

# Passive Measurement of Vertical Transfer Function in Ocean Waveguide using Ambient Noise

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## ABSTRACT

This paper introduces a function of correlation between two hydrophones, basing on the Kuperman-Ingenito ocean ambient noise model. There is a similarity in form between the cross correlation function and the transfer function in ocean waveguide from a point source to a receiver. Thus, the noise cross correlation function between two hydrophones in vertical location can extract actual transfer function, and then the acoustics ray arrival structure of propagation in vertical waveguide can be analyzed. In this paper, the transfer function in vertical ocean waveguide can be obtained from broadband ambient noise cross correlation function of vertical line array. There are some analysis about physical significance of noise interference basing on compared simulation and experiments. This method can be used to research stratification sea floor considering the arrival time structure of each propagation route.

## 1 INTRODUCTION

In the past several decades, scholars had studied the physical characteristics of ocean ambient noise, and put forward many ocean ambient noise models (M. J. Buckingham, 1980, C. H. Harrison, 1997, W. A. Kuperman and F. Ingenito, 1980). This paper introduced the wave number integral form of noise cross correlation function between two hydrophones based on Kuperman-Ingenito (K/I) ocean ambient noise model (W. A. Kuperman and F. Ingenito, 1980). This integral looked like sound field integral form between source-receiver pair, the source-receiver pair corresponded hydrophones pair location that handling noise cross correlation function. Thus the vertical transfer function of ocean waveguide could be extracted from noise cross correlation function. At last, there were some experiments in this paper, the noise data collected by vertical line array (VLA). To compare with simulation results, we used these noise data to obtain the transfer function of ocean vertical waveguide.

## 2 PASSIVELY OBTAINED VERTICAL TRANSFER FUNCTION OF OCEAN WAVEGUIDE

### 2.1 Fundamental theory

According to the K/I model, the ocean ambient noise source is at the  $z'$  position under the sea surface and aroused by the acoustic source which is distributed in the infinite plane, the geometry position of noise source distribution is shown as figure 1.

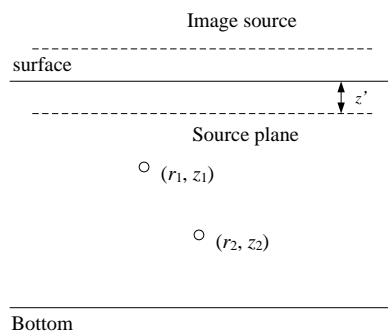


Figure 1. Location of noise sources and receivers

The cross correlation function of ambient noise field between two arbitrary hydrophones can be expressed by,

$$C(\omega, R, z_1, z_2) = \frac{8\pi^2 q^2}{k^2(z')} \times \int_0^\infty [g(k_r, z_1, z') g^*(k_r, z_2, z')] J_0(k_r R) k_r dk_r \quad (1)$$

Where the cross correlation function  $C(\omega)$  which is related with  $\omega$  can be obtained from the Green function of  $z_1$  and  $z_2$ ,  $k_r$  is the horizontal wavenumber,  $k = \omega/c_w$ ,  $c_w$  is the sound speed of sea water,  $q$  is the intensity of sea surface noise source,  $g^*$  is the complex conjugate of the Green function, Bessel function  $J_0$  is relative with the horizontal spacing of the two hydrophones  $R = r_1 - r_2$ .

From the Eq.1, integral form of cross correlation function is similar with the sound field integral from point source  $z_1$  to receiver  $z_2$ . The difference is the transfer function of sound field changing into product of two transfer function of noise field in integral kernel.

$$P(\omega, R, z_1, z_2) = \int_0^\infty g(k_r, z_1, z_2) J_0(k_r R) k_r dk_r \quad (2)$$

The noise cross correlation function of time domain can be used to present the interference structure of the ambient noise

field. The time domain form can be obtained by Fourier transform,

$$C(\tau, z_1, z_2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} C(\omega, z_1, z_2) e^{-i\omega\tau} d\omega \quad (3)$$

According to Eq.3, the cross correlation function of the ambient noise field between  $z_1$  and  $z_2$  can be transformed from frequency domain into time series. From above analysis, cross correlation function in the time domain can be thought as the acoustics propagation process which the source is located at  $z_1$  and receiver is located at  $z_2$ . As a result, the time relation of the vertical transfer function in the waveguide will be obtained.

$$\frac{\partial C(\tau, z_1, z_2)}{\partial \tau} \approx -[G(\tau, z_1, z_2) - G(-\tau, z_1, z_2)] \quad (4)$$

### 2.2 Beamforming and array shape modification

For ocean ambient noise signal measured by the vertical array, the vertical transfer function can be obtained by the cross correlation function of noise field from every array element. For the whole array, the cross correlation function is the output of beamforming at the  $\pm 90^\circ$  direction. The noise vector of the array can be expressed as,

$$\mathbf{p}(\omega) = [p_1(\omega), p_2(\omega), \dots, p_M(\omega)]^T \quad (5)$$

Linear beamforming weight coefficients can be expressed as

$$\mathbf{w} = [w_0, w_1, \dots, w_{M-1}]^T \quad (6)$$

Where  $w_m = e^{-ikmd \sin \theta}$ .

For the vertical transfer function in the waveguide, cross correlation function between array elements is the beamforming result at the up-looking and down-looking direction.

$$C(\omega) = \mathbf{w}^H \mathbf{K} \mathbf{w}^* \quad (7)$$

Where  $w_m = e^{-ikmd}$ ,  $d$  is the spacing of the array element,

$$\mathbf{K} = \mathbf{p} \mathbf{p}^H.$$

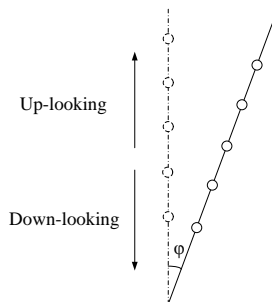


Figure 2. VLA slant angle

If the slant angle of VLA is  $\phi$ , the array shape is modified by weight coefficients  $w_m = e^{-ikmd \cos \phi}$ .

### 2.3 Four simple cross correlation functions in the waveguide

Integral kernel of cross correlation function is thought as the transfer function of the sound field in the waveguide and four simple waveguides are expressed as follow (Martin Siderius, Chris H. Harrison, Michael B. Porter, 2006):

Case 1: Transfer function in the free space

$$g_1 g_2^* = \frac{e^{ik_z(z_1 - z_2)}}{(4\pi k_z)^2} \quad (8)$$

Case 2: Transfer function with only ocean surface reflection

$$g_1 g_2^* = \frac{1}{(2\pi |k_z|)^2} \left[ e^{i(k_z z_1 - k_z^* z_2)} \sin(k_z z_1) \sin(k_z^* z_2) \right] \quad (9)$$

Case 3: Transfer function with only ocean bottom reflection

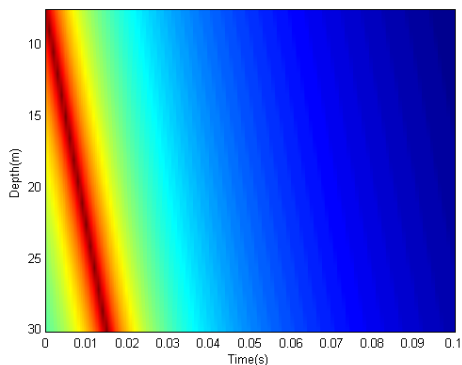
$$g_1 g_2^* = \frac{2}{(4\pi |k_z|)^2} \left\{ \cos[k_z(z_1 - z_2)] - \cos[k_z(2H - z_1 - z_2)] \right\} \quad (10)$$

Case 4: Transfer function with both ocean surface and bottom reflection

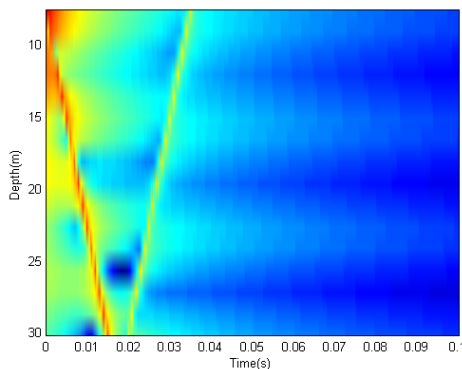
$$g_1 g_2^* = \frac{2}{(2\pi |k_z|)^2} \left\{ e^{ik_z(z_1 - z_2)} \sin(k_z z_1) + \frac{1}{4} e^{-ik_z(z_1 - z_2)} - \frac{1}{2} \cos[k_z(2H - z_1 - z_2)] + \frac{1}{2} \cos[k_z(2H - z_1 - z_2 - 2z_2)] \right\} \quad (11)$$

From above four expressions, in case 1, two point cross correlation function in free space is as same form as sound field transfer function. In case 2, two points cross correlation function with only ocean surface reflection is similar as free space form, the difference is that in this case there is a dipole term  $\sin^2(k_z z_1)$  which is the noise source directional property. In case 3, the first term of cross correlation function is similar as direct wave between  $z_1$  and  $z_2$ , and second term is bottom reflection sound field. In case 4, the cross correlation function is similar as case 3, the difference is both direct wave and bottom reflection wave own a dipole directional factor.

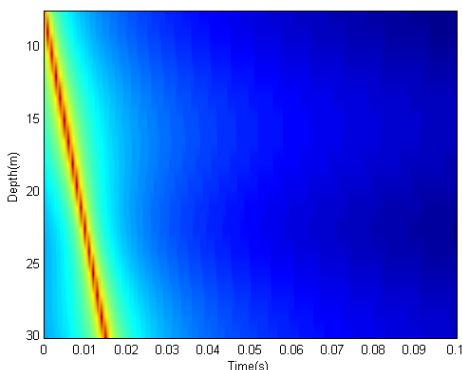
In figure 3, there are cross correlation function of the four simple environments, as well as interference structure of the noise field. The 16 hydrophones are used to simulate and the spacing is 1.5 m. The top hydrophone from the sea surface is 7.76 m and the below one is 30.26 m from the surface. The sea depth was 34m and the sound speed of water column is 1500 m/s.



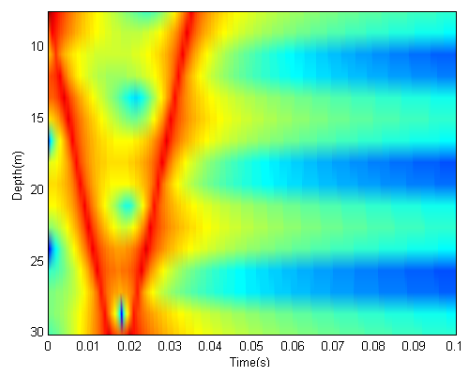
(a) Transfer function in the free space (case 1)



(d) Transfer function with both ocean surface and bottom reflection (case 4)



(b) Transfer function with only ocean surface reflection (case 2)



(c) Transfer function with only ocean bottom reflection (case 3)

**Figure 3.** Cross correlation function of four simple waveguide in time domain

The time domain pictures of cross correlation function are showed in figure 3, which looks like the acoustics ray propagation in vertical waveguide. The free space result (case 1) is similar to the ocean surface reflection result (case2), the only difference is the source form. The former is monopole source and the latter one is dipole source. The case 3 is also similar with case 4 and there is a reflection acoustic ray bounced by bottom. The difference between case 3 and case 4 is also source form.

The above cases are only involved in the simple ocean waveguide where the sound speed profile is invariant. For real ocean environment, sea bottom parameters influence the acoustics propagation obviously. Ocean waveguide can not be only considered water column, in addition the sound speed profiles are different with isothermal in realistic ocean environments. In next section, the experiment data and theory analysis will be used to discuss ocean waveguide included sediment layer influenced on the spatial cross correlation characteristic of ambient noise.

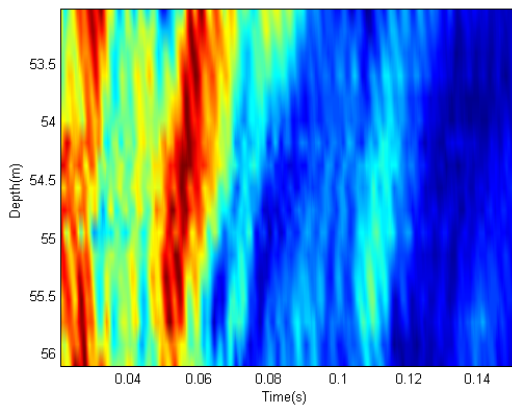
### 3 OCEAN EXPERIMENT AND VERTICAL TRANSFER FUNCTION MEASUREMENT PASSIVELY

#### 3.1 South China Sea experiment

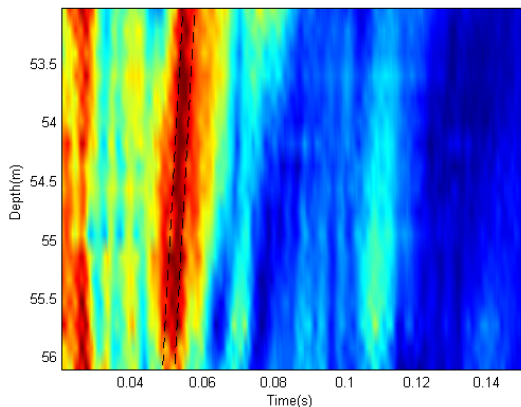
In the South China Sea, the depth of experiment area is 105 m and the sound speed profile is 1515 m/s isothermal. Two VLA are deployed and the each array owns 17 elements. The elements spacing of VLA1 is 0.2 m and the length of VLA1 was 3.2m. The processing frequency scale is 100Hz-3kHz. The element spacing of VLA2 is 0.1 m and the length of VLA2 was 1.7 m. The processing frequency scale is 100Hz-6kHz. The center position of two arrays is 60m below the surface. The slant angle of two arrays is both 25.7° that data come from the pressure sensor in VLAs. The bottom parameters and stratified characteristics are unknown.

#### A. VLA1 result

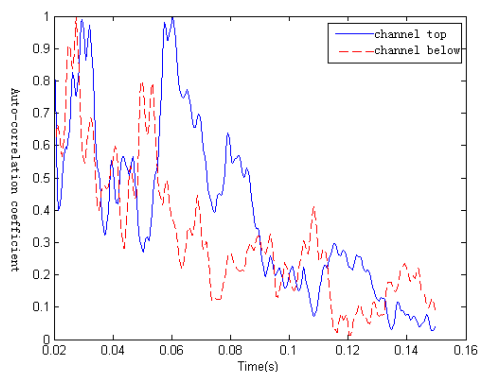
Figure 4 are the time domain results of noise cross correlation function in VLA1. In figure 4 (a) and (b), the x-axis is time scale and y-axis is depth of each channel. Figure 4 (a) is the result without array shape modification, and figure 4 (b) is the result with array shape modification. Figure 5 (a) is the time domain response of cross correlation function of top channel and below channel, and the dashed line in figure 5 (b) indicate the peak value of time domain response.



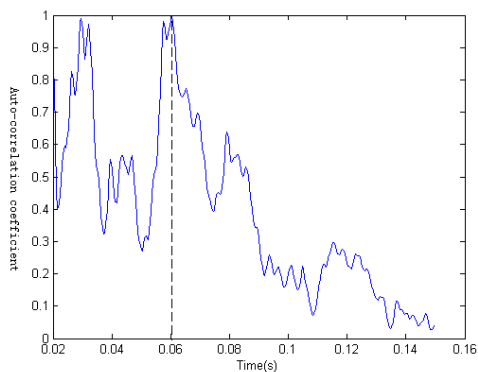
(a) Result without array shape modification



(b) Result with array shape modification  
**Figure 4.** Cross correlation function in VLA1



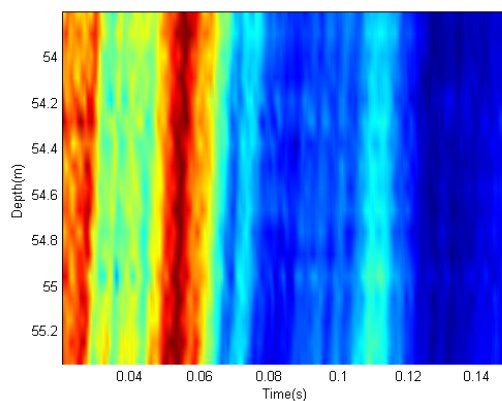
(a) Time domain response of top channel and below channel



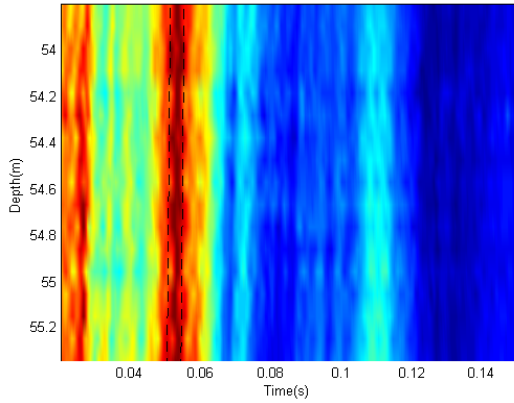
(b) Arrival time of propagation return to top channel  
**Figure 5.** Arrival time of propagation return to top channel and below channel in VLA1

The interference stripe of the dotted line in the figure 4 (b) was the transmission time that the sound ray transmitted from the top hydrophone passing the sea bottom reflecting to the source position, it is as well as the time domain result of the vertical transfer function. The figure 5 (a) is the time domain respond of the transfer function at the top channel. In this figure, the sound ray propagates to the bottom and reflects to the transmitter at 0.0604 s. The arrival time of actual sound ray can be calculated as  $(105-60+1.6)*2/1515=0.0615$  s according to the vertical array position. The time difference of above two results is 0.0011 s. The time difference of respond between the top channel and bottom channel is 0.004 s, amounting to the 6m transmission range.

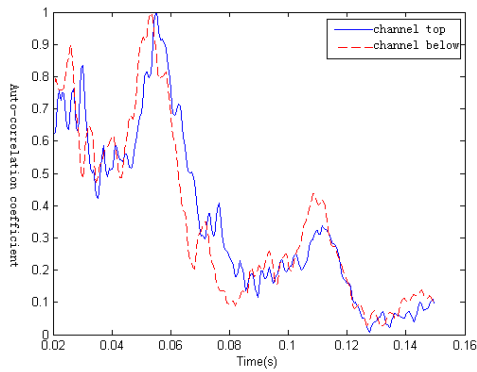
**B. VLA2 result**



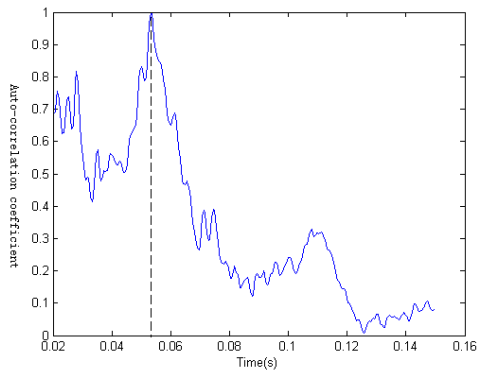
(a) Result without array shape modification



(b) Result with array shape modification  
**Figure 6.** Cross correlation function in VLA2



(a) Time domain response of top channel and below channel



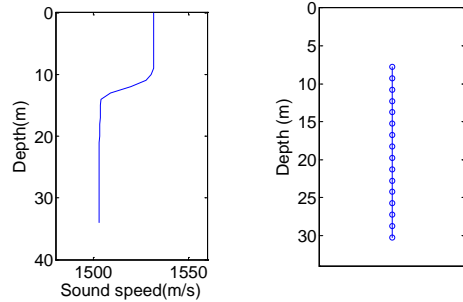
(b) Arrival time of propagation return to top channel  
**Figure 7.** Arrival time of propagation return to top channel and below channel in VLA2

Interpretation of the results from VLA2 is similar to that used for VLA1. In figure 6 the arriving time sound ray of domain respond transfer function at the top channel is 0.0536 s. The arriving time of actual sound ray can be calculated as  $(105-60-1.6)*2/1515= 0.0573$  s according to the vertical array position. The time difference of respond between the top channel and bottom channel is 0.0015 s, amounting to 2.25 m transmission range.

### 3.2 Yellow Sea experiment

#### A. Environment described

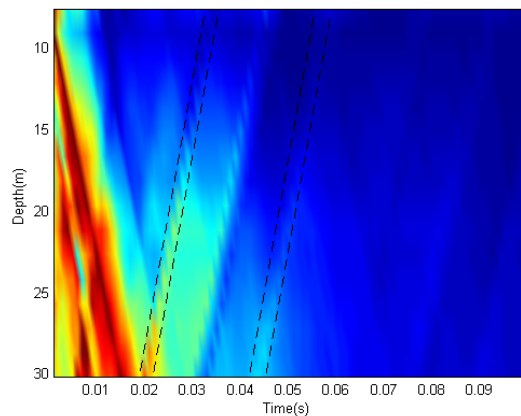
The recorded data described in this paper came from the Yellow Sea experiment in Qingdao. The sound speed profile of the sea experiment area and VLA position are showed in the figure 8. The VLA is consisted in 16 hydrophones. The spacing of the every hydrophone is 1.5 m. The top hydrophone is 7.76 m from the sea surface. The sea depth is 34 m. The bottom parameters describe as: the compressional wave speed is 1649.42 m/s, the density of sediment layer is 1.838 g/m<sup>3</sup> and the attenuation of the sediment layer is 0.5 dB/λ. The thickness of sediment layer is 19.79 m.



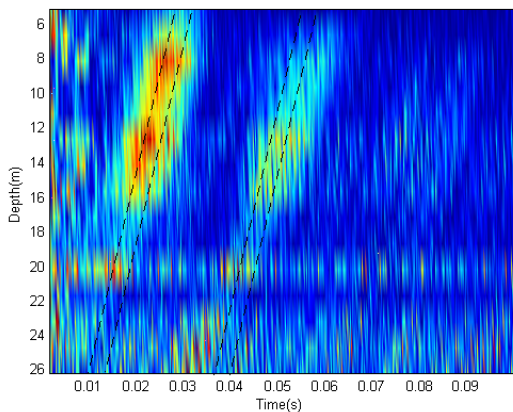
**Figure 8.** Sound speed profile, vertical array location in experiment sea area

#### B. Simulation and experiment result

According to the first section, the cross correlation function of the ambient noise between the each hydrophone element in the vertical array looks like sound ray propagating from one hydrophone to another. For the shallow water waveguide, the ocean ambient noise from every hydrophone is correlated with ambient noise from the top hydrophone. For the whole array, the top hydrophone is assumed as the source and other hydrophones as receivers collected the signal from the top one. From the figure 4, the arrival time structure from the water column to the sediment can be displayed in the vertical cross correlation figure of ocean ambient noise.



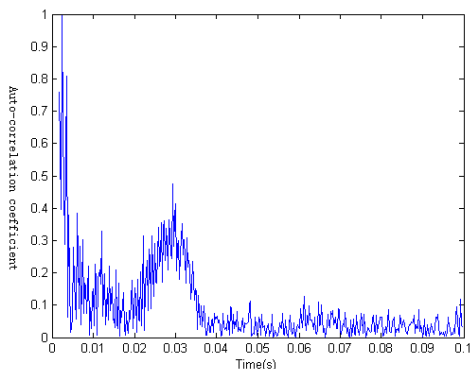
(a) Simulation result



(b) Data result

**Figure 9.** Yellow Sea simulation (a) and experimental (b) result

In the figure 9, (a) is the simulation result and (b) is the experiment data result, both figures own two slant lines. The first slant line corresponds with the sound ray transmission path from top hydrophone transmit to bottom, bounced by bottom and transmit return to the top one. The second slant line is assumed as the reflection result of the first sediment bottom. Considered the bottom inversion result, the arrival time of the reflection sound ray transmission from the first sediment bottom is later 0.024s than transmission from the first sediment top interface. The spacing of the two slant lines approach to the theory estimation result. Figure 10 shows the time domain response of ocean ambient noise cross correlation function at the top hydrophone location in the VLA. According to the theory analysis, the arrival time is 0.032s from the bottom reflection to the transmit source. The peak value occurs at about 0.03s in figure 10. The figure 10 shows that the experiment result is coincidence with the theory analysis.



**Figure 10.** Time domain of vertical correlation in location of top hydrophone

**4 CONCLUSIONS**

The paper compares the ocean ambient noise cross correlation function with sound ray in wave guide, they have so much in common, and thus we can extract the waveguide transfer function from noise cross correlation functions. As a result, the vertical transfer function can be obtained by using the VLA receivers noise signal cross correlation. Fourier transform is used to transform frequency domain form of the cross correlation function into the time domain. According to the experiment data and theory simulation, cross correlation characteristic of ambient noise in the VLA is used to analogy the arrival time structure of sound ray. Furthermore, the depth

from array to the bottom and sediment layer thickness of the sea bottom can be obtained.

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