Periodic stiffness of a cracked shaft

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ABSTRACT

Shaft fatigue crack is one of the most common defects in rotating equipment, due to its extensive operation with continuous heavy loads. Finding an efficient way to evaluate the true stiffness variation due to the crack rotation is the key step to develop both on-line and off-line crack diagnostic techniques. This study analyzed time-variant bending stiffness of elastic shafts with experimentally-induced fatigue, welding and wire cut transverse cracks. It was found that crack gap has a significant effect on the opening and closing behaviour of the transverse crack. As in the case of a cut crack, large crack gap could completely prevent the crack from closing during rotation. A fatigue crack without a clear gap shows a typical opening and closing behavior. Further, it remains fully closed within a small angular range and most of time it is partially closed. It was also observed that both switch and harmonic models cannot describe periodic stiffness variation well enough to represent the actual breathing function of the fatigue crack.

INSTRUCTIONS

Shafts are amongst components subjected to perhaps the most arduous working conditions in high-performance rotating equipment used in process and utility plants such as high-speed compressors, steam and gas turbines, generators, pumps, etc. Although usually quite robust and well designed, shafts in operation are sometimes susceptible to serious defects that develop without much apparent warning. They are prime candidates for fatigue cracks because of the rapidly fluctuating nature of bending stresses, the presence of numerous stress raisers and possible design or manufacturing flaws. Wide variations in temperature and environment during operation also contribute to conditions conducive to eventual fatigue failure.

Consequences of total shaft failure can be catastrophic with enormous costs in down time, consequential damage to equipment and potential injury to personnel. Total failure occurs when the specimen has completely fractured into two or more parts. Operators and maintenance personnel of critical plant machinery are particularly interested in early detection of symptoms that can lead to in-service failure of shafts.

As the shaft rotates, the stiffness, in a fixed direction, changes with time, due to the opening and closing of the crack, which is known as the breathing of the crack. To develop the reliable and accurate detection methods for the shaft crack, crack breathing mechanisms during shaft rotation must be investigated and fully understood. Finding an efficient model of the breathing crack in rotor systems may help in identifying a unique vibration signature of the cracked rotor that assists in the early detection the crack before damage occurs due to further crack propagation.

The breathing fatigue crack has a great deal of attention in literature in recent years as summarized in a review paper (Chris, 2008). The breathing mechanism of the crack that appears in rotating machinery is mainly due to the shaft weight. The technical literature proposes two groups of models for the opening-closing to calculate the flexibility introduced by the crack. The first one calculates the stiffness determining the crack influenced region for each angular position of the shaft. For instance, Jun and Eun (Jun and Eun, 1992) assumed that the open region of the cracked section is confined where the stress intensity factor has positive values. Bernasconi and Xenophontides (Change, Bernasconi and Xenophontides, 1989) exposed that the crack-influenced region is determined by the neutral axis position on the cracked section, according to the instantaneous direction of the bending moment that acts on it.

The second one calculates the stiffness introduced by the crack in two positions, when the crack is completely opened and when it is completely closed, and they assume a stiffness variation function between these values. Gash (Gash, 1993) considered the stiffness variation as a step function. Mayes and Davies (Mayes and Davies, 1984) found experimentally that the stiffness variation could be described by a once-per-revolution sinusoidal function. Sekhar and Balaji (Sekhar and Balaji, 1997) used for the stiffness variation the first four terms of Fourier series of cosines. Nelson and Nataraj (Nelson and Nataraj, 1986) modeled the crack open/close mechanism by a periodic ‘‘switching’’ function, which depends on the bending curvature of the shaft at the cross-section. Recently, Al-Shudeifat and Butcher (Al-Shudeifat and Butcher, 2011) calculated area of moment for a cracked shaft and proposed a new expression for crack open and close behavior.

Previous investigations were mostly based on analytical models, or on a shaft cut instead of actual fatigue crack. There is a lack of reliable experimental data to support the theoretical modeling results. As such, a detailed experimental analysis of the periodical stiffness is necessary. In this study, opening and closing behaviour of three cracks, namely, fatigue crack, welding crack and wire cut crack, was evaluated at a pure bending configuration. Measured periodic stiffness values were compared with those predicated by the switch and harmonic models to examine the accuracy and reliability of these two models.

EXPERIMENTAL

The shaft was made of AISI 1030 steel, which was obtained from Bohler Uddeholm Australia. The standard length and diameter of the shaft are 542 mm and 15.875 mm, respectively. Three types of transverse cracks with a depth of 40% shaft diameter along the radial direction were created in this work.

Fatigue crack

As shown in Figure 1, the specimen shaft was mounted on a three-point bending fixture of a dynamic Instron cyclic machine (model: 8501). To initiate the crack, a 0.254 mm deep V-notch was machined using an Okuma CNC milling machine for fatigue pre-cracking (As per ASTM E399 specifications for fatigue precracks). Tests were performed on a number of shaft specimens, in order to determine the cyclic loading parameters needed to grow the crack to a specified depth (40% of the shaft diameter) without causing permanent deformation of the shaft. The entire test was conducted using various loading forces ranging from a maximum of 11 KN to a minimum of 2 KN. Each test for different applied loading force ranges was carried out for about 2×10^4 cycles. A cyclic force was applied in a sinusoidal waveform with a frequency of 5 Hz at a room temperature to initial the crack. After 2×10^4 cycles, the initial fatigue crack appeared at the notch front, the crack propagation was controlled by further cycling using a lower maximum load. Crack growth was closely monitored during fatigue test by using an Olympus optical microscope. At the end stage, the loading force was reduced to 2 KN. The test was finally stopped when the crack reached 40% of the shaft diameter.

Figure 1. Experimental set-up for fatigue crack growth

Welding crack

This cracked shaft was developed by perfectly welding two pieces of a shaft using a water cooled TIG welder. The material was cut into two pieces with required dimensions and each side was machined by a CNC OKUMA milling machine at the two joining faces. Two machined sides of same material were fixed together by a jig plate with screws to hold it aligned. Within the area of 30 mm length around the crack, the upper part of the material was removed by CNC only leaving the 40% portion which forming the crack, A replacement block was fabricated, using the same material having exactly the same dimensions and shape with the removed portion. The replacement block was then joined with the two pieces of the shaft by TIG welding (without filling material) to create one uniform material structure with a crack depth of 40% of the diameter. The welding conditions, including the welding current and time, were tested carefully in order that a uniform joint is produced without a significant melting into the replacement block or the two joining pieces, and that the crack is kept intact. The shaft was left in the jig plate to cool down slowly, to ensure a minimal effect of a heat treatment to the metal. Finally, the shaft was tested using manual lathe with a live centre at high speed (2500 RPM) to make sure there is no any misalignment caused by welding process. The location and depth of the crack was the same as those of the fatigue-cracked shaft.

Wire-cut crack

A transverse slot was generated by electroerosion. The location and depth of the crack were also identical with those of the fatigue-cracked shaft. Wire cut crack was fabricated using CNC wire cutting machine (Grade A+). The cutting process was known as EDM machining (Electrical Discharge Machining) which is an electro thermal production process, in which a thin (0.1 mm) single strand brass wire in conjunction with de-ionised water allows the wire to cut through metal by the heat generated from electrical sparks. Due to its high precision, EDM is an ideal choice to produce a very thin cut. Another advantage of this process is that it does not apply force during cutting process, which means that the possibility of thermal stress is low. The wire cut EDM equipment was run by OKUMA CNC instruments which controlled the brass wire on a three dimensional axis by reading a G-code software program, which was created from SolidWorks drawing by using SolidCAM software.

Stiffness measurements

Bending stiffness measurements were conducted by Instron universal testing machine, model 5569, under Quasistatic load at the room temperature. A three point bending compression program was set up using Instron Bluehill software, in order to obtain the load vs. displacement curve. Each specimen shaft was rotated and measured at 13 different angular positions from 0° to 360° with a 30° increment. The relative position between crack direction and loading axis is shown in Figure 2.

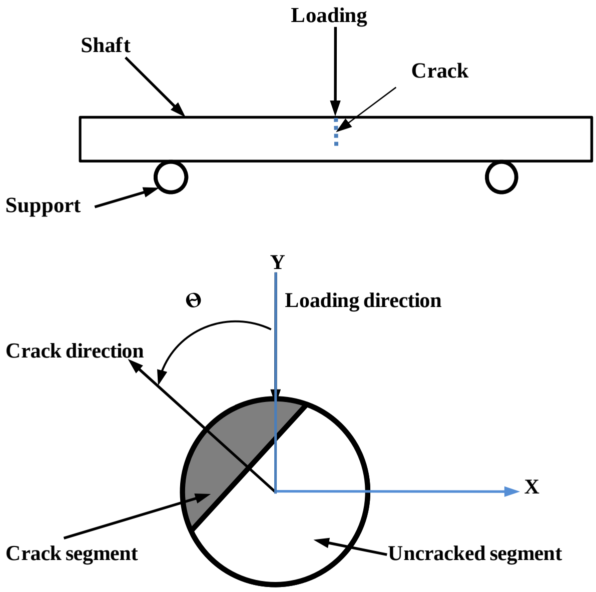

Figure 2. Top: a schematic diagram of the loading direction in three point bending test; bottom: Cross-section view at crack showing crack angular position.

To determine angular positions of the crack accurately for every measurement point, a cloth measuring tape with unit marking was fixed around the crack edge along the shaft, then the circumference of the shaft was subdivided equally into 12 sections, each having 30^0 angular interval. For each measurement, before recording the data, a small initial force was applied to stabilise the measuring process. This was to minimize the error in the displacement caused by the fixture backlash or other set up errors. Loading rate was set at 50 N/min. For safety reason, a 2 KN of maximum force limit was programmed to prevent shaft from accidental overloading and possible damage. Load and displacement data were recorded and processed later using Microsoft Excel. Stiffness was calculated from the slope of the load vs. displacement curve.

RESULTS AND DISCUSSION

Figure 3 shows loading force as a function of displacement for a fatigue-cracked shaft with respect to three crack angular positions. It is clear that the load force/displacement curves have a linear relationship. As the crack rotates gradually from fully closed (0°), partially closed (120°) to the fully open position (180°), the corresponding slope of the curve decreases, indicating a decrease in stiffness.

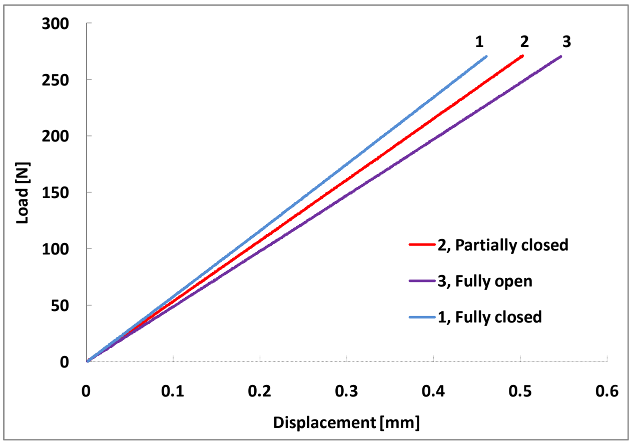

Figure 3. Loading force as a function of displacement for a fatigue-cracked shaft at three crack angular positions.

Stiffness values calculated from the slope of load/ displacement curves are presented in Figure 4. For the intact shaft, the stiffness remains independent of rotating angle with a slight fluctuation due to experimental error. The averaged stiffness is calculated to be 5.8x105 N/m. When a fatigue crack exists, the stiffness behaves totally differently. It can be seen clearly that stiffness at small angles is almost at the same level as that for an intact shaft. This is because the crack is either fully closed or slightly open. Consequently, the fatigue-cracked shaft at these angles behaves in a similar manner as an uncracked one.

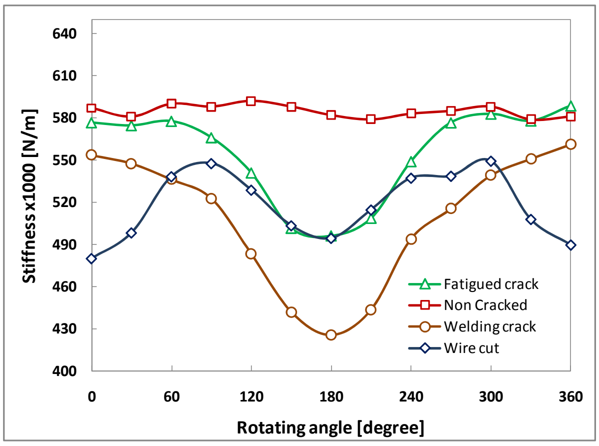

Figure 4. Bending stiffness at middle span for four shafts at various angular positions.

As the crack gradually rotates anticlockwise from its fully closed position at 0°, stiffness does not show any visible change until a certain angle is reached where it starts to decrease rapidly. The minimum stiffness is 4.96x105 N/m and located at 180° where the crack is fully open. As the crack continue to rotate past 180°, a reverse process proceeds. It is evident that a fatigue crack shows typical breathing behaviour.

The welding crack shaft also shows an opening and closing behaviour, similar to that of a fatigue-cracked shaft. However, the stiffness is much lower than that of a fatigue-cracked shaft. This is likely resulted from a change in the material property during welding process, or/and an imperfect bonding between welding surfaces. Consequently, shaft stiffness is much lower than that indicated by the crack nominal depth. To develop a quantitative breathing model, the parameter of crack depth for a welding crack must be used with great care.

For the shaft with a wire-cut crack, stiffness variation with rotating angle follows an "M" shape, which is again totally different from normal crack breathing behaviour. Stiffness at 0° or 360° is more or less the same as that at 180°. This indicates that the two surfaces of the cut do not close together. Figure 5 shows the cut crack gap at the shaft surface.

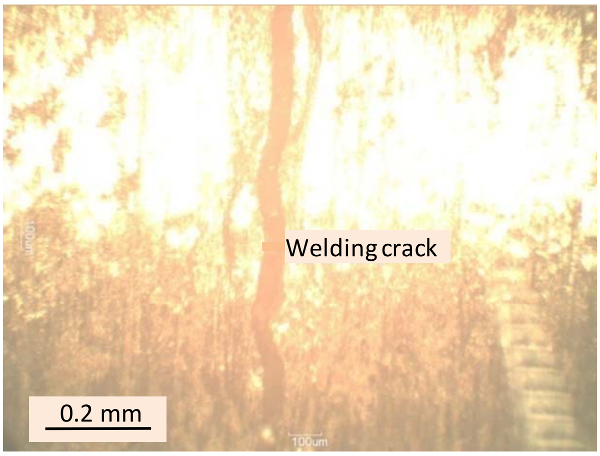

Figure 5. Welding crack observed under Moticam 2300 microscope.

The gap of the cut is approximately 0.15 mm (Figure 6) while that of the welding crack is only about 0.05 mm (Figure 5).

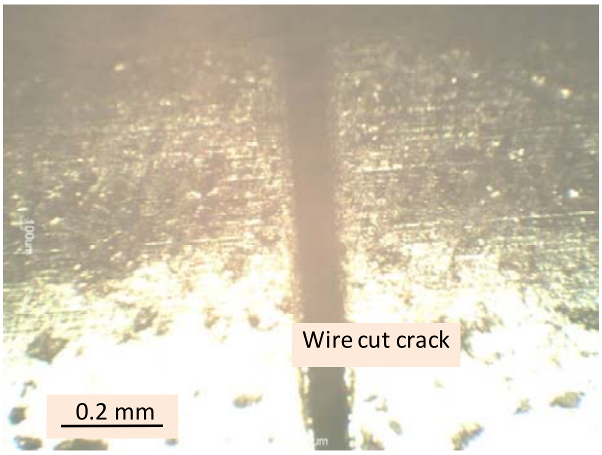

Figure 6. Wire cut crack observed under Moticam 2300 microscope.

It is this large gap of the cut that prevents the two cut surfaces from closing. Basically the cut crack remains at open status during rotation. The higher stiffness observed at 90° (or 270°) is simply due to a larger area moment of inertia at this position. In other words, the thin direction of the uncut segment aligns in the loading direction when the crack is at 0° position. As the crack further rotates to 90° position, the thick direction is parallel to the loading direction. Observation in this study is in agreement with previous experience which has shown that even the narrowest practical machined notch cannot simulate a natural crack well enough to provide a satisfactory measurement of KIC.

As the shaft rotates, the fatigued crack opens and closes alternatively. The tension stress, existing below the neutral axis of the cracked element trends to keep the crack open. The compression stress field that exists above the neutral axis tends to close the crack. The angle at which the crack starts to open or close depends on the shaft diameter and crack depth. Formulas to calculate these two critical angles are presented in (Al-Shudeifat and Butcher, 2011). For a 40% crack depth and a shaft diameter of 15.875 mm used in this study, these two angles are calculated and the angular ranges in which the crack remains fully open, partially closed or fully closed, are presented graphically in Figure 7. The crack starts to open at 11.5° and becomes fully open at 152.1°. As the crack continues to rotate, it starts to close at 207.9° and becomes fully closed at 348.5°. It can be seen clear that the crack only remains fully closed within a very small angular range of 23°. On the other hand, the crack is partially closed most of the time.

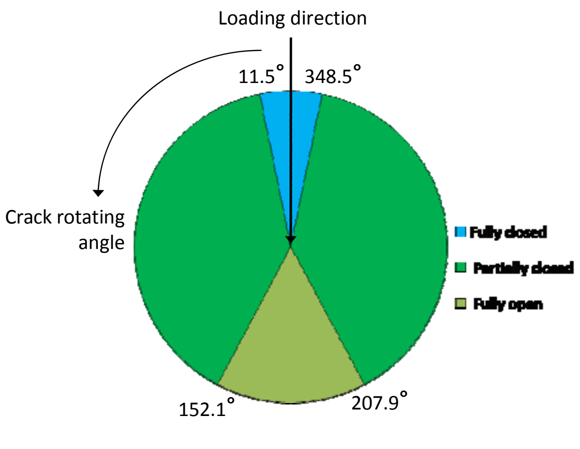

Figure 7. Relationship between crack angular position and its opening/closing status.

Periodic stiffness variation was usually simulated using a switch model (Jun and Eun, 1992) or a harmonic model (Mayes and Davies, 1984). The crack in the switch model was assumed to have only two states, either open or closed during each revolution. The mathematic description of the switch model is given in Equations 1 and 2, where Ko and (Ko-ΔK) are the stiffness values of a shaft when the crack close and open fully, respectively and ω is the rotating frequency of the shaft and t is time. The harmonic model assumed a time-variant stiffness of the shaft which can be described by Equation 3.

$$K=K_0, \qquad 0^0 \le \omega t \le 90^0 \text{ and } 270^0 \le \omega t \le 360^0. \tag{1}$$

$$K = K_o - \Delta K \qquad 90^0 \le \omega t \le 270^0. \tag{2}$$

$$K = K_o - \frac{\Delta K}{2}(1+\cos(\omega t + 180)) \tag{3}$$

Using experimental values for Ko = 5.8x105 kN/m and Ko-ΔK=4.96x105 kN/m, respectively, stiffness values predicted by two models are calculated and presented in Figure 8, together with experimental results for a fatigue crack. It was found that the true stiffness profile due to a transverse fatigue crack cannot be described accurately by either the switching model or the harmonic model. The most significant discrepancies between experimental results and theoretical models lie in the angular ranges where the crack is partially closed. In a recent study (Han, 2007), it was also observed that the true stiffness variation with rotating angle for a transverse shaft crack is neither the switching model nor the harmonic model.

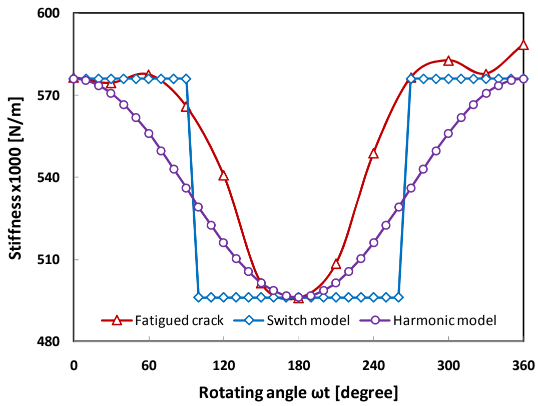

Figure 8. A comparison of measured periodic stiffness values and those calculated using theoretical models.

CONCLUSIONS

This study has examined breathing mechanism for three types of shaft cracks based on experimental results. It was found that the width of the crack opening gap plays an important role in controlling the crack breathing behaviour. A fatigue crack without a clear gap shows a well-known opening and closing behaviour. A thicker gap for a cut crack can change periodical stiffness from a "V" shape to an "M" shape. A welding crack does show a breathing function. However, to develop quantitative crack modelling and crack diagnostic technique, crack nominal depth must be considered carefully as welding process may produce much lower stiffness values. This study also shows that neither the switching model nor the harmonic model can describe the periodic stiffness of a transverse shaft crack accurately.

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