Sound radiation of a plate into a reverberant water tank

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ABSTRACT

This paper presents a study on sound radiation from a finite plate into a reverberant water tank and its dependence on the modal coupling of the plate's modes through their radiating field. The results of this study are compared with that of sound radiation of the plate into a free space for establishing a general understanding of the effects of fluid loading on the structural vibration and sound radiation into various spaces.

INTRODUCTION

The acoustics of underwater structures are an interesting topic because the effect of structural modal coupling through underwater sound radiation on the structural response and sound radiation involves rich physical mechanisms. Previous experimental investigation (Liu *et al.* 2010) into the vibration and sound radiation of a torpedo-shaped structure in air aimed at searching for experimental evidence to: (1) support the results from FEA modelling (Merz *et al.* 2009); (2) identify new problems which are important to the structural vibration and sound radiation but are overlooked in the existing models; (3) measure the modal characteristics of the structure in air (defined as the dry structure) for comparison with that of the same structure in water.

Recent work (Pan *et al.* 2011) on the measurement of the underwater vibration response and near-field radiated sound pressure of the torpedo-shaped structure has demonstrated changes in the structure's natural frequencies and mode shapes due to modal coupling through fluid loading. The vibration characteristics of a finite plate mounted on a rigid baffle and radiating sound into a semi-infinite space were used to illustrate their dependence on the fluid-loading and the participation of modal couplings through their sound radiation.

One of the issues involved in studying underwater structural vibration and sound radiation is that their characteristics are strongly dependent on the nature of the underwater space where the structure radiates sound. Even in open sea conditions, reflection of sound by the sea bed, surface or shore may lead to difficulties in interpreting the results. There are always practical limitations involved in obtaining full information about structural vibration and sound radiation in an open sea environment.

On the other hand, experimental tests in a water tank allow for the possibility of measuring the distributed vibration velocity vectors on the structure's surface using a 3D laser vibrometer and the radiated sound field using scanning hydrophones. Such a laboratory experiment is also unaffected by environmental conditions. However, due to the limitations of the size of the water tank and the surface reflections, the modal coupling between the structural modes may be dominated by a different physical mechanism, *i.e.,* coupling between structural-acoustical modes. Therefore a question is raised about whether a study of the structural radiation into a reverberant tank has any practical benefits for the understanding of structural sound radiation in more realistic underwater conditions (such as in free spaces or large water channels).

This paper presents a preliminary analysis of a plate's sound radiation into a reverberant water tank. By comparing the results with the structural vibration and sound radiation into a free space, we are able to demonstrate that the effects of different spaces on the sound radiation and structural vibration can be described by a set of self- and cross-modal radiation impedances of the plate. We found that:

(1) The modal radiation impedances are determined by the modal characteristics of both the plate and the acoustical space where the plate radiates sound, and by the plate-acoustical coupling.

(2) The modal radiation impedances determine the structural vibration and sound radiation.

Thus it appears that a theoretical study of sound radiation into water tanks and its experimental verification may lead to some progress in this area of research.

MODEL DESCRIPTION

A two-dimensional finite plate of thickness h , coupled with the water in an enclosed region of $(-\frac{L_x}{2} \le x \le \frac{L_x}{2}, 0 \le y \le L_y)$ of a water tank, is shown in Figure 1. The plate is simply supported at $x = 0$ and $x = L$ ($L \le L_x / 2$) by rigid baffles of infinite size and is excited by a line force $F_o e^{-j\omega t}$ at $x = x_o$.

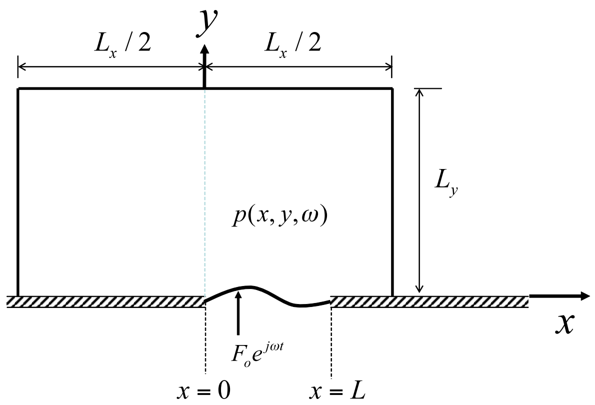


Figure 1: A finite plate in an infinite baffle loaded by a reverberant sound pressure in a water tank (top view).

We express the displacement of the plate by the dry mode shapes of the plate vibration:

$$w(x,\omega) = \sum_n W_n \sin(k_n x) \quad (2)$$

and the sound pressure in the water tank by rigid wall mode shapes:

$$p(x,y,\omega) = \sum_{lm} P_{lm} \cos(k_l x + l\pi/2)\cos(k_m y) \quad (3)$$

where $k_n = n\pi / L$, $k_l = l\pi / L_x$ and $k_m = m\pi / L_y$ are the modal wavenumbers of the plate and water tank. Using modal coupling analysis of the plate and sound wave equations (Pan and Bies 1990a), the coefficients of the plate displacement are described by the following coupled equation (4):

$$(k_n^4 - k_p^4)W_n - \mu k_p^4 \sum_{n'} I_{nn'} W_{n'} = \frac{2F_o}{DL}\sin(k_n x_o), \quad (4)$$

where $\mu = \rho_o / \rho_p h$ represents fluid-loading and $k_p = (\rho_p h\omega^2 / D)^{1/4}$ is the flexural wavenumber of the plate in a vacuum. Other system parameters are $D = Eh^3 / [12(1-\nu^2)]$ as the plate's bending stiffness, and ρ_p, E and ν as the plate's density per unit area, Young's modulus and Poisson's ratio, respectively. The internal damping of the plate is included in the system model by using a complex Young's modulus $E^* = E(1 + j\eta)$, where η is the structural loss factor. The important terms in Equation (4) are the plate's modal coupling factors, which represent the interaction between the n^{th} and n'^{th} dry modes through their interaction with the enclosed sound field:

$$I_{nn'} = \frac{2}{L}\sum_{lm} \frac{J_{lm,n} J_{lm,n'}}{\varepsilon_l \varepsilon_m (k^2 - k_{lm}^2)}, \quad (5)$$

where $k_{lm} = \sqrt{k_l^2 + k_m^2}$, $k = \omega / c_o$ and:

$$\varepsilon_l = \begin{cases} L_x/2, & l \neq 0 \\ L_x, & l = 0 \end{cases}, \text{ and} \quad (5a)$$

$$\varepsilon_m = \begin{cases} L_y/2, & m \neq 0 \\ L_y, & m = 0 \end{cases}. \quad (5b)$$

The coupling coefficient between the $(l,m)^{th}$ tank mode and the n^{th} plate mode is determined by the following mode shape integration:

$$J_{lm,n} = \int_0^L \cos(k_l x + l\pi/2)\sin(k_n x)dx. \quad (6)$$

Then the $(l,m)^{th}$ modal amplitude of the enclosed sound pressure is determined by the plate displacement as:

$$P_{lm} = -\frac{\rho_o \omega^2}{\varepsilon_l \varepsilon_m (k^2 - k_{lm}^2)} \sum_{n'} J_{lm,n'} W_{n'}. \quad (7)$$

The sound power (per unit length in the *z*-direction) radiated from the vibrating plate can be determined by integrating the complex sound intensity over the plate surface:

$$\overline{W} = \frac{j\rho_o \omega^3 L}{2} \sum_n \sum_{n'} I_{nn'} W_n^* W_{n'} \quad (8)$$

where * represents complex conjugate.

It is worthwhile to note that the same expression of complex sound power exists for sound radiation of the same plate into a free space. For this case, the plate modal coupling factors should be replaced by the following terms as described in (Pan *et al.* 2011):

$$I_{nn'} = \frac{1}{\pi L}\int_{-\infty}^{\infty} \frac{\phi_{n'}(\gamma_x)\phi_n^*(\gamma_x)}{\gamma_y} d\gamma_x, \quad (9)$$

where $\phi_{n'}(\gamma_x) = \int_0^L \sin(k_{n'} x)e^{-j\gamma_x x}dx$ and $\gamma_y = \sqrt{\gamma_x^2 - k_o^2}$.

RESULTS AND DISCUSSION

Coupling Factors between Plate Modes

The importance of the effects of the fluid loading on the plate vibration is measured by the fluid-loading parameter μ and the plate's modal coupling factors. The former determines if the fluid-loading is important and the latter determines details regarding fluid-loading on various plate modes. If the plate is loaded with water, then μ increases by nearly 1000 times greater than when the plate is loaded with air. As a result, the plate modal coupling in Equation (3) becomes important.

To generate numerical results, we have used the plate and water parameters as listed in Table 1. The driving location of the point line force is $x_o = 0.33$ m .

Table 1 Water, plate and tank parameters for simulation.

Water	*Plate*	*Water Tank*
$\rho_o = 1000\text{kg/m}^3$	$\rho_p = 7800\text{kg/m}^3$	Large Tank
$c_o = 1460\text{m/s}$	$E = 21.6\times10^{10}\,\text{Pa}$	$(L_x, L_y) = (11,12.5)\text{m}$
	$\nu = 0.3$	Small Tank
	$\eta = 0.01$	$(L_x, L_y) = (3.3,3.54)\text{m}$
	$h = 0.01\text{m}$	$\xi_{lm} = 0.001$
	$L = 1.2\text{m}$	

Figure 2 shows the modal coupling factors of the first plate mode when radiating into a free space and a large reverberant space of size $(L_x, L_y) = (11,12.5)\text{m}$. The damping ratio of the acoustical modes in the tank is 0.001.

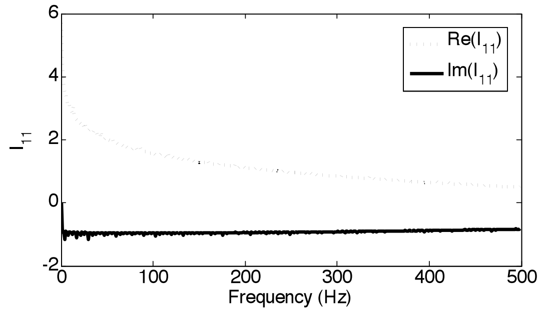
The coupling factor between the plate modes n and n' represents the radiation impedance between the mode n and the mode n', and a loading pressure radiated by the mode n on the mode n'. Figure 2(a) shows the self-coupling factor of the first plate mode ($n = 1$). As the mode radiates into free space, the coupling factor $I_{1,1}$ calculated by Equation (9) has a positive real part in the low frequency range, exponentially decaying with frequency (see Figure 2(a)). This corresponds to the mass loading of the radiated sound and causes a significant reduction in the natural frequency of the first mode. The negative imaginary part of the coupling factor linearly increases with frequency. It corresponds to the energy loss due to sound radiation of the mode. As the mode radiates into a large reverberant space, the coupling factor calculated by Equation (3) has a huge positive real part at very low frequencies (below 10Hz), where the undamped response of the first acoustic mode in the tank is under the mass-controlled condition. Unlike the coupling factor for free space, the real part of $I_{1,1}$ going into the reverberant space changes from negative (stiffness-controlled response) to positive (mass-controlled response) as the frequency passes a resonance frequency of the water tank. Therefore depending on the relative location of the resonance frequency $f_{1,dry}$ of the dry plate mode with respect to the natural frequency f_{lm} of the acoustic mode, the resonance frequency $f_{1,wet}$ of the mode of the wet plate will:

(1) reduce if $f_{1,dry} < f_{lm}$,

(2) increase if $f_{1,dry} > f_{lm}$, and

(3) have a significant change if $f_{1,dry} \approx f_{lm}$ under the condition that the spatial coupling between the $(l,m)^{th}$ tank mode and the plate mode is strong and $J_{lm,1}$ is large.

The imaginary part of $I_{1,1}$ only has negative values near the natural frequencies of the reverberant space, indicating that sound radiation from the plate into the reverberant space is realized by energy transfer into acoustical modes and dissipation by modal damping.



(a)

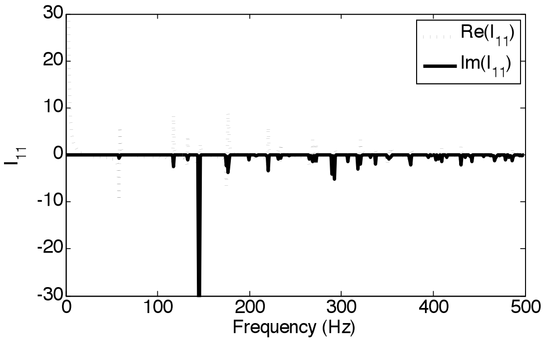


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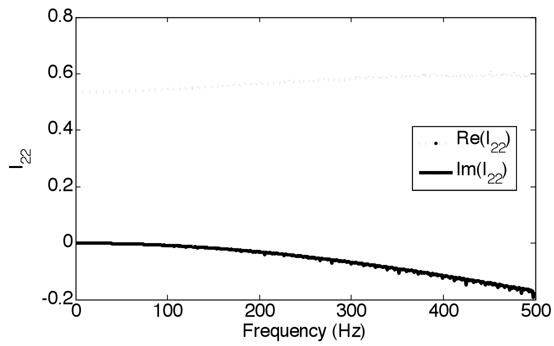
Figure 2: Real and imaginary parts of $I_{1,1}$ when (a) radiating into the free space, and (b) radiating into the large reverberant space.

Although the above discussion and conclusion are for the first plate mode, the same modal coupling mechanisms control the higher order natural frequencies of the wet plate. In general the effect decreases for higher order modes and for non-volume displacement modes ($n = even$). This statement is supported by the self-coupling factors of the $n = 2$ and $n = 3$ modes in Figures 3 and 4.

Cross-modal coupling factors will also contribute to the changes in the natural frequencies of the wet plate. Coupling between the lower-order odd-odd plate modes is the most significant due to their efficient spatial coupling capacities.



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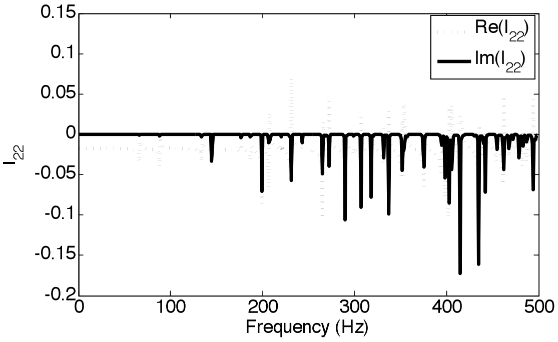

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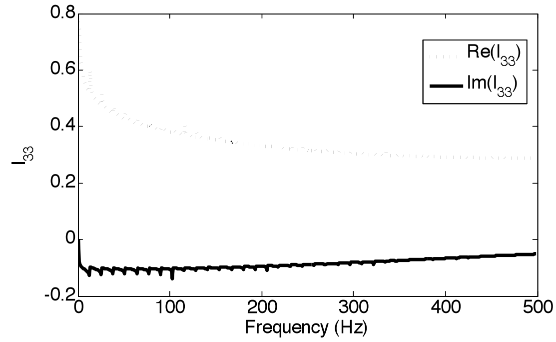
Figure 3: Real and imaginary parts of $I_{2,2}$ when (a) radiating into the free space, and (b) radiating into the large reverberant space.



(a)

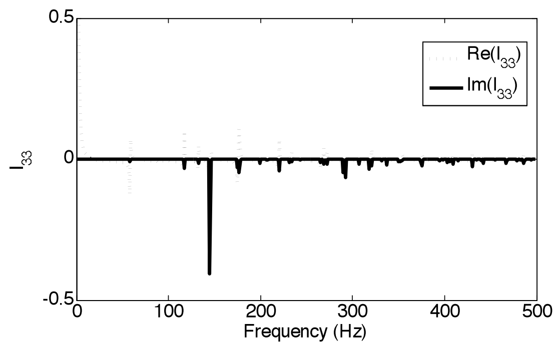


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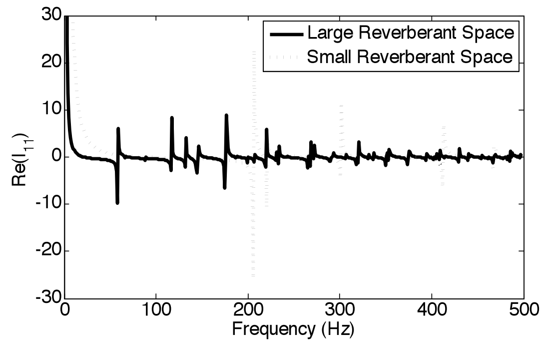
Figure 4: Real and imaginary parts of $I_{3,3}$ when (a) radiating into the free space, and (b) radiating into the large reverberant space.

When radiating into the reverberant space, the modal coupling factors are also dependent on the dimensions of the space. If the dimensions of the space are reduced, the number of modal coupling pairs between the sound field and plate vibration will also reduce. In Figure 5, we compare the self-coupling factors of the first plate mode when it radiates into the large reverberant space $(11, 12.5)$ m and a smaller one $(3.32, 3.54)$ m. Much fewer peaks were found in the coupling factor with the small-reverberant space.



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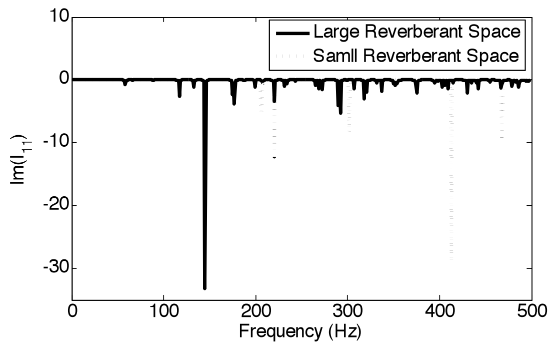


(b)

Figure 5: (a) Real part and (b) imaginary part of $I_{1,1}$ for two different reverberant spaces.

Spatial Averaged Plate Vibration

The dependence of the plate modal coupling upon the fluid loading of different radiation spaces will result in different plate responses. Figure 6 shows the spatial averaged plate velocity, which becomes the plate vibration energy per unit area when multiplied with the plate's surface mass density. The three curves correspond to three different pressure loadings by the fluids in free, large reverberant and small reverberant spaces. Previous analysis (Pan *et al.* 2011) on the natural frequencies of plate radiation into a free space showed a significant reduction in the natural frequencies of the wet plate. This observation is supported by the results in Figures 2(a), 3(a) and 4(a), where mass loading (the real part of $I_{n,n}$) exists consistently in the whole frequency range of interest and for all of the modes.

However, when radiating into reverberant spaces the properties of the fluid loading on the plate modes vary with frequency and with the size of the space. The frequency shift of the wet plate modes could be towards either end of the frequency scale depending on whether the loading's pressure is mass- or stiffness-controlled. The frequency range of effective mass or stiffness loading is limited to a close neighbourhood around the natural frequencies of the acoustical modes that are capable of effectively coupling with the plate modes concerned. As a result, the natural frequencies of the higher order modes of the wet plate experience much less shift when compared with those radiating into the free space.

The fluid loading is more obvious for the lower order modes because the coupling factors are significantly larger in the low-frequency range and for the lower order volume displacement modes ($n = 1$ and $n = 3$). Much of the high structural response is observed at the lower frequency peaks. This can be explained by: (1) large resonance coupling by lower-order/lightly damped acoustic modes, and (2) reduced fluid loading effects on plate vibration when there are no well-coupled acoustical modes located in the neighbourhood of the dry plate mode.

In addition to the peaks contributed by the wet plate modes, peaks are also found in the plate vibration response at the natural frequencies of the acoustical modes. Previous analysis [Pan and Bies 1990a, Pan and Bies 1990b, Pan *et al.* 1990] of acoustical and structural coupled systems has demonstrated that the peaks of the wet plate modes are the structurally controlled modes with most of the system energy in the structure, while the latter are the water tank-controlled modes with most of the system energy in the sound field in the tank.

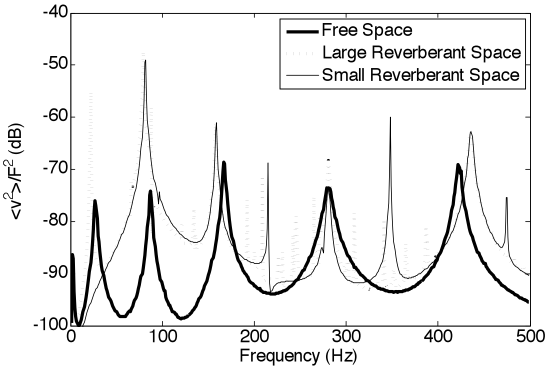


Figure 6 Squared vibration velocities of the plate excited by a unit point force at x_o and pressure loading by the fluid in free space and large and small reverberant spaces.

The frequencies of the first four peaks of the wet and dry plates are listed in Table 2 to support the above statements. The table includes three different spaces (free space, large and small reverberant spaces) where the plates radiate sound.

Table 2 Frequencies of the first four peaks of the dry plate ("Dry") and the wet plates in: free space ("Wet 1"), large reverberant tank ("Wet 2"), and the small reverberant tank ("Wet 3").

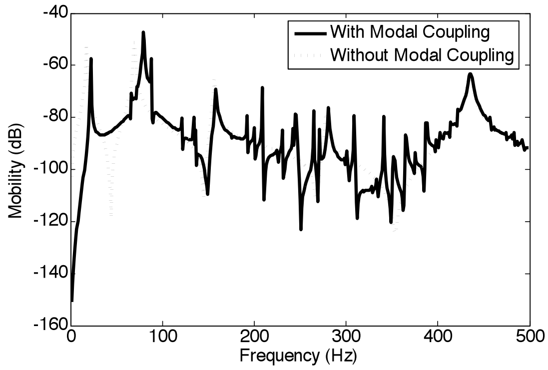
No.	*Dry (Hz)*	*Wel (Hz)*	*Wet 2 (Hz)*	*Wet 3 (Hz)*
1	17	2	22	-
2	69	27	79	82
3	156	87	158	159
4	278	167	281	281

It is interesting to note that when radiating into the small reverberant space, the plate response does not show the peak at the first mode. This may be due to the large separation distance between the natural frequency of the first dry plate mode and the lower frequency acoustical modes.

This feature of the frequency response of the plate vibration in reverberant space is further demonstrated by the driving point mobility of the plate and by comparing it with the mobility of the plate without fluid loading ($I_{n,n'} = 0$). The plate displacement without fluid loading is determined by:

$$w(x,\omega) = \sum_n \frac{2F_o \sin(k_n x_o) \sin(k_n x)}{DL(k_n^4 - k_p^4)} . \qquad (10)$$

The plate mobilities with and without the fluid loading are compared in Figure 7. For both reverberant spaces, the modal coupling caused by the fluid loading has little effect on the peak responses of the wet plate modes (*i.e.*, frequency and response) for $n \geq 3$. For the $n = 1$ and $n = 2$ plate modes, the coupling still demonstrates a significant impact.



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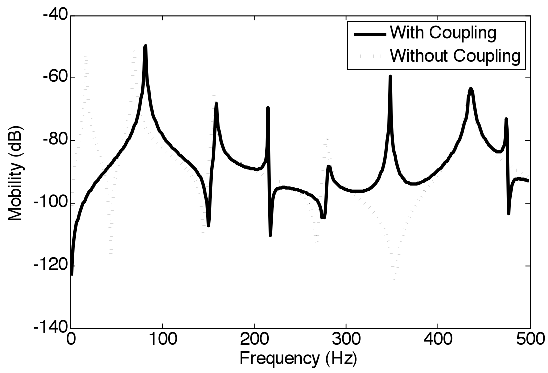

(b)

Figure 7: Plate driving point mobility with and without modal couplings through (a) the large reverberant space, and (b) the small reverberant space.

Distribution of Plate Vibration

The distributed plate displacements at the first four peaks of the vibration responses are illustrated in Figure 8. The vibration response at the first peak frequency of the plate radiating into the free space is only contributed to by the first mode of the dry plate as illustrated previously [3]. The first peak frequency of the plate radiating into the large reverberant space is higher than that of the first natural frequency of the dry plate. This indicates that the modal coupling is dominated by the stiffness-controlled sound pressure. The middle plot in Figure 8(a) shows the participation of the second mode of the dry plate. The constructive superposition of the first and second modal responses in the region of $L/2 \leq x \leq L$ makes the response higher than that in $0 \leq x \leq L/2$. No peak dominated by the first mode of the dry plate was found in the vibration response of the wet plate radiating into the small reverberant space. For this case, the plate displacement below 100Hz is mainly dominated by the second mode of the dry plate. However, at very low frequencies, as shown in Figure 8(a), the non-resonant response still has the distribution of the first mode of the dry plate.

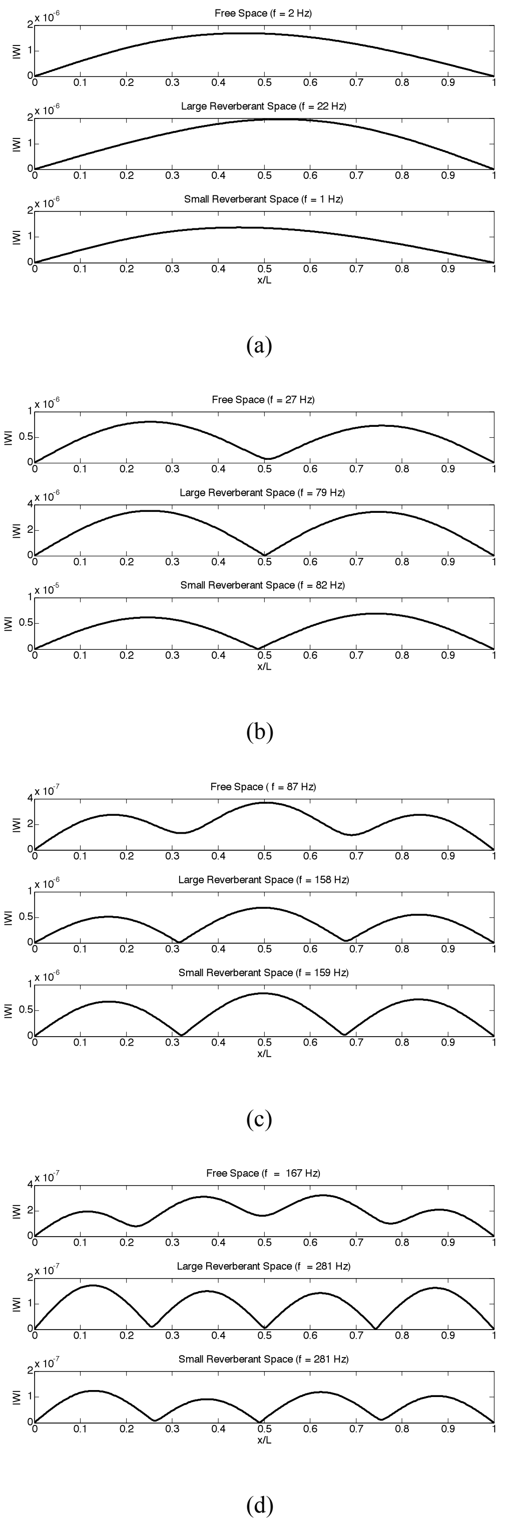

Figure 8: Distributed plate displacement at the (a) first, (b) second, (c) third and (d) fourth peak frequency of the plate vibration response.

When radiating into the reverberant spaces, Figure 8(b) shows no evidence of any contribution from other dry plate modes to the response at the second peak frequencies. However, the second peak response of the wet plate when radiating into the free space includes the contribution of the first mode of the dry plate.

The vibration responses at the third and fourth peaks, as shown in Figures 8(c) and 8(d), all show different degrees of modal coupling through the fluid-loading pressure in the acoustical spaces. Such differences are caused by the different modal coupling between the plate modes radiating into different spaces.

Radiated Sound Power

Since the plate vibration and radiated sound pressure in the spaces belong to the two parts of one coupled system, the characteristics of plate vibration and of modal coupling factors should also be associated with the radiated sound field. Equation (8) precisely describes the contribution of the modal amplitudes and modal coupling factors of the dry plate to the complex sound power radiated into the space.

The real part of the complex power is positive. It represents the energy radiated into the spaces and dissipated by wave propagation to infinity (for radiation into the free space) or by the modal damping of the acoustical modes (for radiation into the reverberant spaces). Figure 9 shows the radiated sound power into the free space, large and small reverberant spaces. The sound power radiated into the free space can be characterised by the resonant frequencies and frequency bandwidths in the vibration responses, while the sound power into the reverberant spaces is characterised by both resonances of the plate and spaces. The peaks in the sound power are mostly in narrow bandwidth. This demonstrates that the sound power is dissipated by the lightly damped acoustical modes, noting that in this analysis the modal damping constant of the acoustical modes is much smaller than that of the dry plate modes. The contribution of the wet plate modes with a broader frequency bandwidth is only viewed by a cluster of several peaks with narrow bandwidth (for radiating into the large reverberant space) or by peaks with broader bandwidths at the bottom and a narrow bandwidth at the top (for radiating into the small reverberant space). Revisiting Equation (8), the frequency characteristics of sound power can be represented by the contributions of two parts:

(1) The modal coupling factor $I_{nn'}$ which behaves as a narrow-band filter at the natural frequencies of the acoustical modes in the reverberant space and as a broad-band filter for sound radiation into the free space.

(2) The modal vibration amplitudes of the plate which behave as the input signal to the modal coupling filters. The vibration amplitudes carry the resonance characteristics of the wet plate, which include both plate-controlled modes and water tank-controlled modes.

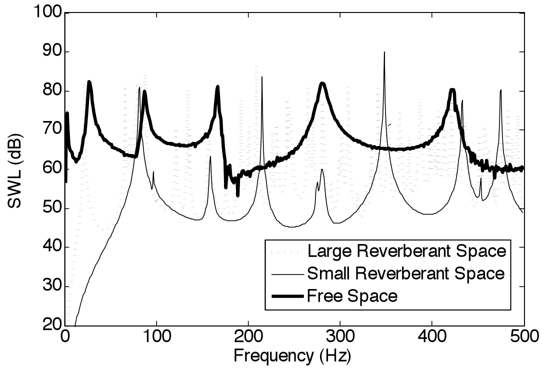

Figure 9: Radiated sound power of the plate into the free space and large and small reverberant spaces.

CONCLUSIONS

Determining how much we can learn by studying structural sound radiation into a water tank was the motivation of this paper. A typical water tank with horizontal cross sections of $11 \times 12.5\ \mathrm{m}^2$ has a wavelength of about 3 m at 500 Hz. Thus for a 1.2 m long structure, the radiated sound field in the tank will neither satisfy the far-field condition nor the diffused field condition.

The analysis presented in this paper demonstrated that the vibration of the submerged plate and its radiated sound power can be described by a general expression in terms of the mode shapes of the dry plate and the modal coupling factors between the plate modes. The modal coupling factors are physically related to the self- and cross- modal radiation impedances of the plate and are determined by the properties of the sound field and of the spatial coupling between the plate and acoustical modes. The free-field condition can be regarded as a specific example of a radiated space and can be described by a specific set of modal coupling factors.

Because of the coupling between the acoustical modes and the plate modes, the plate vibration and radiated sound power revealed the characteristics of the coupled system. As a result, a better understanding of structural sound radiation into any acoustical space was obtained. This was useful as a practical underwater sound radiation environment may not be a perfectly free field.

However, this paper is only a preliminary study on plate sound radiation into a reverberant water tank. The work was limited by the following two conditions:

(1) It was a two dimensional model, where the modes in the *z*-direction of Figure 1 were ignored; and

(2) Static water loading and its effect on the bending modes of the dry plate were also ignored.

Therefore, it is necessary to confirm experimentally the observations made in this paper.

Nevertheless, the paper provided useful guidance for:

(1) Prediction of underwater structural vibration and sound radiation if the modal characteristics of the dry structure and the sound field characteristics are available.

(2) Identification of the modal coupling factors if the modal characteristics of the dry structure and a measurement of the vibration response of the wet structure are available, which in turn can be used for predicting the sound power using Equation (8).

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