Toward a simple model for peak pressure of underwater signals from offshore impact pile driving

Marshall V. Hall

9 Moya Crescent, Kingsgrove NSW 2208, Australia

ABSTRACT

A simple model is derived for the peak pressure of the underwater sound radiated when an offshore pipe pile is struck by a hammer. A pipe pile is modelled as a semi-infinite cylindrical shell of an elastic solid. Neglecting coupling between the axial and radial particle velocities results in the former being a solution of the wave equation. The impact generates a pulse of vibration that travels down the pile at the longitudinal sound-speed. At a given time after impact, the axial particle velocity increases exponentially as a function of axial distance, until the peak at a distance proportional to the time is reached. At a given distance, the axial velocity after the peak has arrived decreases exponentially with time. The radial velocity is estimated from the axial velocity using the definition of the Poisson ratio. The radiated sound pressure (which is proportional to the radial acceleration) is found to be proportional to the Poisson ratio and Young's Modulus of the solid, hammer velocity, contact area between hammer and pile, and square of the pile radius; and inversely proportional to the hammer's mass and the sound-speed along the pile. This model is applied to a published scenario for which the radiated sound pressure had been computed using a Finite Element Model. The simple model yields a sound pressure only one-tenth of that produced by the Finite Element Model. Some assumptions used in the model are identified that may explain the disparity.

INTRODUCTION

Offshore pile driving radiates regular pulses of loud noise underwater, and a substantial amount of data has been presented in the literature on the peak pressure and energy of these pulses. The peak pressure at a distance of 10 m can be of the order of 1 atmosphere (220 dB re μPa^2). When an incompressible hammer longitudinally strikes a vertical steel pipe pile with no cushion, the velocity of the pile head rises to a maximum almost instantaneously, and then decays exponentially with a time constant of a few milliseconds. An exponential pulse of axial strain is generated whose depth constant is typically between 10 and 20 m. This pulse travels down the pile at 5 m/ms. If the pile length is at least 30 m, then shallow vibrations prior to 12 ms will be unaffected by reflection from the pile toe. By virtue of the Poisson ratio of steel, the axial strain pulse generates a radial strain pulse (a transient bulge). The radial acceleration of the pile wall associated with this transient bulge radiates sound waves into the surrounding medium, which is assumed here to be water only. At an underwater receiver some distance from the pile, the peak sound pressure will be negligible until the leading edge of the bulge (LEB) reaches the sea surface, and then increase as the LEB departs from that surface to an asymptote as the LEB depth exceeds several times the depth-constant of the bulge. According to standard theory the peak sound pressure radiated by a pulsating pile will be proportional to the water density, the pile wall radial acceleration, the pile radius, and the vertical aperture of the pile; and inversely proportional to the range (providing the range is in the far-field of the pile aperture over the frequency band of interest).

Although the quantity of descriptive data on noise from pile driving is large, there have been few papers that attempt to model the physics of the impact and the consequent sound radiation. A significant such paper was that by Reinhall & Dahl (2011), who used a Finite-Element Model for the sound generated by a simple impact hammer. Their results are entirely numerical and the published paper cannot be examined with a view to ascertaining the relative importance of the input parameters.

The objective of the present paper is to present a simple analytic model for the sound radiated from a semi-infinite pile, since such a model can be examined for the relative importance of the individual input parameters. The various assumptions and approximations that need to be made are explained during the derivation.

THEORY

Assumptions

A pipe pile is modelled as a thin cylindrical shell. The following simplifying assumptions will also be made:

- The pile is semi-infinite in length. Conventional analyses treat finite lengths and thus include echoes from the pile toe; this aspect is beyond the scope of the present analysis.
- The hammer is incompressible
- The hammer strikes the pile instantaneously and uniformly over its face, and does not cause the pile to twist or bend
- Only the sound radiated shortly after the impact will be addressed. The height of the pile face above the water surface is taken to be small.

Solving the Equations of motion

To describe the geometry of the pile and its vibrations, cylindrical co-ordinates are used: the axial and radial distances are denoted by z and r, and the corresponding displacements are denoted by u and w. Azimuthal variation (torsion) is assumed to be negligible. Although the problem of a cylindrical shell being struck longitudinally has not been addressed analytically in the literature, the similar problem of a solid cylindrical rod being struck by an incompressible mass has been addressed by Love (1944, page 431) and Timoshenko & Goodier [henceforth denoted by 'T&G'] (1970, page 497). Both of these analyses neglected the generation of radial waves. In a prior general analysis, Love (page 289) obtained a solution for the case in which torsion vibration is absent and the axial and radial vibrations are independent of azimuth angle. Using the boundary condition that the normal and shear stresses on the cylinder's curved surface are zero, Love showed that for a very slender solid rod the propagation speed is given by

$$c = \sqrt{E/\rho} \tag{1}$$

where E and ρ are the Young modulus and density of the solid. In a later analysis of a thin cylindrical shell, Love (page 546) presented equations of motion for each of the three types of vibration (axial, radial and torsional). The equations for the axial and radial displacements are as follows (except that Love's replacement of the second-order time derivative by minus the square of angular frequency reverts here to the derivative):

$$\frac{\partial^2 u(z,t)}{\partial z^2} - \frac{\upsilon}{a}\frac{\partial w(z,t)}{\partial z} = \frac{1-\upsilon^2}{c^2}\frac{\partial^2 u(z,t)}{\partial t^2} \tag{2}$$

$$\frac{\upsilon}{a}\frac{\partial u(z,t)}{\partial z} - \frac{w(z,t)}{a^2} = \frac{1-\upsilon^2}{c^2}\frac{\partial^2 w(z,t)}{\partial t^2} \tag{3}$$

in which z is distance along the pile from its face, t is time from impact, c is the (constant) longitudinal sound-speed, a is the cylinder radius, and ν is the Poisson ratio. Radial displacement (w) is measured inward from the shell's exterior curved surface, and axial displacement (u) is measured in the direction of positive z. Equations (2) and (3) are consistent with corresponding equations presented by Leissa (1993, page 37), after allowing for Leissa's measurement of w outward from the cylinder axis. The shell wall thickness does not appear in either of these equations, which are accurate providing the wall thickness is no more than around 5% of the radius. One difference between a shell and solid is that, as may be seen from Eqs. (2) and (3), the propagation speed of longitudinal vibrations along a shell is higher by a factor of $1/\sqrt{(1-\nu^2)}$. For steel (ν = 0.29) the propagation speed along a thin shell is 4.5% higher than along a solid rod. This propagation speed will be denoted by q.

Equation (2) has a second order time derivative and its solution will therefore require two initial conditions. At any distance z we will need to specify two combinations of two variables (displacement and particle velocity) at one time. The pertinent initial conditions (when t = 0) are that the displacement is zero for all z and the particle velocity is zero except at z = 0 (the impact). The initial conditions are therefore expressed as:

$$u(z,0) = 0, \; z \geq 0 \tag{4}$$

$$\partial u(z,0)/\partial t = 0, \; z > 0 \tag{5}$$

$$\partial u(0,0)/\partial t = V_0, \; z = 0 \tag{6}$$

where V_0 is the velocity of impact.

As shown by T&G (page 499) for a solid rod, the stress pulse at the cylinder face is an exponential decay with time that commences suddenly when t = 0. Stress (σ) is given by (T&G page 492):

$$\sigma(z,t) = E\,\partial u(z,t)/\partial z \tag{7}$$

where ∂u/∂z is the elastic strain. This pulse will propagate along the pile with little or no dispersion and, at a fixed time t, the stress magnitude will increase with z until the value of z = qt is reached.

Axial vibration

Since Eq. (2) also has a second order distance derivative, its solution will need two boundary conditions. Equation (2) would be a wave equation in u if it did not include a term in w. Equation (3) is an equation of motion in w (but not a wave equation), and includes a term in u. Although coupled partial differential equations may usually be solved simultaneously by taking Laplace Transforms (LT) of each, such a process is beyond the scope of the present paper. Equation (2) has therefore been simplified by neglecting the term in w. The resulting wave equation means that u(z,t) is a function of (z – qt) and/or (z + qt). This constraint on the nature of u(z,t) is equivalent to a boundary condition for the simplified Eq. (2).

The simplified Eq. (2) is solved by assuming that the displacement waveform is travelling in the direction of increasing z and is therefore a function of (z – q t). Only if the pile were of finite length would we need to add a reflection from the toe, which would be a function of (z + q t). As q is a constant, the displacement is an exponential function:

$$u(z,t) = B\{1-\exp[(z-qt)/Z]\}, \; qt > z \tag{8}$$

where B and Z are constants to be determined from the initial and boundary conditions. At any distance z, u = 0 until t = z/q, after which u rises to an asymptote (B).

The second boundary condition arises from the mutual equation of motion for the hammer and pile (which remain in contact for a semi-infinite pile, since there is no reflection). The hammer compresses the pile, which in return decelerates the hammer. The equation of motion for the incompressible hammer is

$$M\,\ddot{u}(0,t) = -AE\,\partial u(0,t)/\partial z \tag{9}$$

where M is the hammer mass, A is the cross sectional area of contact, and the double dot denotes second-order partial differentiation with respect to time.

Substitution of Eq. (8) into (9) yields

$$MB(q/Z)^2 = AE\,B/Z. \tag{10}$$

If we define

$$\Omega = AE/Mq, \tag{11}$$

then Eq. (10) yields

$$Z = q / \Omega . \tag{12}$$

From Eq. (8) the particle velocity is

$$\dot{u}(z,t) = B\Omega \exp[(z - qt)/Z]$$

It follows from Eq. (6) that B = V_0/Ω. (13)

With these values for B and Z, Eq. (8) becomes

$$u(z,t) = (V_0 / \Omega)\{1 - \exp[\Omega(z/q - t)]\} \tag{14}$$

At this stage, the stress predicted by Eq. (14) will be compared with that presented by T&G (page 499) for a mass striking a rod. From Eqs. (7) and (14):

$$\sigma(z,t) = (E\,V_0 / q)\exp[\Omega(z/q - t)] \tag{15}$$

The peak stress magnitude is E V_0/q. In order to adapt this result to a solid rod, we replace q by c and thus obtain ρc V_0, which is equivalent to the peak stress presented by T&G (page 499). Similarly, 1/Ω would equal the time constant presented by T&G if q were replaced by c.

Radial Vibration

If Eqs. (2) and (3) were solved simultaneously without neglecting any terms, then it would be possible to obtain an accurate expression for w(z,t) from Eq. (3). Although an attempt has been made to solve Eq. (3) by substituting Eq. (14) for u, it has been found that the result for w is a sum of sinusoidal functions of time with no damping. If this were correct, a pulse of finite energy would produce an eternal radial vibration of constant amplitude. For the present paper, an alternative approximate result is obtained by applying the definition of the Poisson ratio: radial strain = ν × axial strain.

Since axial strain is given by the ratio of particle velocity to sound-speed (T&G, pages 492 - 494) it follows that:

$$w(z,t)/a = -\upsilon\,\dot{u}(z,t)/q\,,$$

in which the left-hand-side is the radial strain at distance z. The radial acceleration would therefore be

$$\ddot{w}(z,t) = -\upsilon(a/q)\,\dddot{u}(z,t)\,.$$

(The triple derivative of displacement with respect to time is known as the “jerk”). From Eq. (14) it follows that

$$\dddot{u}\,(z,t) = \Omega^2 V_0\, \exp[\Omega(z/q - t)],$$

and the radial acceleration would become, on replacing Ω by q/Z:

$$\ddot{w}(z,t) = -\upsilon a V_0\, q/Z^2\, \exp[\Omega(z/q - t)]. \tag{16}$$

Radiated sound pressure

As remarked by Sherman & Butler (2007, page 445) for a source emitting a mono-tone signal at a frequency of ω = ck:

> the pressure field for the pulsating sphere does not change form as the distance from the sphere increases. However, most acoustic radiators have more complicated pressure distributions in the near field that become approximate spherical waves with directional dependence at sufficient distance from the radiator. A simple example is a uniformly vibrating cylindrical line source of length L and radius a where a is much smaller than both L and the wavelength. It can be considered to consist of a large number of adjacent infinitesimal point sources each of length dz as shown in Fig. 10.4. The differential contribution to the pressure field from each point source is given by Eq. (10.15b) as
>
> $$dp = j\,\frac{\rho c k}{4\pi r}\,dQ\exp(-jkr) \tag{10.21}$$
>
> where dQ = 2πa u dz is the differential element of source strength and u is the radial velocity.

This expression can be expressed in terms of a time derivative if we replace jck u = jω u by ∂u/∂t. In the notation of the present paper, the sound pressure at horizontal range r from a semi-infinite cylinder of radius a in a uniform medium (and neglecting the surface reflection) is given approximately by

$$p(r,t) = \frac{\rho_w\, a}{2}\int_0^\infty \frac{\ddot{w}(z,t)}{R(z)}\,dz\,, \tag{17}$$

where ρ_w is the density of the medium (water) and R is the range from a point on the pile at depth z below the receiving hydrophone ($R^2 = r^2 + z^2$). The peak pressure at a given instant of time will be due to a finite aperture of the pile that will correspond to a spread of travel times no more than a fraction of the period of the highest frequency of interest. It is assumed here that the depth constant of the bulge will be similar to the appropriate aperture. Since the stress pulse is an exponential transient function of depth z, Eq. (17) can be simplified to

$$p(r,t) = \frac{\rho_w\, a}{2\,r}\,\ddot{w}(0,t)\,Z\,. \tag{18}$$

If it is also assumed that the maximum radial acceleration occurs when t = 0 (when R = r), then the peak pressure that corresponds to the peak acceleration given by Eq. (16) is:

$$p\_peak(r) = \rho_w\, a^2\, \upsilon\, V\, q/(2\,r\,Z)\,. \tag{19}$$

MEASURED DATA

The preceding model is now compared with data presented by Reinhall & Dahl (2011). They examined underwater sound pressures from the driving of a hollow steel pile, 31.9 m long with diameter 76.2 cm and wall thickness 2.54 cm (the area of contact was thus 0.061 m^2). The pile was driven approximately 14 m into the sediment in water of depth 12.5 m. The piles were driven using a Delmag D62-22 Diesel Hammer with an impact weight of 6200 kg and energy of 180 kNm. The water sound-speed was set to 1485 m/s.

Reinhall & Dahl reported the following observations of their FEA results:

(i) "rise time in stress at the interface after impact is several orders of magnitude smaller than the time constant". This is consistent with a steel-on-steel impact. Including a cushion would result in a rise time of the order of 1 millisecond.

(ii) "It was found that the average pressure across the top of the pile during impact could be approximated by $p(t)=2.1 \times 10^8 \exp(-t/\tau)$ Pa, .."

Elasticity theory (T&G p 499) gives:

(i) peak stress = EV/c. Adapted to a thin cylinder, this would become $EV/q = \rho c^2 V/q = 7800 \times 5010^2 \times 7.6 /5235 = 2.84 \times 10^8$ Pa. This is 2.6 dB greater than the FEA result of 2.1×10^8 Pa.

(ii) Time constant $1/\Omega = Mq/AE = Mq/(A \rho c^2) = 6200 \times 5235 /(0.061 \times 7800 \times 5010^2) = 0.0027$ s. This is 33% smaller than the FEA result of 0.004 s.

According to Reinhall (personal communication, 2012): "I assumed no explosion effects. The hammer fell freely and had a velocity of approximately 7.6 m/s when contact was initiated. I assumed both the hammer and pile to be elastic (which might partially explain the lower pressure). The contact pressure and decay time was determined using FEA (Finite-Element Analysis)".

Reinhall & Dahl reported sound pressures at horizontal ranges of 8, 12 and 15 m, with most data at 12 m. They deployed nine hydrophones at depths from 4.9 to 10.5 m, and found that the FEA peak sound pressure at 12-m range increased from 50 kPa at 4.9 m to 100 kPa at 9.8 and 10.5 m. The FEA peak pressures were generally in good agreement with the measured data.

The highest frequency at which data were presented in the paper was 2 kHz, for which the period is 0.5 ms. The spread of travel times that would add coherently to increase the peak pressure would therefore be around 0.2 ms, which corresponds to a spread of 0.3 m in travel distance through the water. At a distance of 12 m, a vertical aperture of 3 m would provide signals that arrive at the hydrophone within a window of 0.2 ms from a static source. Since the source is moving (downward) at 5 m/ms, the aperture is increased by a further 1 m. Even though the depth constant Z was around 14 m, the integral of the exponential decay from 0 to 4 m would be around 40% of the integral from 0 to Z.

Substitution of the parameter values from Reinhall & Dahl into Eq. (19) yields a peak pressure of 5 kPa, only one-tenth of their corresponding peak pressure.

DISCUSSION AND CONCLUSIONS

Using a simple model to predict peak pressure of the signal radiated in water from a pile struck by a hammer has underestimated the result predicted by Finite Element Analysis by a factor of 10. This disparity cannot be due to underestimating the stress at the pile face, since this was (slightly) over-estimated. Approximations and assumptions have been made, and it is apparent that the treatment of the travelling bulge as a sound source has been less than rigorous. One weakness in the derivation is that vibration internal to the pile was assumed to be unaffected by the external medium, whereas continual radiation would cause the internal peak displacements to decay with increasing depth. Nevertheless, it is difficult to see how more rigorous treatments of these factors would increase the radiated pressure at all, let alone by a factor of 10. Although the presence of a stiff cushion between pile and hammer does not appear to be relevant to the scenario examined here, it is of interest that such a cushion can result in a higher peak pressure.

In principle, a feasible cause of the disparity could be the neglect of resonances in the radial vibration. Such resonances may arise from a complete solution of Eqs. (2) and (3), and this possibility will be analysed in further studies.

REFERENCES

Leissa, A, 1993, *Vibration of shells*, Acoustical Society of America, New York, USA.

Love, AEH 1944, *A treatise on the mathematical theory of elasticity* (fourth edition), Dover Publications, New York, USA.

Reinhall, PG & Dahl, PH 2011, 'Underwater Mach wave radiation from impact pile driving: Theory and observation', *Journal of the Acoustical Society of America*, vol. 130, pp. 1209 – 1216.

Sherman, CH & Butler, JL 2007, *Transducers and arrays for underwater sound*, Springer, New York, USA.

Timoshenko, SP & Goodier, JN 1970, *Theory of elasticity* (third edition, international student edition), McGraw-Hill Kogakusha Ltd, Tokyo, Japan.