Prediction of radiated sound power from vibrating structures using the surface contribution method

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ABSTRACT

A common measure for near-field acoustic energy of a vibrating structure is the acoustic intensity, which usually has positive and negative values that correspond to energy sources and sinks on the surface of the radiating structure. Sound from source and sink areas partially cancel each other and only a fraction of the near-field acoustic energy reaches the far-field. In this paper, an alternative method to identify the surface areas of a vibrating structure that contribute to the radiated sound power is described. The surface contributions of the structure are based on the acoustic radiation modes and are computed for all boundaries of the acoustic domain. In contrast to the sound intensity, the surface contributions are always positive and no cancellation effects exist. To illustrate the method, the radiated sound power from a resonator is presented.

INTRODUCTION

Prediction and control of interior and exterior structure-borne sound is important in many engineering applications such as aircraft, aerospace vehicles, automobiles and marine vessels. For interior noise problems, a method to predict the contribution to radiated sound from individual components of a vibrating structure was developed by identifying the contribution of each node of a boundary element model to the total sound pressure (Ishiyama et al. 1988).

For exterior noise problems, the sound intensity is commonly used to analyze contributions of vibrating surfaces to the radiated sound power. Other methods to identify acoustic energy source areas on a vibrating structure include the inverse boundary element technique (Ih, 2008) and near-field acoustic holography (Maynard, 1985). The concept of the supersonic acoustic intensity was introduced by Williams (1995, 1998) to identify only those components of a structure that radiate energy to the acoustic far-field. Since subsonic wave components of the vibrating structure only contribute to evanescent acoustic energy in the near-field, these wave components are filtered out. Only the remaining supersonic wave components, which correspond to the resistive part of sound intensity, radiate acoustic energy to the far-field.

This paper presents a new method to compute the surface contributions to the radiated sound power from a vibrating structure. The surface contributions are based on the acoustic radiation modes (Cunefare and Currey, 1994; Chen and Ginsberg, 1995), and are computed for every node of a boundary element mesh of the radiator. In contrast to the sound intensity which can be either positive or negative and as such results in cancellation effects of energy on the surface of the vibrating structure, the surface contributions are always positive. Hence the surface contributions will directly indicate which parts of the surface contribute to the radiated sound power, while the sound intensity may yield much different values over similar surface regions due to the cancellation effects and thus falsely predict the surface contributions to the radiated sound power. To illustrate the difference between the sound intensity and the continuous surface contribution to the radiated sound power from a vibrating structure, a numerical example corresponding to an open resonator composed of two parallel plates is presented.

RADIATED SOUND POWER

Sound Power and Sound Intensity

For exterior acoustic problems, the well-known Helmholtz equation is given by

$$\left(\nabla^2 + k^2\right)p = 0 \tag{1}$$

where p is the acoustic pressure and k is the wave number. Discretisation of the acoustic domain leads to the following linear system of equations (Marburg and Nolte 2008)

$$\mathbf{Hp} = \mathbf{Gv} \tag{2}$$

where $\mathbf{p}$ is the acoustic pressure vector, $\mathbf{v}$ is the particle velocity vector in the normal direction, and $\mathbf{G}$, $\mathbf{H}$ are the boundary element matrices. The radiated sound power P is defined as (Marburg et al. 2013)

$$P = \frac{1}{2}\Re\left\{\int_\Gamma p v_n^* \,\mathrm{d}\Gamma\right\} = \int_\Gamma \mathbf{I}\cdot\mathbf{n}\mathrm{d}\Gamma \tag{3}$$

where $\Re$ denotes the real part of a complex number, * denotes the complex conjugate, v_n is the particle velocity in normal direction, $\mathbf{I}$ is the sound intensity and $\mathbf{n}$ is the outward normal on the boundary Γ pointing into the complementary domain. Γ is taken to be the surface of the radiating structure. The discretised sound power can be written as a sum of all nodal sound power contributions by

$$P = \sum_{k=1}^{N} P_k = \sum_{k=1}^{N}\int_\Gamma \mathbf{I}_k \cdot \mathbf{n}_k \mathrm{d}\Gamma_k . \tag{4}$$

The nodal contributions in terms of the sound power P_k or the sound intensity $\mathbf{I}_k$ can be either positive or negative, which results in cancellation effects of energy on the boundary Γ. Thus P_k and $\mathbf{I}_k$ are not suitable to visualise the surface contributions to the radiated sound power from a vibrating structure.

Surface Contributions to Radiated Sound Power

In what follows, the radiated sound power is described in terms of the sum of only positive sound power contributions of the radiating surface. When all the contributions are positive, the cancellation effects observed in Eq. (4) are eliminated, thus delivering a tool to visualize surface contributions to the radiated sound power.

Defining the surface contribution to the radiated sound power as η, the total radiated sound power is expressed by the following boundary surface integral

$$P = \frac{1}{2}\int_{\Gamma} \eta(\mathbf{x})\mathrm{d}\Gamma(\mathbf{x}). \tag{5}$$

Similar to the sound intensity, the physical unit of η is W/m^2. Let

$$\eta(\mathbf{x}) = \beta(\mathbf{x})\beta(\mathbf{x})^* \tag{6}$$

where β is a vector without physical significance. For any interpoloation node $\mathbf{x}_k$ on the boundary Γ, η_k is given by

$$\eta_k = \beta_k\beta_k^*. \tag{7}$$

From Eq. (7) it is observed that η_k is always real and positive for any complex β. Discretisation of Eq. (5) leads to (Marburg et al. 2013)

$$P = \frac{1}{2}\boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{\Theta}\boldsymbol{\beta} \tag{8}$$

where T denotes the matrix transpose, $\boldsymbol{\Theta}$ is the boundary mass matrix, $\boldsymbol{\beta}$ is expressed in terms of known boundary values and is given by (Marburg et al. 2013)

$$\boldsymbol{\beta} = \boldsymbol{\Psi}\sqrt{\boldsymbol{\Lambda}}\boldsymbol{\Psi}^{T}\boldsymbol{\Theta}\mathbf{v}. \tag{9}$$

$\boldsymbol{\Psi}$ are the acoustic radiation modes and $\boldsymbol{\Lambda}$ is a diagonal matrix with the corresponding radiation efficiencies. Substitution of Eq. (9) into Eq. (8) allows the express the surface contributions to the total radiated sound power in terms of only real positive values.

NUMERICAL EXAMPLES

The method has been implemented using the boundary element code Akusta (Marburg et al. 2003, 2005). Additional subroutines are written using FORTRAN 90. Eigenvalue problems have been solved using a simple simultaneous vector iteration procedure. A residual tolerance of 10^{-5} was required. Constant and linear discontinuous boundary elements have been used for the models. In the case of linear elements, collocation points are selected for the zeros of the Legendre polynomials (Marburg et al. 2003).

An open resonator consisting of two square parallel plates is modelled. The open resonator is presented in two configurations – with and without a Helmholtz resonator, which is an acoustic equivalent for a tuned vibration absorber. Figure 2 shows the configuration of the two parallel plates without a Helmholtz resonator. Figure 3 shows the two parallel plates with a Helmholtz resonator embedded in the lower plate. The lower plate is fixed and has a thickness of 0.4 m. The upper plate is flexibly mounted and has a thickness of 0.3 m. Both plates have a top surface area of 1.5 m^2 and are 0.915 m apart. The upper plate oscillates in the vertical direction with a surface normal particle velocity of v_n = 1 mm/s. Damping only exists in the form of radiation damping.

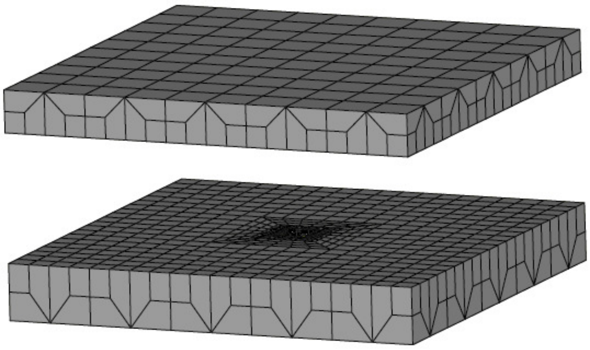

Figure 2. Open resonator consisting of two parallel plates

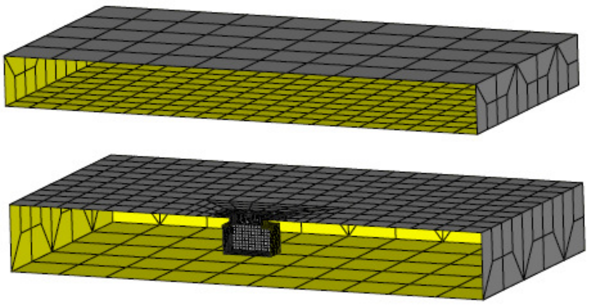

Figure 3. Open resonator consisting of two parallel plates with a Helmholtz resonator embedded in the lower plate

The total radiated sound power for both configurations is calculated using Eq. (3) and presented in Figure 4. Note that Eqs. (4) and (8) would yield exactly the same result for the total radiated sound power. For the first configuration consisting of two parallel plates and an air gap, the vibro-acoustic system has resonances at 60 Hz, 198 Hz and 379 Hz in the considered frequency range. The resonance of the open resonator system at 60 Hz corresponds to a rigid body mode similar to the rigid body mode that occurs at 0 Hz in a closed fluid-filled box (Marburg et al. 2006). The resonances at 198 Hz and 379 Hz correspond to half a wavelength and one full wavelength between the plates, respectively. Adding a Helmholtz resonator tuned to a resonant frequency of 198 Hz to the lower plate of the open resonator significantly reduces the sound power at this frequency, while other frequencies remain mostly unaffected. This is typical behavior of a tuned vibration absorber or Helmholtz resonator.

The normalized normal sound intensity $\mathbf{I}\cdot\mathbf{n}$ and normalised continuous surface contribution η are compared for the two plate configurations without a Helmholtz resonator (Figure 5) and with a Helmholtz resonator (Figure 6). In both figures, the normal intensity (left) and surface contribution (right) is shown at two viewing angles to show the top surface of the lower plate (top) and bottom surface of the upper plate (bottom). The normal sound intensity is always zero for the lower

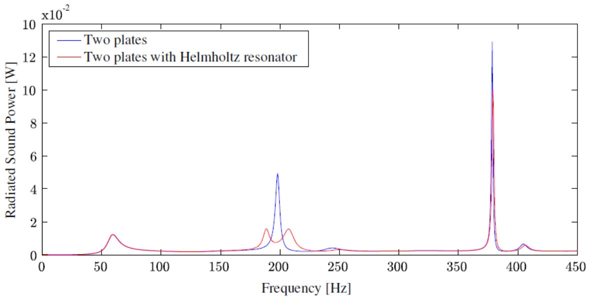

Figure 4. Radiated sound power by the open resonator consisting of two parallel plates with an oscillating upper plate – with and without a Helmholtz resonator embedded in the lower plate

plate because of zero particle velocity (the plate is fixed). On the upper plate, the normal sound intensity is positive on the inner side facing the lower plate (see lower pair of plates) and negative on the outer side (see upper pair of plates). In contrast, the surface contribution is distributed over both plates and is always positive.

It is important to note that the lower plate contributes to the radiated sound despite being fixed in space. The fact that the lower plate contributes to the radiated sound becomes obvious if the plate were removed from the vibro-acoustic system in which case the frequency response of the system would change significantly.

The localized effect in the results for the surface contribution of the Helmholtz resonator on the fixed bottom plate can be clearly observed in the section view in Figure 7. Thus, the surface contribution is more appropriate for visualization of the actual contributions of the lower and upper plates to the radiated sound power.

SUMMARY

A method to identify the surface contributions to the radiated sound power of a vibrating structure has been presented. The surface contributions to the far-field radiated sound power can be observed at the fluid boundary on the surface of the structure. An expression for the sound power is derived in terms of the acoustic radiation modes. The surface contributions are then computed for every node of a boundary element mesh of the radiator. In contrast to the sound intensity, using surface contributions, the radiated sound power is described as the sum of only positive sound power contributions of the vibrating surface, thus avoiding cancelation effects. A numerical example has been used to illustrate the method, corresponding to an open resonator composed of two parallel plates. Using the surface contribution method, the individual contributions of the lower and upper plates of the open resonator to the radiated sound power were identified. This is particularly valuable for the fixed lower plate of the resonator, for which sound intensity wrongly indicates zero contribution to the radiated sound. The technique presented here provides a new method to localize the relevant radiating surface areas on a vibrating structure.

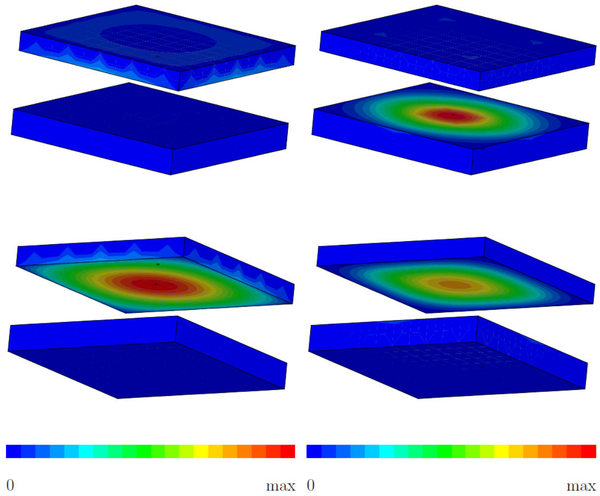

Figure 5. Normalized normal sound intensity (left) and surface contribution (right) for the two parallel plates at 198 Hz

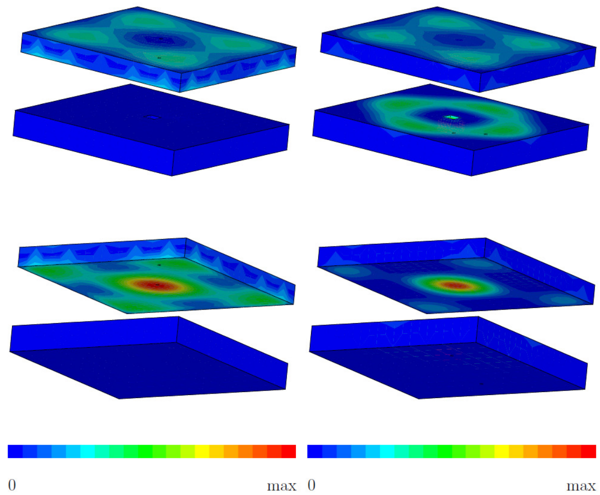

Figure 6. Normalized normal sound intensity (left) and surface contribution (right) for the two parallel plates with a Helmholtz resonator at 198 Hz

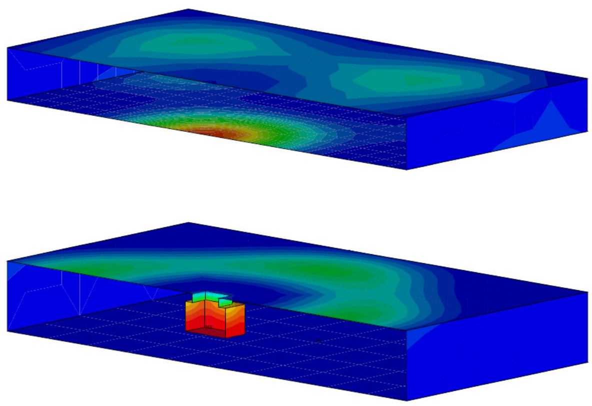

Figure 7. Section view of the two parallel plates with a Helmholtz resonator showing the normalized continuous surface contribution at 198 Hz

REFERENCES

Chen, PT & Ginsberg JH 1995, 'Complex power, reciprocity, and radiation modes for submerged bodies', *Journal of the Acoustical Society of America*, vol. 98, pp. 3343-3351.

Cunefare, KA & Currey, MN 1994, 'On the exterior acoustic radiation modes of structures', *Journal of the Acoustical Society of America*, vol. 96, pp. 2302-2312.

Ih, J-G 2008, *Computational acoustics of noise propagation in fluids*, chapter 20, Inverse boundary element techniques for the holographic identification of vibro-acoustic source parameters, Springer, Berlin, Germany, pp. 547–572.

Ishiyama, S, Imai, M, Maruyama, S, Ido, H, Sugiura, N & Suzuki, S 1988, 'The applications of ACOUST/BOOM - a noise level predicting and reducing computer code', *Proceedings of the Seventh International Conference on Vehicle Structural Mechanics*, Society of Automotive Engineers, Warrendale, Pennsylvania, pp. 195–205.

Marburg, S & Amini, S 2005, 'Cat's eye radiation with boundary elements: Comparative study on treatment of irregular frequencies', *Journal of Computational Acoustics*, vol. 13, pp. 21-45.

Marburg, S, Dienerowitz, F, Horst, T & Schneider, S 2006, 'Normal modes in external acoustics. Part II: Eigenvalues and eigenvectors in 2D', *Acta Acustica united with Acustica*, vol. 92, no. 1, 97-111.

Marburg, S, Lösche, E, Peters, H & Kessissoglou, N 2013, 'Surface contributions to radiated sound power', *Journal of the Acoustical Society of America*, vol. 133, pp. 3700-3705.

Marburg, S & Nolte, B 2008, *Computational acoustics of noise propagation in fluids*, chapter 0: A unified approach to finite and boundary element discretization in linear acoustics, pp. 1-34, Springer, Berlin, Germany.

Marburg, S & Schneider, S 2003, 'Influence of element types on numeric error for acoustic boundary elements', *Journal of Computational Acoustics*, vol. 11, pp. 363-386.

Maynard, J, Williams, E & Lee, Y 1985, 'Nearfield acoustic holography: I. Theory of generalized holography and the development of NAH', *Journal of the Acoustical Society of America*, vol. 78, no. 4, pp. 1395-1413.

Williams, E 1995, 'Supersonic acoustic intensity', *Journal of the Acoustical Society of America*, vol. 97, no. 1, pp. 121-127.

Williams, E 1998, 'Supersonic acoustic intensity on planar sources', *Journal of the Acoustical Society of America*, vol. 104, no. 5, pp. 2845-2850.