2011). In addition to considering the number of transducer elements, the spacing between elements determines a spatial aliasing wavelength (with a corresponding frequency) – in short, there must be at least two sampling points per wavelength. A third consideration is the radius of the array, because radiation efficiency becomes vanishingly small for high order spherical harmonics when the product of wave number and radius is small (e.g., <1). In this study we examine the potential of a small dodecahedral loudspeaker array for directivity control using spherical harmonics. The unusual characteristic of our loudspeaker is that it has just a single driver, and we create the virtual loudspeaker array by conducting successive measurements with the loudspeaker oriented onto each of the twelve faces.

Examples of previously reported high order directivity-controlled loudspeakers can be found in the work of Avizienis et al. (2006), Zotter (2009), Pollow and Behler (2009), Pasqual et al. (2010), Rafaely (2009), and others. Many of these approaches use Platonic solid loudspeaker driver configurations, especially the dodecahedron (12 faces) and icosaehedron (20 faces). One departure from this is found in the work of Avizienis et al. (2006), who developed a 120-element loudspeaker (albeit configured on an icosahedron).

With a high order microphone, a single impulse response measurement delivers all of the spatial information that the microphone can gather from the soundfield. However, with a high order loudspeaker, successive measurements must be made, because the sound from each element mixes with that of the others. These successive measurements could be done in a number of ways, and the most straightforward is to measure from one face at a time. Another possibility is to synthesise a succession of spherical harmonics – and this approach has the potential to reduce the number of measurements (for example, in the case of a dodecahedral array, nine measurements would be needed, instead of twelve). For our prototype loudspeaker, only the first option is available, and the spherical harmonic radiation patterns are derived post hoc. The inevitability of successive measurements means that this measurement technique is necessarily vulnerable to time variance in the environment around the loudspeaker, even if the loudspeaker is not repositioned between successive measurements. Repositioning the loudspeaker increases the risk of time-variance error, and so this must be done with great care.

A loudspeaker with a single driver has advantages: it is very easy and inexpensive to construct, and so it provides easy access to the possibility of controlled directivity source measurements of acoustic environments. A further potential advantage of the single driver loudspeaker is that there is no possibility of ‘cross-talk’ (i.e., the movement of one driver affecting the movement of other drivers), and so there is no need for a matrix of cross-talk cancellation filters.

Used in conjunction with a high order microphone, a high order loudspeaker could be used to derive a matrix of room impulse responses from source to receiver. For example if both source and receiver were configured for second-order Ambisonics, then the resulting matrix would have nine loudspeaker radiation patterns and nine microphone reception patterns (expressed as spherical harmonics), yielding 81 room impulse responses in the matrix. With such a matrix, the directivity of both source and receiver could be manipulated post hoc, and it would be a simple matter to have these directivities varying in time for the purpose of source/receiver simulation (e.g., to simulate the time varying directivity of a musical instrument or human voice, together with the time varying directivity of a listener’s ears with incidental head rotations). This matrix grows rapidly as the Ambisonics order
is increased (e.g. 256 RIRs for 3rd order, 625 RIRs for 4th order, 1296 RIRs for 5th order).

While the icosahedron is the Platonic solid with the most faces, and is capable of synthesising spherical harmonics for 3rd order Ambisonics, the dodecahedron does have some advantages (even though it is limited to 2nd order Ambisonics). As shown by Pasqual et al. (2010), the dodecahedron is the Platonic solid with the greatest proportion of its surface available for radiation from circular transducers, and so has greatest radiation efficiency for a given radius.

**PROTOTYPE DESIGN AND CONSTRUCTION**

We chose to construct our loudspeaker enclosure as a dodecahedron, rather than as a sphere, as a plastic dodecahedral box was readily available to us. An advantage of the dodecahedron is that it is easy to see how it should be rotated during the twelve successive measurements. The plastic box is 77 mm between opposite faces. One of the faces was almost entirely replaced by the loudspeaker driver. Given these dimensions, the spatial aliasing frequency is about 3.44 kHz. Prior to the insertion of the driver, mass and stiffness was added to the box by gluing large steel washers on the interior eleven remaining faces and adding a layer of resin over the entire interior surfaces. Sound absorptive material was inserted into the box. Steel (ferromagnetic) screws were mounted on each of the twenty vertices for the magnetic suspension system described below.

The driver used was an Aurasound type 4190 microphones. For each measurement orientation, six sweeps, each 58 s in duration, were recorded, and the final impulse response for each receiver position was derived from the synchronous average of the six. Measurements were made in a quiet anechoic room. From these measurements, using 64-bit floating point processing, we derived impulse responses with a signal to noise ratio of about 80 dB.

The loudspeaker has an axis of symmetry that runs through the centre of the driver to the centre of the opposite face. Hence, in a free field there are four non-redundant face orientations if the loudspeaker is to be repositioned onto each of its twelve faces. Therefore, we conducted free field measurements on the four non-redundant orientations. The six microphones were arranged so that one was in line with the centre of a face, and the remaining five microphones were towards each of the five vertices of the same face (Figure 2). As a result, there are some redundant microphone positions, which depend on the orientation of the loudspeaker, shown in Fig. 2. In total there are twelve non-redundant measurements.

**COMPUTER SIMULATION**

Computer simulation was done using a ‘spherical caps’ concept similar to the approaches of Zotter et al. (2007), Pollow and Behler (2009) and Aarts and Janssen (2011). That is, each driver on the virtual dodecahedral array was modelled as a convex circular piston on a sphere. This was done by using an analytic expression of the transfer function between a point on a sphere (approximating the dodecahedron) and a point 2 m from the sphere centre, taking into account the diffraction around a solid sphere, after Williams (1999). The surface area of the piston was created by making a dense array of such points, forming a circle that approximated the dodecahedron (shown as a partial net). Redundant microphone positions are linked by red lines.

**LOUDSPEAKER MEASUREMENTS**

Measurements of the loudspeaker response were made at a distance of 2 m from the dodecahedron centre. Impulse responses were derived by analysing a swept sinusoid signal (logarithmic, 50 Hz – 20 kHz) that was played by the loudspeaker and recorded simultaneously by six Brüel & Kjær type 4190 microphones. For each measurement orientation, six sweeps, each 58 s in duration, were recorded, and the final impulse response for each receiver position was derived from the synchronous average of the six. Measurements were made in a quiet anechoic room. From these measurements, using 64-bit floating point processing, we derived impulse responses with a signal to noise ratio of about 80 dB.

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**Figure 1.** Photograph of the prototype sound source, showing the magnetic suspension system and the laser-guided positioning (the red spots).

We developed an acoustically transparent suspension system for the loudspeaker. Three neodymium magnets, attached to nylon lines, were suspended so that the magnets could support the loudspeaker consistently in any of the twelve orientations. In order to verify that the loudspeaker was suspended correctly after reorienting it, two fixed lasers were used – one horizontal and one vertical (from below). In the absence of the loudspeaker, these laser beams intersected. In the presence of the loudspeaker, the horizontal laser shone in the centre of a face, and the vertical laser shone on a vertex.

**Figure 2.** Microphone positions (red dots) expressed as equivalent angles from the dodecahedron centre on each of the four non-redundant faces of the dodecahedron (shown as a partial net). Redundant microphone positions are linked by red lines.
RESULTS

If the measurements explained in the Loudspeaker Measurements section were to be distributed around a sphere, seventy-two receiver positions would be obtained (six measurement positions per dodecahedron face). These seventy-two measurement positions were replicated in the computer simulation. In order to test the performance of the simulation and the actual loudspeaker, comparisons are made with the values obtained for theoretical first and second order spherical harmonics.

Impulse responses were obtained in both the simulation and loudspeaker measurements at each of the seventy-two receiver positions by adding the impulse responses from on the actual driver or simulation points on the modelled driver. Prior to this, gain corrections were applied in order to create the desired spherical harmonic patterns to each driver or, in the case of the computer simulation, clusters of points representing each driver.

Correlation coefficient was used as a measure of similarity between the theoretical response and the actual response of the simulation and loudspeaker measurements. The correlation coefficients were calculated on an octave band basis, performed on the value calculated by obtaining the RMS of the entire impulse response and then normalising to the highest value (thus obtaining values ranging from 0 to 1). Correlation coefficients were not calculated for the 0th order harmonic as the ideal values are constant and therefore correlation cannot be determined.

For the simulation case, this allows us to understand the theoretical usable frequency range for a loudspeaker array with the characteristics of the one presented here. For the measurement case, the correlation coefficient allows us to understand the performance of the array, bearing in mind that spherical harmonics will be used as the basis for directivity control in the future.

For reference, Figure 3 illustrates the spherical harmonics up to the second order, which are usually referred to by $Y_n^m$, where $n$ is the order, and $m$ is the degree.

![Figure 3. Visualisation of spherical harmonics, up to the second order. Colour indicates phase.](Image)

### Table 1. Correlation coefficients calculated for first and second order harmonics between the computer simulation and the theoretical values.

<table>
<thead>
<tr>
<th>Freq(Hz)</th>
<th>$Y_0^0$</th>
<th>$Y_1^0$</th>
<th>$Y_1^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>125</td>
<td>0.0003</td>
<td>0.4331</td>
<td>0.4676</td>
</tr>
<tr>
<td>250</td>
<td>0.9999</td>
<td>0.9998</td>
<td>0.9897</td>
</tr>
<tr>
<td>500</td>
<td>0.9999</td>
<td>0.9999</td>
<td>0.9942</td>
</tr>
<tr>
<td>1k</td>
<td>1</td>
<td>1</td>
<td>0.9982</td>
</tr>
<tr>
<td>2k</td>
<td>1</td>
<td>1</td>
<td>0.9991</td>
</tr>
<tr>
<td>4k</td>
<td>0.9999</td>
<td>1</td>
<td>0.9993</td>
</tr>
</tbody>
</table>

### Table 2. Correlation coefficients calculated for first and second order harmonics between spherical harmonics synthesised from the measurements and the corresponding theoretical values.

<table>
<thead>
<tr>
<th>Freq(Hz)</th>
<th>$Y_0^0$</th>
<th>$Y_1^0$</th>
<th>$Y_1^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>125</td>
<td>0.4275</td>
<td>0.3848</td>
<td>0.0175</td>
</tr>
<tr>
<td>250</td>
<td>0.6749</td>
<td>0.5304</td>
<td>0.5556</td>
</tr>
<tr>
<td>500</td>
<td>0.6208</td>
<td>0.5415</td>
<td>0.438</td>
</tr>
<tr>
<td>1k</td>
<td>0.994</td>
<td>0.9264</td>
<td>0.9928</td>
</tr>
<tr>
<td>2k</td>
<td>0.9967</td>
<td>0.9926</td>
<td>0.9977</td>
</tr>
<tr>
<td>4k</td>
<td>0.9929</td>
<td>0.9943</td>
<td>0.9914</td>
</tr>
</tbody>
</table>

A more detailed characterisation of the computer simulation was also performed by calculating impulse response to a grid of 961 points around the source, also at 2 m. The deviation from ideal values was calculated, again, by comparison with the theoretical response of the first and second order spherical harmonics. As in the correlation coefficient calculation, values were obtained by calculating RMS of each impulse response and then normalising to the maximum value. Re-
Results are presented in Figures 4-6. The figures show the shape of the response obtained. The colour-coding represents absolute error from the ideal value, also normalised to 1. Error is shown in green to gray for values from 0 to 0.5, and errors greater than 0.5 are coded monotonically in red. Results are presented for three orders of spherical harmonics and one degree for each order; results are presented at 125 Hz, 500 Hz and 2 kHz. The zeroth order spherical harmonic is well rendered at all three frequencies (Fig 4). The first order spherical harmonic is well rendered at 500 Hz and 2 kHz, but the directivity pattern at 125 Hz bears little resemblance to the desired pattern (Fig 5). The second order spherical harmonic pattern is only well rendered at 2 kHz (Fig 6).

DISCUSSION

As demonstrated in the simulation and measurement results, a loudspeaker as compact as ours will have problems at lower frequencies when trying to reproduce spherical harmonic patterns, making the start of the range of usability the 250 Hz octave band for 1st order harmonics, and 1000 Hz for 2nd order harmonics. This is taken from the correlation coefficient results and graphically shown in the results from the simulation at a high-density sampling grid (Fig 4-6). It should be noted that this could be corrected with individual frequency dependent filtering for each driver. For this paper only gain has been taken into account to produce the spherical harmonic patterns investigated.

Figure 4. Directivity patterns for 0th order spherical harmonic simulation. Colour-coding shows absolute error from normalised values.

Figure 5. Directivity patterns for 1st order spherical harmonic simulation. Colour-coding shows absolute error from normalised values.

Figure 6. Directivity patterns for 2nd order spherical harmonic simulation. Colour-coding shows absolute error from normalised values.
The small size of our loudspeaker means that it has relatively poor radiation efficiency at low frequencies, especially for the second order spherical harmonics. In quiet environments, perhaps this can be overcome by making long duration measurements with optimised gain structure. However, it should be borne in mind that the measurements presented here were 6 minutes in duration (each), and the anechoic room was very quiet – and yet the rendering of second order spherical harmonics was poor. These errors could be caused by several other factors, including misalignment of measurement microphones, misalignment of the loudspeaker as it is rotated, differences in driver directivity from theoretical approximation and inappropriate driver and enclosure damping. These imperfections would be hard to quantify and correct with signal processing, however improvements could be made that include correction of off-axis response of the driver and time misalignment. Ultimately, a larger radius loudspeaker would be preferable as a directivity-controlled measurement source.

CONCLUSION

A loudspeaker capable of providing directivity pattern control by using a single driver and multiple measurements has been presented. The theoretical constraints of the loudspeaker configuration presented have been explored by a computer simulation. Also, measurements of the device have been presented and areas of improvement to the device under test have been identified. Possible avenues for future research include exploring the beam-forming capabilities of the device in its current configuration, physical improvements of the device and exploring the possibility of constructing a loudspeaker with different characteristics that could help improve its low frequency beam-forming capabilities.

REFERENCES


