# Geoacoustic perspectives: what have we learned about modelling sound propagation in the ocean bottom

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## ABSTRACT

This paper reviews the development of geoacoustic inversion as a statistical inference process to estimate geoacoustic model parameter values and their associated uncertainties. Nonlinear inversion methods are examples of model-based signal processing techniques that were enabled by the introduction of efficient numerical techniques for searching multi-dimensional model parameter spaces. Applications of inversions based on acoustic pressure field data (matched field processing methods) are discussed and analysed. The paper concludes by pointing out limitations in the present day inversion techniques that can severely limit performance, and discusses some new approaches that provide robust performance without compromising the accuracy of the estimated model parameters.

## INTRODUCTION

The interaction of sound with the ocean bottom is generally acknowledged to have a significant impact on the acoustic field in the ocean, especially in shallow water. Over the past several decades, there has been a concentrated research effort in underwater acoustics to understand the physics of sound propagation in the ocean bottom. This work has led to the general practice of using geoacoustic models, – profiles of the sound speed and attenuation, and density of ocean bottom materials (Figure 1) – to describe the bottom for applications such as prediction of transmission loss, analysis of sonar performance, environmental impact assessments of sound on marine fauna, etc. Much of the research was focused on developing inversion methods to determine geoacoustic model parameter values from the information about the model contained in measurements of the acoustic field – or quantities that can be derived from the field – in the water.

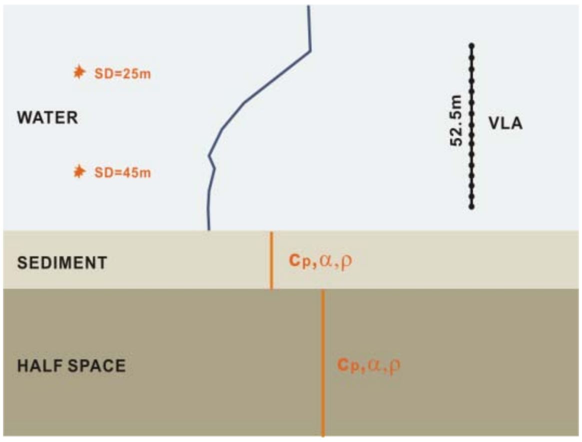


**Figure 1**. Geoacoustic model indicating a simple layered structure of sound speed, $c_p$, attenuation, $\alpha$, and density, $\rho$.

This paper reviews the stages in the development of geoacoustic inversion as a statistical inference process to estimate geoacoustic model parameter values and their associated uncertainties. The inversion methods fall into two main categories, linear methods that assume small changes from an initial profile, and methods that are fully non-linear (Chapman, 2008). The non-linear methods are examples of model-based signal processing techniques that were made possible by the introduction of efficient numerical techniques for exploring multi-dimensional model parameter spaces. Inversion methods based on both approaches have been benchmarked in exercises with simulated data (Tolstoy et al., 1998; Chapman et al., 2003), and have also been applied for use with data from experiments in many different ocean bottom environments - with varying degrees of success (e.g. Special issue on Geoacoustic Inversion, IEEE J. Oceanic Eng., 2003; Caiti et al., 2006).

The focus in this paper is on the development and application of non-linear inverse methods that make use of acoustic pressure field data (matched field processing (MFP) methods). The technique of matched field inversion (MFI) is analysed in terms of its performance in estimating realistic geoacoustic models in shallow water waveguides. Examples are also presented that demonstrate severe limitations of MFI in waveguides in which the water sound speed profile is uncertain. The paper concludes with a discussion of some new approaches that provide robust performance without compromising the accuracy of the estimated model parameters.

## GEOACOUSTIC INVERSION METHODS

### Matched field processing

The acoustic field measured in the ocean contains information about the properties and structure of the water column and the ocean bottom. Knowledge of the physical properties of both systems, the ocean and its boundaries, is fundamentally important. The relationship between the physical properties of the ocean medium and the acoustic field in the ocean is expressed by the acoustic wave equation. It is a non-linear mapping, and analytic solutions for the field can be obtained only for very simple ocean waveguide models (e.g. Jensen et al, 1995). Sophisticated numerical methods for calculating acoustic fields in realistic ocean waveguides were developed primarily in navy laboratories in the 1970s, and research continues in improving these techniques to the present time.

Unlike the forward problem of calculating the field from a set of ocean environmental parameters, the inverse problem of inferring the properties of the ocean medium from acoustic field data is inherently non-unique. The origins of MFI in underwater acoustics trace back to the paper by Homer Bucker, who suggested that source location in the ocean could be determined by matching measured data with calculated replicas of the acoustic field (Bucker, 1976). MFP was viewed as a correlation process; the maximum in the correlation between the data and replicas occurred for the correct source position in range and depth in the ocean (Porter, 1993). The relationship between the measured and modelled fields at an array of hydrophones is conveniently expressed in terms of the Bartlett processor, $B_f(r,z)$

$$B_f(r,z) = |\boldsymbol{Q}_f^{\dagger}(r,z)\boldsymbol{P}_f(r,z)|^2/|\boldsymbol{Q}_f(r,z)|^2|\boldsymbol{P}_f(r,z)|^2. \qquad (1)$$

Here $\boldsymbol{Q}_f$ is the vector of modelled fields for a set of waveguide environmental parameters, $\boldsymbol{m}$ (assumed to represent the 'true' waveguide); $\boldsymbol{P}_f$ is the vector of measured data and $f$ is the sound frequency. The inverse problem of source localization by MFP was initially implemented as a straightforward grid search over range and depth, and several examples were reported of successful applications in experiments.

## Matched field inversion

The more general application of MFP as an inversion technique for determining environmental properties of the waveguide was not as direct. The basic premise was similar: for a known experimental geometry, the best match between measured and modelled fields was obtained for the waveguide model that represented the true ocean environment. However, the process of evaluating possible models of the environment involved an extensive search over a multi-dimensional model parameter space. Although numerical methods for calculating sufficiently accurate replica fields were available in the 1980s, it was not feasible to carry out grid searches with the available computing resources at the time.

The breakthrough that enabled geoacoustic inversion with MFP occurred in 1990 when Frazer introduced simulated annealing as an efficient global search technique (Basu and Frazer, 1990). MFI was thus formulated as an optimization problem with four basic components:

- A prior geoacoustic model for the waveguide environment
- An accurate method for calculating replica fields
- A cost function for comparing measured and modelled acoustic fields
- An efficient search method for navigating the model parameter space

The prior geoacoustic model was designed based on the best available knowledge of the local environment. This involved assessment of 'ground truth' information from sediment cores and grab samples, and high resolution seismic surveys. The form of the prior model determined the type of geoacoustic model that was inverted. Model structure was generally based on homogeneous or gradient layers to represent the sediment material in the ocean bottom, and the distribution of values for the model parameters was assumed to be uniform within the bounds that were set. The water sound speed profile was usually assumed to be known from measurements at the experimental site.

The cost function was generally based on the Bartlett matched field processor; models tested in the search process were selected or rejected based on the change in the cost function. Convergence was controlled either by pre-selecting the number of iterations, or by a criterion that set a minimum value for the change of the cost function (Lindsay and Chapman, 1993)

Inversions based on simulated annealing as the global search method were reported in the early 1990s (Collins et al., 1992; Lindsay and Chapman, 1993). Simulated annealing is an example of a general approach known as importance sampling for efficiently navigating model parameter spaces. By analogy with a thermodynamic cooling process, SA uses a Boltzmann criterion to allow models that do not decrease the cost function. This feature allows the search to move out of areas of local minima in the model parameter space, thus enabling a more extensive search. The genetic algorithm is another example of a search technique based on importance sampling; this method was introduced for MFI by Gerstoft (1994) and is widely used today. A number of hybrid search methods were also developed that combined global and local search processes such as the downhill simplex method, e.g. simulated annealing and downhill simplex (Dosso et al., 2001); genetic algorithm and Gauss-Newton (Gerstoft, 1995); genetic algorithm and downhill simplex, (Musil et al. 1999). Two benchmark workshops sponsored by the US Office of Naval Research demonstrated that these sophisticated inversion methods were highly successful in tests with simulated data (Tolstoy et al, 1998; Chapman et al., 2003). However, some serious issues about the performance of the methods with experimental data remained unsolved.

## Limitations of optimization inversions

Results of inversions using simulated annealing were conventionally presented in terms of the annealing history of each model parameter during the search process. The example in Figure 2 shows results from a well-designed inversion based on a hybrid search method (simulated annealing with downhil simplex): the allowed values for each model parameter were well sampled during the initial phase of the search (to about 10000 steps). Subsequently, the process fixed on subsets of the values which optimized the cost function and remained in those regions. The search was completed in about 30,000 steps, during which the 'temperature' decreased from an initial high value. The spike at the end of the search results from a final 'quenching' of the local downhill simplex algorithm to refine the optimal values.

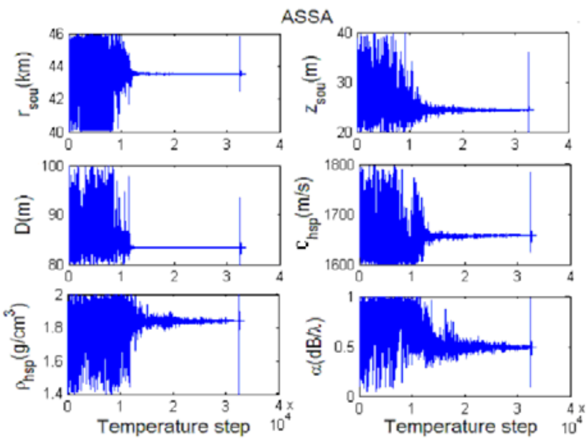


**Figure 2**. Annealing history from a hybrid search process for estimating model parameters of a half-space geoacoustic model (Chen et al., 2006).

Note that this example shows that the search included geometrical parameters of the experimental arrangement in addition to the geoacoustic parameters.

The annealing history provides only an indication of the optimal values of the search process. A sense of how well each parameter was estimated is obtained from a scatter plot of the cost function values for each model that was tested. Figure 3 shows the scatter plots for the same model parameters. This display provides an indication of the hierarchy of sensitivity of the model parameters, and a sense of which ones were well estimated in the inversion. Scatter plots that appear as 'tornadoes' ( e.g. the range, $r$, source depth, $z_{sou}$, water depth, $D$, in the top three panels) indicate well-estimated parameters, whereas those that appear broader at the base (e.g. those in the lower panels: density, $\rho$, sound speed, $c$, and attenuation, $\alpha$, of the half-space) are less well estimated. The flatness of the display also indicates that the parameter is not sensitive, i.e. the data from the experiment do not contain any useful information about the parameter.

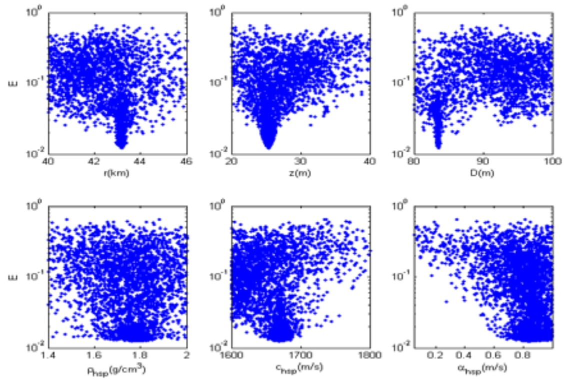

**Figure 3**. Cost function values $(1 - B_f(r,z))$ for the geoacoustic model parameters from the optimization inversion in Figure 1 (Chen et al., 2006).

Figure 3 reveals the inherent weakness of the optimization approach. Optimization inversions always generate an 'optimal' estimated value for each parameter. However, it is usually the case that some model parameters are insensitive, so that the 'optimal' values of such parameters do not significantly affect the acoustic field. As a result, inversions were often over-parameterized, with meaningless values for some of the model parameters. Optimization inversions did not provide a statistically valid measure of the uncertainty of the estimated values.

### Model parameter correlations

An inherent problem in geoacoustic inversion arises due to correlations that exist between model parameters. Optimization inversions addressed this issue by re-parameterizing the model parameter set during the initial stages of the inversion (Collins and Fishman, 1995). Although this enabled more efficient navigation of the model parameter space in the search process, it did not eliminate the basic problem. The fundamental issue is that, due to the model parameter correlations, errors in the estimate of one parameter will impact the estimates of all the others.

A simple but striking example of this effect is the acoustic 'mirage' in source localization by MFP. D'Spain et al. (1998) showed that the range and water depth were strongly correlated in matched field source localization. Since water depth and source range are not ever known exactly in experiments, the uncertainty in these parameters generates errors in all other estimates in the inversion. Another well known example is the correlation between source range and sound frequency through the waveguide invariant (Lysanov and Brekhovskih, 1993). The common practice of using multi-frequency data in inversions mitigates the impact of this effect to some degree, but does not eliminate the basic problem.

## INVERSION AS STATISTICAL INFERENCE

### Bayesian inference

The complete solution of the inverse problem involves providing an estimate for the model parameters and a measure of the uncertainty of the estimates. Some researchers reported attempts to generate probabilities of the estimated parameter values that were generated in the search process (Gerstoft and Mecklenbrauker, 1998; Jaschke and Chapman, 1999). However, the full resolution of the inverse problem as a statistical inference process was provided by Dosso (2002a; 2002b) who introduced Bayesian inference (Sen and Stoffa, 1996) for geoacoustic inversion in underwater acoustics.

Bayes' relationship between measured data, $\boldsymbol{d}$, and a set of environmental model parameters, $\boldsymbol{m}$, is expressed in terms of conditional probabilities:

$$P(\boldsymbol{m}|\boldsymbol{d})P(\boldsymbol{d}) = P(\boldsymbol{d}|\boldsymbol{m})P(\boldsymbol{m}). \qquad (2)$$

Here, $P(\boldsymbol{m}|\boldsymbol{d})$is the conditional probability of the model given the data, $P(\boldsymbol{d}|\boldsymbol{m})$ is the conditional probability of the data given a model $\boldsymbol{m}$, and $P(\boldsymbol{m})$ is the prior information about the model $\boldsymbol{m}$.

The complete solution of the inverse problem is given by $P(\boldsymbol{m}|\boldsymbol{d})$, the *a posteriori* probability distribution (or PPD) of model parameter values. It is evident from (2) that Bayesian inversion involves an interaction between the information about the model that is contained in the data and the prior knowledge about the model. If there is no information in the data about a model parameter, the probability of that parameter is close to the original prior probability distribution. Otherwise, the final probability distribution is determined by the information contained in the data.

The relationship between the data and the set of environmental model parameters can be interpreted in terms of the mismatch between the measurement and a prediction of the measurement, $\boldsymbol{q}$, based on the model:

$$\boldsymbol{d} - \boldsymbol{q}(\boldsymbol{m}) = \boldsymbol{n}. \qquad (3)$$

Here, the mismatch $\boldsymbol{n}$ can be interpreted as noise arising from either the noise in the experimental data itself or theory errors due to differences between the environmental model and the real earth, or differences caused by an inaccurate model of the physics of the problem (in this case, the wave equation). The distribution of $\boldsymbol{n}$ is generally not known.

Bayesian inversion is implemented by assuming that the conditional probability of the data for a given model, $P(\boldsymbol{d}|\boldsymbol{m})$, in (2) can be expressed in terms of a likelihood function of the data and model mismatch, $E(\boldsymbol{m},\boldsymbol{d})$:

$$E(\boldsymbol{m},\boldsymbol{d}) = [(\boldsymbol{d} - \boldsymbol{q}(\boldsymbol{m}))^{\dagger} C_d^{-1} (\boldsymbol{d} - \boldsymbol{q}(\boldsymbol{m})), \qquad (4)$$

where $C_d$ is the data error covariance matrix (Dosso, 2002). In many applications, the assumption is made that the covariance matrix is diagonal. However, this condition is not usually correct, and some attempt must be made to evaluate $C_d$ in the inversion. This involves making assumptions about the statistics of the data mismatch distribution, and these must be verified by statistical tests (Dosso et al., 2006; Jiang and Chapman, 2007).

Although the complete solution of the inverse problem is given by the PPD, it is a multi-dimensional distribution that is difficult to visualize. Its interpretation in terms of model parameter estimates and their uncertainties involves computation of the properties of the PPD, such as the maximum *a posteriori* estimate (MAP), the mean values and covariances, and marginal probability distributions. Parameter uncertainties can be quantified in terms of credibility intervals, i.e. the $\gamma$% highest probability density interval that represents the minimum width interval that contains $\gamma$% of the marginal probability distribution.

### Limitations of MFI

Inversions based on the Bayesian formulation were applied to experimental data from various different experiments, with remarkable success in estimating geoacoustic profiles that compared favourably with ground truth information. However, most of the experiments were carried out at sites where the ocean environment was benign for MFI: constant water depth and minimal variability of the ocean sound speed profile and the sediment materials and structure over the track of the experiment. For these conditions, the inversions could be carried out assuming that the sound propagation was independent of range. An example of Bayesian inversion with experimental data is discussed here that demonstrates the performance of the method, and reveals the fundamental limitations of MFI in strongly variable ocean environments (Jiang and Chapman, 2009).

The experiment was carried out in 2006 near the edge of the continental shelf break off the New Jersey coast of the eastern USA (Tang et al., 2007). The site is strongly influenced by internal waves, eddies and fronts that are shed from the Gulf Stream that passes offshore. These features create a highly variable sound speed profile in the ocean, with short time scales of the order of minutes and spatial variability scales of the order of a few km. An example of the sound speed variability at the site during the experiment is shown in Figure 4. The profiles shown were measured over a 4-hour period at stations along an 8-km track.

The data used in the experiment consisted of multiple continuous wave (CW) tones that were transmitted from a ship that held station at a distance of 1 km from a moored vertical line array. The array consisted of 16 hydrophones at spacings of 3.75 m, with the bottommost sensor about 8.2 m above the sea floor. The water depth was ~79 m over the propagation path. Data from 7 CW tones from 53–703 Hz were used simultaneously in the inversion.

The data from this experiment presented a significant challenge for MFI due to the strong variability of the sound speed profile in the water over the experimental track. To account for the variability, the sound speed profile was included as an unknown in the inversion, using empirical orthogonal functions (EOFs) to account for the observed variability in the profile. Consequently, a total of 17 parameters were required in the inversion: 4 geometrical parameters of the experimental arrangement (source range and depth, water depth, and array tilt); 4 EOFs for the sound speed profile in the water; and 9 geoacoustic parameters of a single layer model of the bottom in which the sediment was modelled as a gradient layer for the sound speed and density.

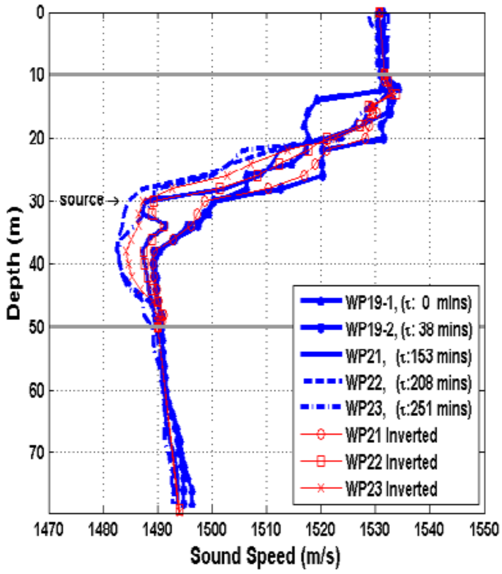


**Figure 4**. Sound speed profiles measured along the experimental track at the New Jersey site.

The marginal densities for the geometrical parameters are shown in Figure 5. These parameters are highly sensitive in the inversion, and the estimated values compared very well with ground truth data from the experiment. The vertical lines represent the 95% HPD limits.

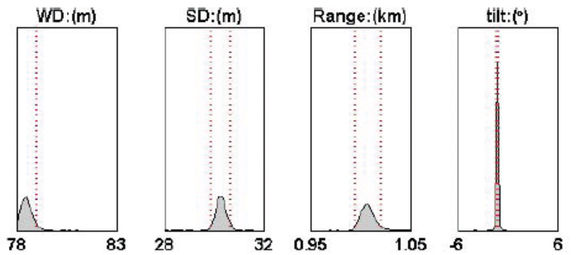


**Figure 5**. Marginal probability densities for the water depth, *WD*, source depth, *SD*, Range and array tilt.

Similar results were obtained for the 4 EOFs, and the estimated sound speed profile derived from the EOFs is shown in Figure 6. The assumption in the inversion was that a single profile could account for the changes in the water sound speed along the propagation path.

Marginal densities for the geoacoustic model parameters are shown in Figure 7. The vertical dotted lines show the 95% HPD limits. The densities for the most sensitive parameters, sediment layer thickness, *H*, the sound speeds in the sediment, $c_{p1}$ (top) and $c_{p2}$ (bottom) and basement, $c_{pb}$, and the sediment density, $\rho_1$, are peaked within the parameter bounds, indicating that these parameters were well estimated in the inversion. However, the marginal densities for the other parameters were relatively flat, indicating that the data did not contain significant information about these parameters.

These results are typical of those from other matched field inversions: the most sensitive parameters are generally the sound speeds in the uppermost layers of sediment (within a few wavelengths of the sea floor). A particularly striking result from the inversion is the accurate estimate of sediment thickness. Ground truth surveys revealed a strong sub-bottom reflector at a depth of about 20 m that was ubiquitous over the experimental area. The inversion was also sensitive to a slow sound speed layer within the sediment above the basement reflector (Ballard et al, 2010). Although the detailed structure within the sediment could not be resolved with these data, the presence of the low speed layer was inferred from the negative gradient of sound speed within the sediment.

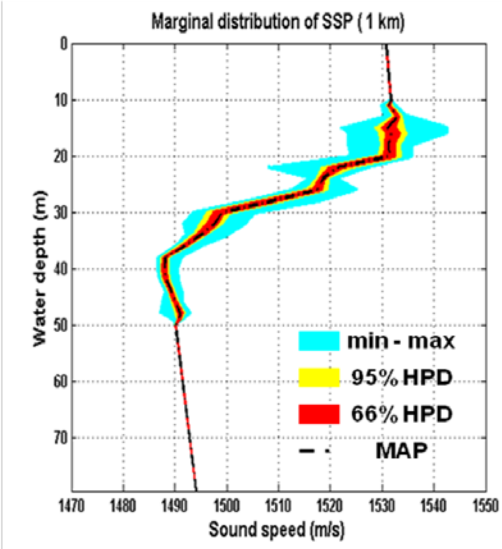


**Figure 6**. Estimated sound speed profile in the water.

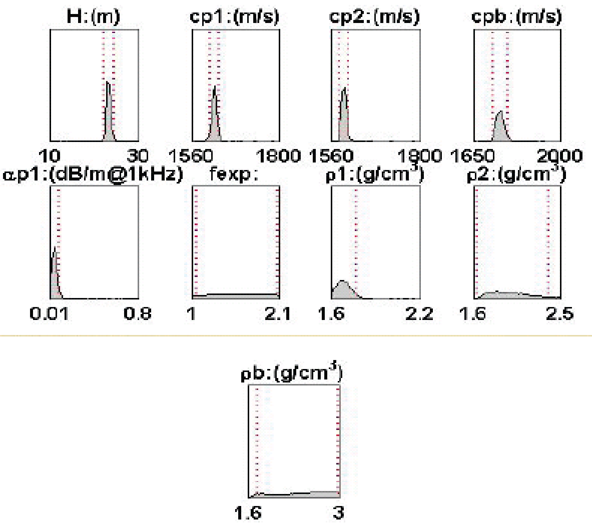


**Figure 7**. Marginal densities for the geoacoustic model parameters estimated in the inversion.

Attenuation is interpreted as an intrinsic loss in the sediment, and was modeled in this inversion as frequency dependent, $\alpha_0(f/f_0)^\beta$, where $f_0$ = 1 kHz. The results indicated that the inversion with data from a range of 1 km was not sensitive to attenuation: the marginal density for the constant, $\alpha_{p1}$, was shifted to the lower bound, and the density for the exponent, $f_{exp}$, was flat. However, the data are sensitive to other mechanisms that remove energy from the propagation plane, such as scattering. Since the loss accumulates with range, data from greater ranges likely contain more information about this parameter.

Although the inversion was successful in providing accurate estimates of the geoacoustic model, the overall success of the same approach for other data sets at longer ranges was not repeated. The success of the inversion reported here depended on the assumption that the sound speed variation in the water column could be represented by a single profile based on the observed sound speed variations. This assumption was not upheld for data from ranges of 3 km and 5 km from the same experiment. Oceanographic data from moored sensors revealed that internal waves passed through the experimental site when the longer range data were obtained. Knowledge of the full range dependence of the sound speed profile is required for inverting these data.

This example indicates the fundamental weakness of model-based inversions such as MFI. If the environmental variation cannot be modelled, the inversion will fail. However, the degree of variability that will allow simple assumptions such as a single profile is not known. And even for simple assumptions, the increased computational load of including additional model parameters as unknowns in the inversion is a significant issue.

Apart from the issues mentioned above, there are other challenges that need to be addressed. Most of the inversions reported to date have been restricted to low frequencies (< 1 kHz) for which the sea floor and sub-bottom layer interfaces are assumed to be smooth. Inversions at higher frequencies must address rough surface scattering losses in modelling the acoustic field. The impact of shear wave propagation in the bottom has been addressed in some inversions, but this issue is generally ignored. Another important issue is the assumption of 2-D sound propagation. In most cases, this assumption is valid. However, in experimental geometries that involve propagation across a sloping sea bottom, 3-D propagation effects must be considered. An example reported by Jiang et al. (2006) demonstrated the impact of 3-D sound propagation on MFI at a site in the Florida Straits. In this case, sound refracted along the slope could be removed by spatial filtering; otherwise, a 3-D sound propagation model is required (Sturm et al., 2009).

Ocean sediments are porous media, and there has been significant research effort in developing theories of sound propagation sediment materials. Among the most well known are the Biot theory (Biot, 1956; 1962), and the more recent theories based on grain shearing by Buckingham (1997; 1999; 2007). The critical issue is the dispersion of sound speed and attenuation in sediments: experiments show that the frequency dependence of attenuation in sand sediments is non-linear within the low frequency band less than 5 kHz. However in most applications of MFI, sound propagation has been modelled using viscous fluid models (e.g. normal mode; parabolic equation), or in some cases visco-elastic models. These methods inherently assume linear frequency dependence for attenuation.

The impact of using more appropriate models for sound propagation in marine sediments has not been examined extensively in MFI. One of the benefits of using the grain shearing theory, for instance, may be in obtaining a more efficient set of model parameters for sampling the PPD. The theory provides analytic expressions for the sound speed, attenuation and density in terms of more fundamental physical parameters (such as porosity, compressional and shear grain contact stress) that are independent (Buckingham, 1999; 2007).

## OTHER APPROACHES

There is no simple remedy to fix model-based approaches such as MFI for conditions in which there is insufficient knowledge of the waveguide environment. A reasonable alternative approach is to use quantities derived from the acoustic field in the inversion, instead of the measured pressure. Although this usually requires special signal processing to extract the observable, there are clear benefits if modelling the observable is not sensitive to variability in ocean waveguide properties. One example is the use of travel time. Jiang et al. (2010) reported an inversion of relative travel times between sub-bottom and sea floor broadband signal arrivals to estimate sound speed and attenuation in the sediment. The experiment was designed to provide a tomographic sampling of the sediment using multiple source depths and a vertical hydrophone array at very short range. The data (shown in Figure 8 for a single source/receiver pair) are more robust to uncertainty in the water sound speed profile due to the relatively short range (~ 200 m), assuming that the sound speed profile is adequately sampled at the site during the experiment.

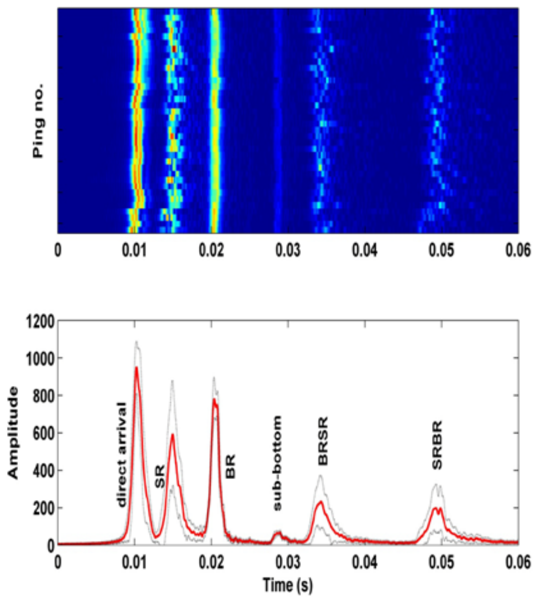


**Figure 8**. Matched filtered broadband data from 1-s linear frequency modulated sweeps transmitted for one minute. The sub-bottom reflection is clearly seen about 10 ms after the sea floor reflection (BR).

Other quantities such as the sea bottom reflection coefficient derived from broadband data (Holland and Osler, 2000; Holland et al., 2005), modal wave numbers extracted from CW data (Ballard et al., 2010), and modal dispersion from time-frequency analysis of broadband signals (Potty et al., 2003) have been used successfully.

The approach by Ballard et al. (2010) is particularly interesting because it provides an estimate of the range dependence of the geoacoustic profile along the experimental track. The method extracts the wave numbers of propagating modes and follows the change in wave number as the source opens range along the track. Although it requires independent information about the layered structure (obtained from two way travel time data from a chirp sonar survey along the track), the method is one of few that addresses range dependence successfully. It is worth emphasizing that the Bayesian formalism can be applied for inversion of all these different types of data.

Perhaps the most promising new approaches are those that make use of ambient noise. The use of ambient noise measured on a vertical array as a fathometer has been demonstrated by Siderius et al. (2006). Recently, Quiano extended this approach for geoacoustic inversion using the wind noise measured by the array as the sound source (Quiano et al., 2012). The method inverts the broadband reflection coefficient that is estimated from wind noise data on the array. The estimate of reflectivity is self-calibrated, and the reflection coefficient inversion is robust to uncertainty in the water sound speed profile. This is also true of the reflection coefficient inversions of controlled source data as proposed by Holland (Holland et al., 2005).

Finally, a promising technique that is robust to uncertainty in both the experimental geometry and the water sound speed profile was reported by Bonnel (Bonnel et al., 2011). The method is based on estimating the modal dispersion from single hydrophone data using a signal processing technique known as warping. Although the use of modal dispersion data for estimating geoacoustic model parameters is not new, warping enables the inversion of relatively short range data for which the modes are not clearly separated in time. Warping transforms the non-linear dispersion relationship in the original time-frequency domain to single tones at frequencies near the modal cut-off frequencies in the warped domain (Figure 9). The warping operation is reversible, so that the modes that are resolved in the warped domain can be filtered and transformed back to the original time-frequency space.

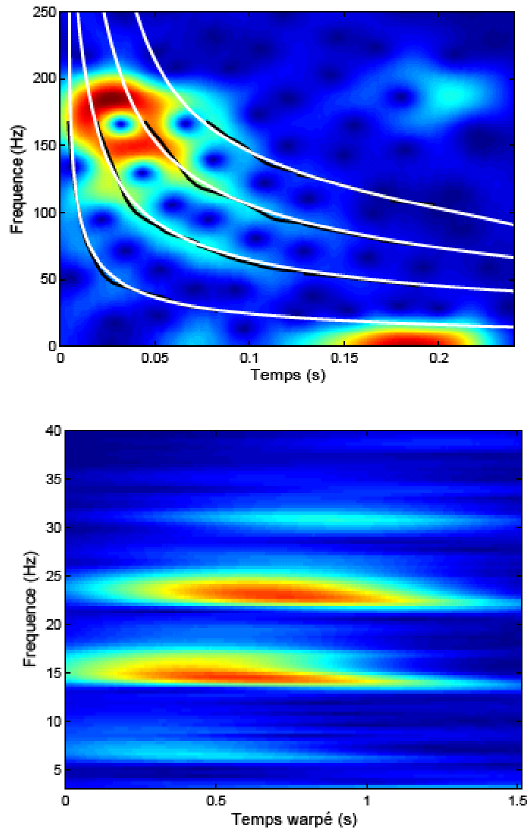


**Figure 9.** Modal dispersion in the original time-frequency domain (top panel) and in the warped domain (bottom panel) for a broadband light bulb signal at a range of 7 km.

The method was applied successfully to broadband light bulb data from the New Jersey shelf experiment (Bonnel and Chapman, 2011) to estimate the parameters of the single layer geoacoustic model that was used for MFI as discussed previously. Figure 9 shows the initial time-frequency display of the modal dispersion (upper panel) and the subsequent display in the warped domain; four modes are resolved in the warped domain (lower panel). The black curves in the upper panel indicate the estimated modal dispersion curves, and the white curves are the modelled dispersion curves based on the estimated sediment model parameters.

## SUMMARY

This paper reviewed the development of geoacoustic inversion in underwater acoustics as a statistical inference method. The widely used technique of matched field inversion was examined to display its advantages and discuss its fundamental limitations. MFP is an example of model based inversion: model parameters are estimated by comparing measured data with calculated replicas of the data. In underwater acoustics, the wave equation describes the physical interaction of sound with the ocean medium, and efficient and accurate numerical techniques have been developed for modelling the acoustic field for realistic ocean environments. The Bayesian formalism for MFI provides the complete solution to the inverse problem: estimates of the model parameter values and statistically valid measures of their uncertainties are derived from the *a posteriori* probability density. The marginal probabilities derived from the PPD indicate the degree to which the data contain information about the model parameters. However, if there is uncertainty due to variability in the properties of the ocean environment, model-based inversions such as MFI can fail.

New approaches that are robust to uncertain knowledge of the ocean properties and the experimental geometry provide some options for alternative methods for model-based inversion of geoacoustic model parameters. A few of these methods, such as time-frequency analysis of broadband data, reflection coefficient inversion and travel time tomography were briefly discussed.

## ACKNOWLEDGEMENTS

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