The loss mechanisms of plane-wave reflection from the seafloor with elastic characteristics

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ABSTRACT

As an integral part of the waveguide, the seafloor plays an important role in underwater acoustic propagation, particularly in shallow water environment where the acoustic waves strongly interact with the seafloor. It is therefore critical to understand the fundamental loss mechanisms from the interaction between acoustic waves and the seafloor for both forward and inverse underwater acoustics problems in shallow water. This paper reviews the classic theory of plane-wave reflection from layered solid media, and its application on revealing the mechanisms of the reflection loss from the seafloor with a specific geoacoustic structure consisting of a sediment layer of weak elasticity overlaying a solid substrate. The significant reflection loss mechanisms include the compressional-shear wave conversion, the possible sediment-substrate surface wave excitation, and the resonances in the sediment layer.

INTRODUCTION

In the shallow water region covering ocean portion from the shore to the continental shelf, and in the relatively low frequency range from a few Hz to a few kHz, seafloor serves as a complicated yet fascinating boundary of underwater acoustic waveguide, and the interaction between acoustic waves and seafloor has always been a prime concern for shallow water acoustics (Katsnelson et al, 2012).

The development of seafloor geoacoustic model, i.e. the effective seafloor structure and acoustic properties, are critical for both forward and inverse underwater acoustics problems. Generally, the seafloor top-layer sediments (e.g. clay, sand, gravel etc.) are unconsolidated and their elasticity is limited, therefore they can be modelled approximately as fluids which only support compressional wave. For certain seafloor materials such as basalt and limestone, however, the elasticity is considerable and they must be modelled as solid media which support both compressional and shear waves.

As the elastic characteristics in the geoacoustic model induce different yet more complex mechanisms for acoustic wave generation and propagation than fluid characteristics, attention has been attracted to incorporate seafloor geoacoustic model with elastic characteristics into various underwater acoustics propagation models, such as wavenumber integration (Schmidt et al, 1985; Goh et al, 1996), normal mode (Porter, 2001; Westwood et al, 1996&1999) and parabolic equation (Collins, 1989&1991; Wetton et al, 1990; Jerzak et al, 2005; Kusel et al, 2007; Outing et al, 2006; Collis et al, 2008). Experimental and theoretical investigations have been conducted on seafloor shear-induced effects on underwater acoustic propagation (e.g. Tollefsen, 1998 and Duncan, 2009). Inversion exercises on seafloor elastic properties also have been carried out (Dong et al, 2006&2010).

By reviewing relevant acoustics theory in solid media and its application on geoacoustic models, this paper aims to gain understanding of the possible loss mechanisms of plane-wave reflection from seafloor, particularly with typical geoacoustic structures applicable for shallow water environment.

PLANE WAVE REFLECTION FROM LAYERED SOLID MEDIA

Wave propagation in anisotropic solid media (i.e. physical properties of the media are directional dependent) is a considerably complicated process (Auld, 1973; Nayfeh, 1995). The isotropic solid media, a much simplified case, is of present interest, and the wave equations in such media generally can be obtained analytically. Comprehensive wave theories in isotropic solid media can be found in Ewing et al (1957) or Brehovskikh (1980).

The frequency-domain elastic plane waves in a homogeneous isotropic solid media in xz plane can be described by the compressional wave potential φ and shear wave potential ψ;

$$\varphi = A_c \exp[i(\alpha z + \xi x)] + B_c \exp[i(-\alpha z + \xi x)] \quad (1)$$

$$\psi = A_s \exp[i(\beta z + \xi x)] + B_s \exp[i(-\beta z + \xi x)] \quad (2)$$

Wavenumbers for compressional and shear waves $\alpha = \sqrt{k_c^2 - \xi^2}$, $k_c = \omega/c_c$; $\beta = \sqrt{k_s^2 - \xi^2}$, $k_s = \omega/c_s$, where ξ is horizontal wavenumber. The compressional and shear wave velocities $c_c = \sqrt{(\lambda + 2\mu)/\rho}$ and $c_s = \sqrt{\mu/\rho}$ respectively, where λ and μ are the Lame constants, ρ the medium density.

At the interface of two solid media of half space 1 and 2, as shown in Figure 1, the following boundary conditions regarding the continuities of both normal particle displacement and certain stress tensors have to be satisfied;

$$(i\xi\varphi - \partial\psi/\partial z)|_1^2 = 0 \quad (3)$$

$$(\partial\varphi/\partial z + i\xi\psi)|_1^2 = 0 \quad (4)$$

$$[-2\mu\xi(\xi - k_s^2/2\xi)\psi - i\partial\varphi/\partial z]|_1^2 = 0 \quad (5)$$

$$[2\mu\xi(\xi - k_s^2/2\xi)\varphi + i\partial\psi/\partial z]|_1^2 = 0 \quad (6)$$

In Figure 1 the incident wave is either compressional or shear, and identified as A_c^1 and A_s^1 respectively. By substituting compressional and shear wave potentials of the two media (1) and (2) into the boundary conditions (3) to (6), the amplitudes of the body waves reflecting and transmitting from the interface are linearly connected with incident wave amplitudes on the boundary:

$$\begin{pmatrix} B_c^1 \\ B_s^1 \\ B_c^2 \\ B_s^2 \end{pmatrix} = [S] \cdot \begin{pmatrix} A_c^1 \\ A_s^1 \end{pmatrix} \quad (7)$$

The scattering matrix

$$[S] = \begin{pmatrix} R_{cc} & R_{sc} \\ R_{cs} & R_{ss} \\ T_{cc} & T_{sc} \\ T_{cs} & T_{ss} \end{pmatrix} \quad (8)$$

Among the elements of the scattering matrix, R_{cc}, R_{ss}, T_{cc} and T_{ss} present the reflection and transmission coefficients of the compressional and shear waves respectively, and R_{cs}, R_{sc}, T_{cs} and T_{sc} the conversion coefficients between compressional and shear waves during reflection and transmission process. The detailed solutions of the elements are presented in Brekhovskikh and Godin (1990).

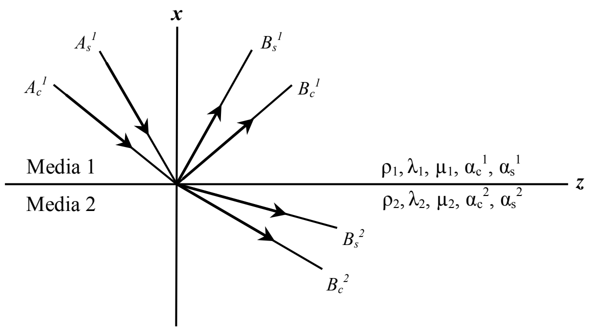

Figure 1. Elastic waves reflection and transmission at a boundary of two elastic half-spaces.

For waves incidence to an elastic half-space from a fluid, as only compressional waves exist in the fluid, i.e. $\mu = 0$, the scattering matrix (8) has three valid elements, and they are the compressional wave reflection coefficient R_{cc}, the compressional-compressional wave transmission coefficient T_{cc} and the compressional-shear wave transmission coefficient T_{cs}.

For waves reflecting from a system of layered solid media, direct global matrix approach (Schmidt and Tango, 1986) can be pursued by constructing wave potential equations for all the layers with the satisfaction of boundary conditions upon the interface between layers, then the set of equations being solved via matrix inversion. The alternative approach is the method of the matrix propagator which applies recurrence formulas relating the amplitudes of waves in adjacent layers. Details of the second method, which is widely used in seismology, are presented in Gilbert and Backus (1966).

The wave reflection from layered solid media also may cause the excitation of surface waves at the media interfaces. Surface waves (i.e. the Scholte wave at a fluid-solid interface and the Stoneley waves at a solid-solid interface) have the characteristics with their amplitude decaying rapidly with increasing distance from the media interface. They can occur only for limited combinations of media parameters, along with incoming wave requirements (Rauch, 1980; Ainslie, 2003).

SHALLOW WATER GEOACOUSTIC MODEL – A CASE STUDY

The geoacoustic properties in shallow water environment can vary significantly amongst different regions. A typical geoacoustic model for the shallow water coastal sediments consists of distinct vertical layers, such as the top layer of unconsolidated sediments of order 1 to 10 m thick, a possible layer of semi-consolidated sediments of tens of meters, and a consolidated sediment base (Katsnelson et al, 2012). The reflection characteristics of such shallow water geoacoustic model have been well investigated (e.g. Hovern and Kristensen, 1991; Tollefsen, 1998; Ainslie, 2003).

Uncommon shallow water geoacoustic models, e.g. a sediment structure consisting of a thin hard-cap overlaying a limestone half space as an example, can lead to peculiar features in sound propagation, and have been occasionally identified. (Gavrilov etc., 2012).

Table 1. A shallow water geoacoustic model with gravel sediment and basalt substrate

Layer	Thickness (m)	Sound speed (m/s)	Attenuation (dB/λ)	Density (g/cm^3)
Water column	∞	c - 1500	α - 0.0	1.00
Sediment - Gravel	10	c_c - 1700	α_c - 0.6	1.5
		c_s - 180	α_s - 1.5	
Substrate - Basalt	∞	c_c - 4750	α_c - 0.1	2.7
		c_s - 2500	α_s - 0.2	

Table 1 lists geoacoustic parameters for a layered shallow water seafloor system consisting of a 10-m gravel sediment and a basalt substrate. According to Ohta & Goto (1978) as cited by Hamilton (1987), the regression equation for shear speed in gravel is $c_s(z) = 178.1\, z^{0.312}$, where z is depth below the water-seafloor interface (m). This function increases from 0 at z = 0 to approximately 360 m/s at z = 10 m. An average value of 180 m/s has been selected as the representative shear wave velocity for gravel sediment.

The reflection coefficients as the function of grazing angle and frequency, for the seafloor system as in Table 1, were calculated using the plane-wave reflection coefficient program BOUNCE (Porter, 2007) and are presented in Figure 2, considering situations with and without elastic characteristics in sediment and substrate layers. The BOUNCE program applies the direct global matrix approach to calculate the plane-wave reflection from a system of layered solid media (Schmidt and Tango, 1986).

Panel (a) in Figure 2 considers the sediment and substrate layer without elastic characteristics. As can be seen from the panel, the sediment layer is thin compared with the incident wavelength at frequencies below 100 Hz, making the layer transparent to the incident wave. The reflection coefficient has the apparent critical angle around 71.6^o. With frequency increases, the sediment layer gradually overtakes the substrate as being predominant in determining the reflection coefficient. The critical angle of the reflection coefficient is around 28^o for frequencies above 200Hz. The panel also reveals the evident angle-dependent resonance pattern, relating to the quarter and half-wavelength layer effects in the sediment layer.

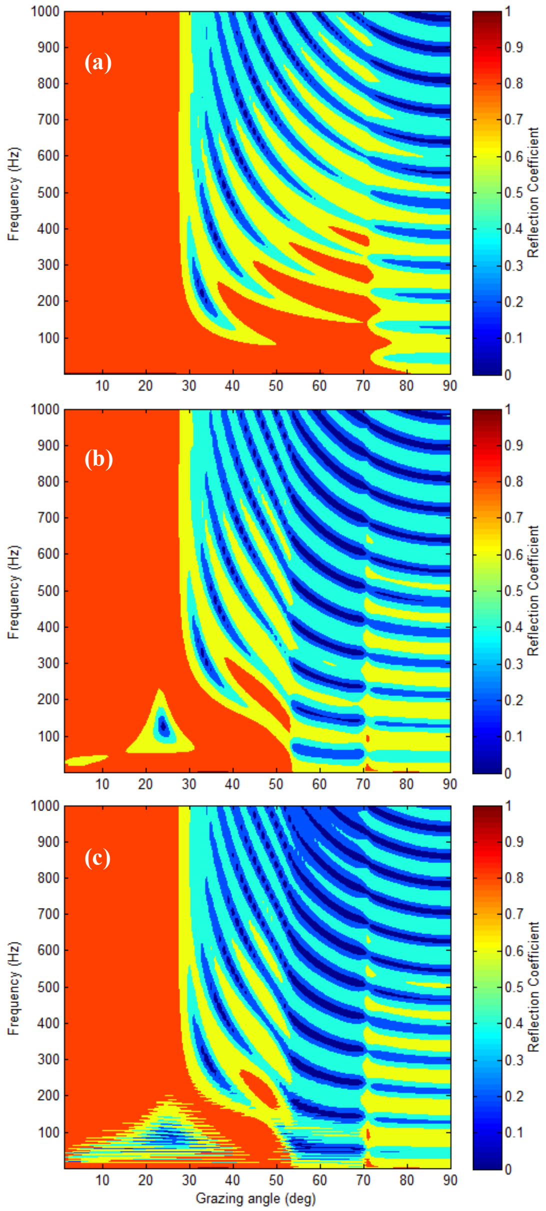

Figure 2. Plane wave reflection coefficient versus grazing angle and frequency from a layered shallow water seafloor system with geoacoustic parameters given in Table 1. Panel (a) considers the model without elastic properties in both sediment and substrate layers, Panel (b) without elastic properties in sediment layer, and Panel (c) with elastic properties in both sediment and substrate layers.

Panel (b) in Figure 2 includes the elastic characteristics for substrate layer only, while Panel (c) considers the elastic characteristics in both sediment and substrate layer. Apart from having similar features as shown in Panel (a), Panel (b) & (c) also have an extra critical angle of around 53^o which relates to the shear speed in the substrate layer. Compared with Panel (a), the higher reflection loss within the two grazing angles (53^o~71.6^o) in Panel (b) & (c) is due to the compressional-shear wave conversion for the wave incidence on the sediment-substrate interface. The excitation of the surface waves at the sediment-substrate interface also induces higher reflection loss within the region with low frequencies and low grazing angles, as being evident in both Panel (b) and Panel (c). The introduction of the weak elastic characteristics in the sediment layer leads to small but noticeable variation of the reflection coefficient.

CONCLUSIONS

This paper provides a brief review of the theory of plane-wave reflection from layered solid media, and investigates the reflection characteristics of a typical shallow water geoacoustic model, i.e. a sediment structure consisting of a sediment layer of weak elasticity overlaying a solid substrate. The compressional-shear wave conversion and resonances in the sediment layer are significant loss mechanisms for the plane wave reflection. The possible excitation of the sediment-substrate surface wave may also be able to induce significant reflection loss.

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