On Robustness of Constrained Least Mean Square Beamformer

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ABSTRACT

Adaptive beamformers suffer from performance degradation when the assumptions made of the environment, signal sources, or sensor array, are violated. This paper investigates the robustness of the constrained least mean square beamformer with respect to its adaptive step size, in the presence of model errors and direction of arrival mismatches. A sacrifice in convergence speed can be used to improve the robustness of the algorithm, as simulation results show improved performance in the presence of errors as the algorithms step size is made small. A comparison is made between the effect of the popular diagonal loading method and a reduced step size on the robustness of the algorithm, showing improved results with a reduced step size.

INTRODUCTION

The constrained least mean square beamformer is a widely used adaptive beamforming algorithm, effective in producing a signal originating from a direction of interest with high fidelity in the presence of strong interferences. Fields in which the technique is applicable include wireless communications and networking, sonar, radar, microphone array speech processing, and medical imaging. A disadvantage exists as adaptive beamformers suffer from performance degradation when the assumptions made of the environment, signal sources, or sensor array, are violated. Such violations may be caused by array look direction errors, imperfect sensor array calibration, wavefront distortions and local scattering effects. The error sources other than look direction errors are referred to collectively here as 'model errors', as they cause discrepancies in the signal environment model used originally to develop the adaptive beamformer. That is, the assumption that signals arrive at the sensor array in plane waves is made invalid by these errors.

Recently, there have been many publications on the methods of robust adaptive beamforming (Gershman, 2003; Lin, Li & Jin, 2010). Particular focus is placed on the popular ad-hoc method of diagonal loading (Laseetha & Sukanesh, 2011), while others develop new approaches toward a robust adaptive algorithm (El-Keyi, Kirubarajan & Gershman, 2006; Landau, Lamare, Wang, and Haardt, 2011). The problem with diagonal loading is that there is no rigorous method for determining the magnitude of the loading, whereas new algorithms necessitate a significant performance improvement over existing algorithms to justify implementation.

In this paper, we investigate how the widely used constrained least mean square beamformer can be made robust by reducing the algorithms adaptive step size. A sacrifice in convergence speed can be used to improve the robustness of the algorithm. The authors analyse the effectiveness of the algorithm in the presence of both look-direction errors and model errors as the step size is made small, and compare the performance to that produced through the use of the diagonal loading method.

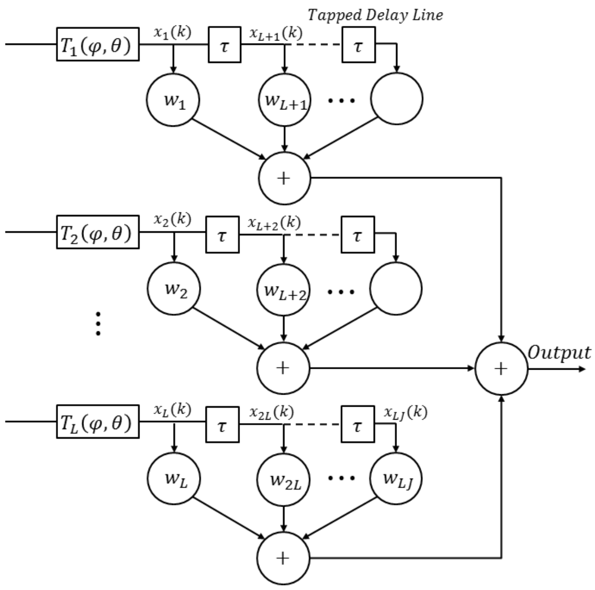

Figure 1. Broadband Beamformer Structure

BEAMFORMING PROBLEM FORMULATION

Constrained Least Mean Square Beamformer

The beamforming algorithm used in this study is the Constrained Least Mean Square or Constrained LMS algorithm developed by Frost (1972). This is a simple stochastic gradient-descent algorithm that in the adaptive process progressively learns the statistics of noise arriving from directions other than the look direction, minimising the noise power in these directions. Noise arriving from the look direction may be filtered out by a suitable choice of the frequency response characteristic in that direction.

The beamforming processor structure, depicted in Figure 1, with L sensors and J taps per sensor has a vector of tap voltages $X(k)$ such that

$$\boldsymbol{X}^T(k) = [x_1(k), \quad x_2(k), \quad \dots, \quad x_{LJ}(k)]. \qquad (1)$$

The vector of weights at each tap is

$$\boldsymbol{W}^T = [w_1, \quad w_2, \quad \cdots, \quad w_{LJ}]. \tag{2}$$

The output of the array is then

$$y(k) = \boldsymbol{W}^T \boldsymbol{X}(k). \tag{3}$$

The stochastic constrained LMS algorithm developed in Frost (1972) is

$$\boldsymbol{W}(0) = \boldsymbol{F} \tag{4}$$

$$\boldsymbol{W}(k+1) = \boldsymbol{P}[\boldsymbol{W}(k) - \mu\, y(k)\boldsymbol{X}(k)] + \boldsymbol{F} \tag{5}$$

where P is an LJ x LJ matrix determined by the constraints on each vertical set of weights, F is an LJ-dimensional vector specifying the frequency response in the look direction, and μ is the algorithms convergence coefficient.

Diagonal Loading

Diagonal loading is a popular and effective ad hoc method of imparting robustness on an adaptive beamformer to look direction mismatches and model errors (Elnashar, Elnoubi, & El-Mikati, 2006). The technique involves adding a small constant to the main diagonal of the array covariance matrix. For this algorithm, we use a transient estimate of the covariance matrix; diagonal loading is applied with a modification of the weight update Equation (5)

$$\boldsymbol{W}(k+1) = \boldsymbol{P}[(1-\mu\gamma)\boldsymbol{W}(k) - \mu\, y(k)\boldsymbol{X}(k)] + \boldsymbol{F} \tag{6}$$

where γ is a loading constant.

Algorithm Step Size

The convergence coefficient used in this study is data-dependent, consistent with the common Normalised Least Mean Square algorithm, while incorporating an independent and arbitrary forgetting factor to account for changing signal environments

$$P_X(0) = 0 \tag{7}$$

$$P_X(k+1) = \boldsymbol{X}^T(k+1)\boldsymbol{X}(k+1) + \lambda\, P_X(k) \tag{8}$$

$$\mu = \frac{\mu_c}{P_X(k+1)} \tag{9}$$

where P_x is the power of the array response, λ is the forgetting factor, and μ_c is a constant.

By manipulating μ_c we can select a convergence coefficient as required, and relate this to the performance of the beamformer.

SIMULATIONS

The aim of the simulations is to explore the relationship between the Constrained Least Mean Square algorithm (CLMS) step size and performance under practical conditions where look-direction and model errors will likely exist. The performance metric used is the Signal-to-Interference-and-Noise-Ratio (SINR).

The array considered consists of 16 sensors with a uniform linear inter-element spacing of 0.17m. The speed of sound was taken as 340m/s and a sampling frequency of 4000Hz was used. The design frequency of this array is then 1000Hz. The number of taps in the FIR filters of the beamformer is 16. This gives a sufficiently high level of performance considering the increased computational expense for marginally higher performance that an increased number of taps would provide. The signal incident upon the array consists of four tonal components (200Hz, 400Hz, 600Hz, and 800Hz). This was chosen so as to span the low, mid and high frequencies within the design frequency of the array. There are two interferences, each also with 4 tonal components, (220Hz, 420Hz, 620Hz and 820Hz) and (180Hz, 380Hz, 580Hz, and 780Hz) respectively, both at 10dB relative to the signal. A uniformly distributed random noise of 0 dB relative to the signal is also added. The length of the considered signal is restricted both to ensure that the algorithm converges within a practical length of time, and to minimise the computational requirement of the simulation.

Each of the scenarios described below are simulated for a range of μ_c, with and without a near optimal level of diagonal loading, as the determination of an optimal loading constant without prior knowledge of array response characteristics is still a research area. CLMS(0) denotes the case with no diagonal loading, while CLMS denotes with loading.

Scenario 1

The signal arrives from broadside (perpendicular to the array), with interferences equally spaced either side with an angle of separation of 20° from the signal. This represents a situation with no look-direction mismatch. No model error is introduced, giving a best case scenario for adaptive beamforming.

The results of this scenario are shown in Figure 2. As there are no error sources introduced, the performance of the beamformer is near optimal. This represents the best performance that this beamformer is able to produce, given the levels of noise and interference used here. Loading in this scenario causes a performance reduction, due to the adjustment made to the algorithm distancing filter weights from their optimum. The algorithm step size has no impact on performance in this scenario.

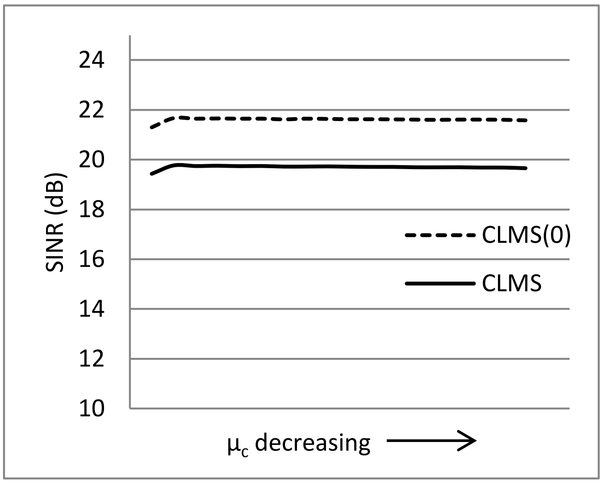

Figure 2. Scenario 1 - No look-direction mismatch, no model error

Scenario 2

The signal arrives from 3.6° (from a line perpendicular to the array), with interferences equally spaced either side with an angle of separation of 20° from the signal. This represents a maximum look-direction mismatch, as the signal arrives from directly between two discrete beamsteering directions. In this situation, the look-direction is perpendicular to the array. No model error is introduced.

The results shown in Figure 3 depict that as the convergence coefficient is reduced, performance improves to the point that a non-optimal diagonal loading causes performance degradation. Performance with diagonal loading is almost constant for decreasing μ_c. Notice that the introduction of a look-direction error produces a significant performance reduction from scenario 1.

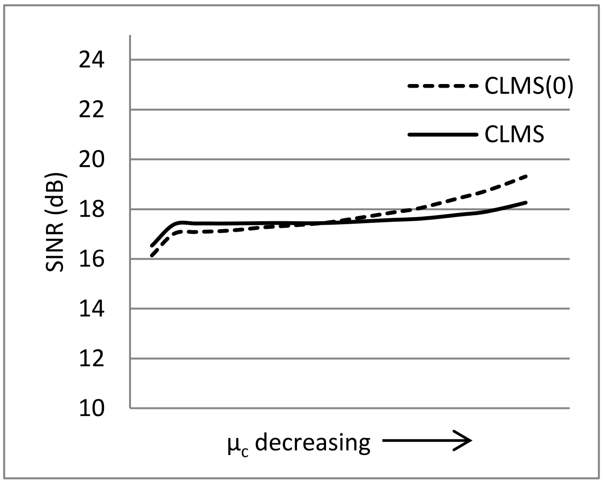

Figure 3. Scenario 2 - Look-direction mismatch, no model error

Scenario 3

The signal arrives from broadside (perpendicular to the array), with interferences equally spaced either side with an angle of separation of 20° from the signal and a maximum model error of 3.6° is introduced. This is chosen to be an error similar in magnitude to the look-direction mismatch. The model error is introduced in simulation by placing a random phase adjustment with a maximum value of 3.6° to the sensor delays, creating non-planar waves incident upon the array.

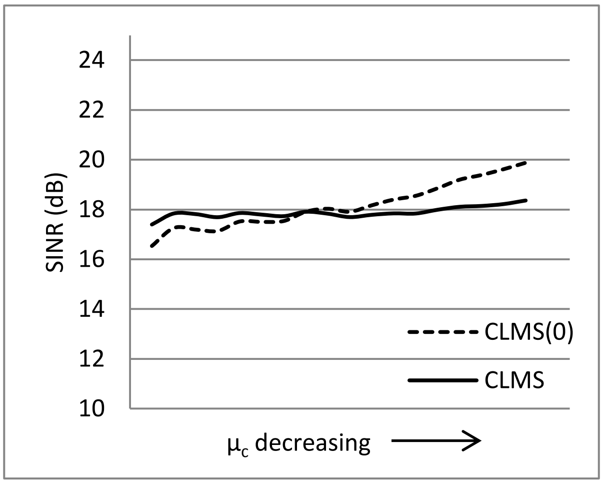

Figure 4. Scenario 3 - No look-direction mismatch, with model error

The results are shown in Figure 4. Similar to scenario 2, when there is no loading, the performance reduction caused by the model error is minimised as the convergence coefficient decreases. Again the performance level with loading is consistent for all μ_c, and there exists a μ_c beyond which diagonal loading becomes detrimental.

Scenario 4

The parameters in this scenario are the same as in scenario 3 except that the signal arrives from 3.6°, as in scenario 2.

The results shown in Figure 5 reflect those in scenarios 2 and 3, except that the cumulative effect of the look-direction mismatch and model error produce a greater performance reduction.

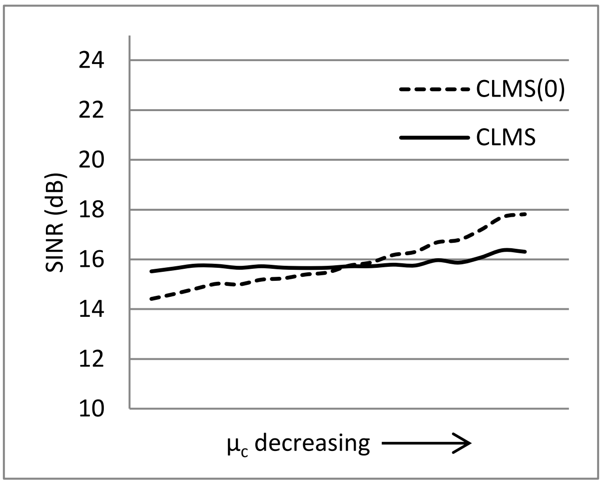

Figure 5. Scenario 4 - Look-direction mismatch, with model error

Convergence Speed

The speed at which the algorithm converges depends upon μ_c. Figure 6 shows the number of samples required for the algorithm to converge, with and without loading. Scenario 4 is used, as the maximum error conditions burden the algorithm with the longest convergence times.

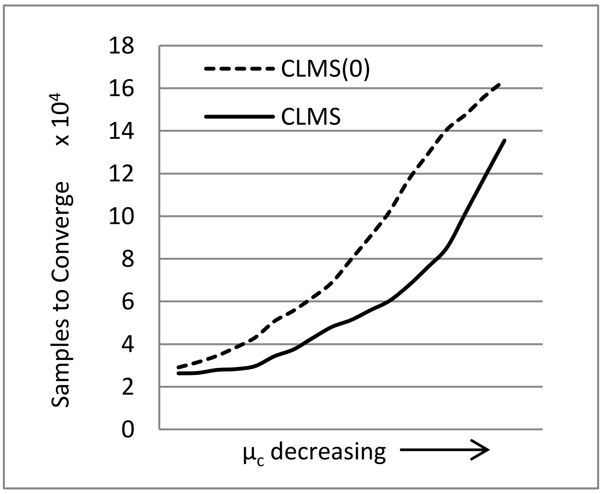

Figure 6. Number of samples required for adaptive weights to converge

The results show that diagonal loading can improve the convergence speed of the algorithm. For example, in Figure 5, where there exists a μ_c where performance is equivalent with and without loading, the loading improves the convergence speed of the algorithm, as indicated by Figure 6.

Optimal Diagonal Loading

A repetition of scenario 4 with optimal diagonal loading constants is performed. An optimal loading constant is determined by simulation, varying the loading until peak performance is consistently achieved over multiple trials. The results given in Table 1 show that as μ_c is reduced, the optimal diagonal loading constant becomes small. The performance improvement also becomes minimal. Assuming that the algorithm is able to converge within an appropriate time as required by the application, this indicates that with a sufficiently small convergence coefficient there is little motivation to determine an appropriate diagonal loading.

Table 1. Optimal diagonal loading

	$\mu_{c\ (max)}$	$\mu_{c\ (min)}$
Loading Constant	2.5	0.3
Performance Increase	11.4%	0.11%

CONCLUSION

This paper has examined the performance of the Frost constrained least mean square beamformer with and without diagonal loading with decreasing adaptive step size. Simulations consider the presence of strong interferences, look-direction mismatches, and adaptive beamformer model errors. Results show that with a sufficiently small adaptive step size, diagonal loading becomes unnecessary, and if not optimal, becomes a source of performance degradation. Again with sufficiently small adaptive step size, the benefit of diagonal loading becomes primarily an increased algorithm convergence speed. This paper has shown that robustness can be improved not only with diagonal loading, but also with a sacrifice in convergence speed, yielding improved performance. Future work would be to determine the mechanism behind this improved performance.

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