Investigation of Sound Radiation from a Water-loaded Cylindrical Enclosure due to Airborne Noise

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ABSTRACT

This paper models and analyses the noise radiation due to internal acoustic excitation of a cylindrical enclosure submerged in a heavy fluid. The enclosure consists of a cylindrical shell filled with air and attached to rigid end plates. To model the airborne noise transmission and radiation, the noise sources were characterised as monopoles. The predicted far-field noise from the shell due to interior airborne noise shows a dominance of sharp peaks due to interior resonances. The effect on far-field sound pressure of different thickness and length of the cylindrical shell, as well as different excitation locations, are discussed. Excellent agreement is obtained between the analytical results and results from numerical finite element / boundary element models.

INTRODUCTION

Water-loaded cylindrical enclosures are widely used as simple examples to demonstrate the acoustic characteristics of underwater vessels. This preliminary study analytically and numerically investigates the transmission and radiation of internal airborne noise from a cylindrical enclosure through modelling. To model the airborne noise, the acoustic emission may be characterised as monopole sources, with transfer functions developed to represent the acoustic radiation.

Sound radiation from a water-loaded vibrating cylindrical shell can be modelled using the following three steps. The first step is to model a finite shell. Detailed discussions and comparisons of different shell equations were given by Pan and Hansen (1997). The second step is to model the excitations. The expression for a point force acting on a shell has been widely reported (see, for example, Pan and Hansen (1997) and Pan *et al.* (2008a, 2008b)). An approximate solution for a monopole source inside a shell was given by James (1985). However, this solution has not been validated either computationally or experimentally. The third step is to model the sound radiation from the shell. An approximation for sound radiation from a vibrating shell was given by Junger and Feit (1986), Tso and Jenkins (2003) and Pan *et al*. (2008a, 2008b), and was based on the spectral radial shell displacement.

The work described here is an extension of the previous work conducted on sound radiation from a vibrating shell by both James (1985) and Pan *et al*. (2008a, 2008b). As part of a comprehensive analytical method for calculating the radiated sound from a cylindrical shell, the excitation stress on the shell due to an interior monopole source is modelled by the approximate solution of James (1985) and the resulting spectral radial displacement due to multiple monopole sources at different locations is evaluated. The far field radiated noise is then determined from the spectral displacement of the cylinder. The main results from the analytical method are validated by two numerical methods, one using the vibroacoustic modelling software Sysnoise and the other a fully coupled Finite Element / Boundary Element (FE/BE) solution.

THEORETICAL METHOD

A finite cylindrical shell model for calculating the far-field pressure, developed by Junger and Feit (1986) and later used by Tso and Jenkins (2003) and Pan *et al.* (2008a, 2008b), is shown in Figure 1(a). The cylindrical shell has two rigid end plates (to form a finite cylinder) which attach to two semi-infinite rigid baffles, so that there is no radiated pressure from the end plates. Figure 1(b) shows the cross-section of the shell with an interior monopole source.

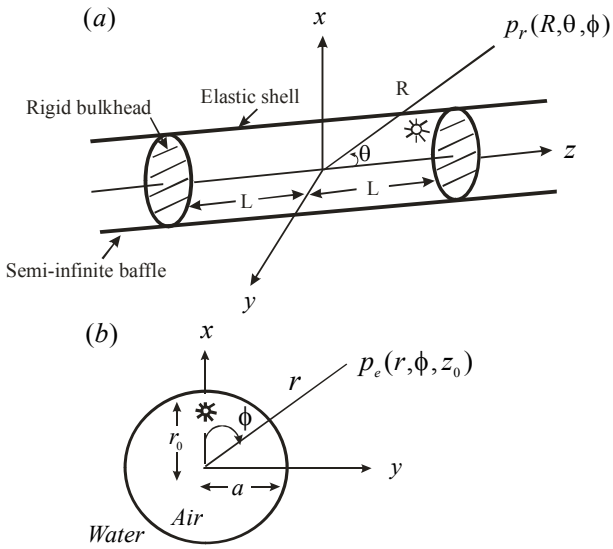

Figure 1. Geometry and coordinate systems of a cylindrical shell: (a) finite cylindrical shell with semi-infinite baffles; (b) cross-section of shell with a monopole source.

Shell Dynamics

The shell equations used in this paper were given by Junger and Feit (1986), who developed the Donnell formulation to include water loading for a thin shell. The water loading of

the shell was obtained by including the acoustic impedance in the formulation. When compared to other thin-shell theories such as Flügge (1973) and Pan and Hansen (1997), the Donnell formulation has fewer terms, which may underestimate the effect of bending. However, it was judged that the Donnell formulation is sufficient for the purpose of demonstrating the methodology in the modelling of sound radiation due to an acoustic source.

The spectral displacements (modal amplitudes) of a submerged shell at a particular mode are obtained from the following matrix equation:

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \begin{bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ F_{mn} \end{bmatrix} \tag{1}$$

where

$$S_{11} = E_1\left[\alpha_m^2 + \frac{n^2}{2a^2}(1-\sigma)\right] - \omega^2\rho_s h$$

$$S_{12} = -E_1(1+\sigma)\frac{\alpha_m n}{2a}$$

$$S_{13} = -E_1\sigma\frac{\alpha_m}{a}$$

$$S_{22} = E_1\left[(1-\sigma)\frac{\alpha_m^2}{2} + \frac{n^2}{a^2} + 2\beta^2(1-\sigma)\alpha_m^2 + \beta^2\frac{n^2}{a^2}\right] - \omega^2\rho_s h$$

$$S_{23} = E_1\left[\frac{n}{a^2} + \beta^2(2-\sigma)\alpha_m^2 n + \beta^2\frac{n^3}{a^2}\right]$$

$$S_{33} = E_1\left\{\frac{1}{a^2} + \beta^2 a^2\left[\alpha_m^2 + \left(\frac{n}{a}\right)^2\right]^2\right\} - \omega^2\rho_s h + \omega^2\rho_e\frac{H_n(\gamma a)}{\gamma H_n^{'}(\gamma a)}$$

$S_{31} = S_{13}$, $S_{32} = S_{23}$, $S_{21} = S_{12}$.

All symbols are defined in the nomenclature at the end of the paper. In Equation (1), U_{mn}, V_{mn} and W_{mn} are the spectral displacements in the axial, circumferential and radial directions of the shell, m and n are the axial and circumferential mode numbers and F_{mn} is the modal force which describes the type and position of an excitation. The time-harmonic factor, $e^{-i\omega t}$, is omitted throughout.

The shell has shear diaphragm or "simply supported" end conditions. The far-field radiated pressure from the shell is determined by the radial displacement. Therefore, only the radial displacement will be presented here. It can be approximated by a double Fourier series as

$$W(\phi,z) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{mn}\sin\left[\frac{m\pi(z+L)}{2L}\right]\cos(n\phi) \tag{2}$$

where W_{mn} may be obtained from Equation (1).

Excitation

For simplicity, the analysis initially considers only one monopole source. The Green's function for the Helmholtz wave equation due to a monopole source $(r_0, 0, z_0)$ can be expressed as:

$$\left(\nabla^2 + k_i^2\right)p_G(r,\phi,z) = -\frac{4\pi P_0}{r}\delta(r-r_0)\delta(\phi)\delta(z-z_0). \tag{3}$$

In the presence of the cylindrical boundary at $r = a$, the interior pressure inside the cylinder is the sum of the Green's function, p_G, and scattering term, p_{ref}. Thus,

$$p_i(r,\phi,z) = p_G + p_{ref} \tag{4}$$

where p_G is given by Skelton (1985) and p_{ref} is given by James (1983).

The acoustic momentum equation is

$$\left.\frac{\partial p_i}{\partial r}\right|_{r=a} = \rho_i\omega^2 W(\phi,z). \tag{5}$$

The excitation stress for the case of an interior fluid is simply

$$F(\phi,z) = p_i(a,\phi,z). \tag{6}$$

The expression for $F(\phi,z)$ can be obtained by solving for p_i using Equations (4) and (5) and then substituting into Equation (6). However, the expression for $F(\phi,z)$ obtained has a coupling term (involving shell radial displacement) which means the interior fluid couples the in-vacuo shell modes. James (1985) gave an approximation for $F(\phi,z)$ which neglects the coupling term because the internal fluid is air. The expression can be summarised as

$$F(\phi,z) = \sum_{q=0}^{\infty}\sum_{n=0}^{\infty}\frac{P_0 e_n e_q}{L\gamma_q a}\cos\left[\frac{q\pi(z+L)}{2L}\right]\cos\left[\frac{q\pi(z_0+L)}{2L}\right] \times\frac{J_n(\gamma_q r_0)}{J_n^{'}(\gamma_q a)}\cos(n\phi). \tag{7}$$

The modal forces, F_{mn}, are the coefficients of the Fourier series expansion of the force excitation, F, viz.,

$$F_{mn} = \frac{e_n}{2\pi L}\int_{-L}^{L}\int_{0}^{2\pi} F(\phi,z)\sin\left[\frac{m\pi(z+L)}{2L}\right]\cos(n\phi)\,d\phi\,dz. \tag{8}$$

Substituting Equation (7) into Equation (8), the modal force due to one monopole source may be written as

$$F_{mn} = \sum_{q=0}^{\infty}\frac{P_0 e_n e_q}{L\gamma_q a}\cos\left[\frac{q\pi(z_0+L)}{2L}\right]\frac{2m[1-(-1)^{m+q}]}{\pi(m^2-q^2)}\frac{J_n(\gamma_q r_0)}{J_n^{'}(\gamma_q a)}. \tag{9}$$

If multiple monopole sources are located inside the cylinder, the modal force is the sum of the individual monopole forces and is given by:

$$F_{mn} = \sum_{q=0}^{\infty} \frac{e_n e_q}{L\gamma_q a} \sum_{s=1}^{j} P_{0s} \cos\left[\frac{q\pi(z_s + L)}{2L}\right] \frac{2m[1-(-1)^{m+q}]}{\pi(m^2 - q^2)} \frac{J_n(\gamma_q r_s)}{J_n'(\gamma_q a)} \quad (10)$$

where j is the number of the monopole sources at positions $(r_s, z_s,\ s = 1,..., j)$.

Far-field Pressure

The pressure radiated from a shell to the far field (see, for example, James (1985), Tso and Jenkins (2003) and Pan *et al.* (2008a, 2008b)) is given by:

$$p_r(R,\theta,\phi) = -i\omega\rho_e c_e \frac{e^{ik_e R}}{\pi R} \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \frac{G_{mn}(\alpha) e^{-\frac{in\pi}{2}}}{\sin\theta \cdot H_n'(k_e a \sin\theta)} \times \cos(n\phi) \quad (11)$$

where

$$G_{mn}(\alpha) = LW_{mn} e^{i\alpha L} \frac{m\pi}{2L^2} \cdot \frac{1 - (-1)^m e^{-2i\alpha L}}{\left(\frac{m\pi}{2L}\right)^2 - \alpha^2} \quad (12)$$

and

$\alpha = k_e \cos\theta$.

The radiated pressure from the shell due to multiple monopole excitations can be obtained by substituting the modal force from Equation (10) into Equation (1) to determine W_{mn} , and then using Equations (12) and (11) to solve for G_{mn} and $p_r(R,\theta,\phi)$ respectively.

NUMERICAL METHODS

For validation of the analytical method, two numerical methods were developed. The first method uses the commercial FE/BE software Sysnoise. Due to currently unresolved issues with a heavy exterior fluid, Sysnoise comparisons will only be presented for air as the exterior fluid. The second method was a fully coupled FE/BE code for a water-loaded cylindrical shell developed by one of the authors (Peters).

Sysnoise Method

For the Sysnoise method, the cylinder boundary conditions were set to match the analytical method (i.e., simply-supported cylinder with rigid ends). The end plates of the cylinder were each attached to rigid cylindrical baffles with lengths equal to the vibrating cylinder, so as to approximate 'semi-infinite cylinder' baffles.

As an interior–exterior coupled solution could not be directly obtained using Sysnoise, the effect of interior air-loading on structural modes was ignored, in a similar manner to the analytical method. Modes of the exterior fluid-loaded cylinder were used as modes for the solution of the interior acoustic problem, using the approximation that the effect of air loading does not appreciably affect the modal response. Shell displacements derived from the interior acoustic problem were then used as vibration boundary conditions to compute the exterior radiation. Due to the issues with a heavy exterior fluid mentioned above, the exterior fluid was restricted to be air.

Using Sysnoise, a short two-metre cylinder with thick and thin shell walls, as listed in Table 1, was modelled. The calculation using this numerical method involved the lowest 400 uncoupled modes, of which only the lowest 50 or so were relevant in the frequency range of interest for air loading. A total of 4452 elements on the non-rigid part of the cylinder were used, which required a computation time of about 11 hours on a modern PC to get the accurate results for each of the following comparisons shown below. Although not shown, less accurate but similar results were obtained at lower resolution with about one hour of computation time.

Fully Coupled FE/BE Method

For the water-loaded cylindrical shell, a fully coupled finite element / boundary element (FE/BE) method was developed to include the effect of water loading. The global system of equations for the FE/BE method is given by:

$$\begin{bmatrix} \mathbf{K} - \omega^2\mathbf{M} & -\mathbf{C}_{e,sf} & -\mathbf{C}_{i,sf} \\ -i\omega\mathbf{G}_e\mathbf{C}_{e,fs} & \mathbf{H}_e & 0 \\ -i\omega\mathbf{G}_i\mathbf{C}_{i,fs} & 0 & \mathbf{H}_i \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \mathbf{p}_e \\ \mathbf{p}_i \end{bmatrix} = \begin{bmatrix} \mathbf{f}_s \\ \mathbf{p}_{e,inc} \\ \mathbf{p}_{i,inc} \end{bmatrix} \quad (13)$$

The matrices **K** and **M** are the FE stiffness and mass matrices of the structure respectively. The matrices **H** and **G** are the frequency dependent BE influence matrices obtained through collocation (Marburg and Nolte 2008). The vectors **p**, **u** and $\mathbf{f}_s$ are respectively the vectors with the nodal values for pressure, displacement and structural forces.

The sound pressure at a specific location R in the fluid domain is given by the discretized representation formula (Marburg and Nolte 2008)

$$p_r(R) = \mathbf{g}^T \mathbf{v}_f - \mathbf{h}^T \mathbf{p} \quad (14)$$

where the vectors **g** and **h** are boundary element influence vectors that depend on R and the geometry of the fluid domain.

The interior fluid was connected to the cylindrical shell and the flat end plates. The external fluid was connected to the cylindrical shell only. Due to the difficulty of matching the shear diaphragm boundary condition of the analytical model, the condition was approximated by fixing the three translational degrees of freedom at both cylinder ends.

Using the fully coupled FE/BE method, models of the short and long cylinders with thin shell walls, as listed in Table 1, were developed. The computation time for each FE/BE run was approximately 5 hours.

For the short cylinder (2 m length), the internal fluid was modelled using 704 linear boundary elements. The external fluid was modelled using 256 linear boundary elements. The cylindrical shell and the flat end plates were discretised using 960 quadratic finite elements.

For the long cylinder (10 m length), the internal fluid was modelled using 496 linear boundary elements. The external fluid was modelled using 576 linear boundary elements. The cylindrical shell and the flat end plates were discretised using 2240 quadratic finite elements.

Fewer boundary elements were used in the long cylinder in the fully coupled FE/BE method in an attempt to further reduce computation time. A convergence analysis of the nu-

merical method was conducted and the mesh densities were found to be sufficient.

RESULTS

The results presented in this paper are based on cylindrical shells with the geometric and material properties, and excitation locations, shown in Table 1. The maximum mode numbers used in the analytical method are also included in Table 1.

Table 1. Shell and fluid parameters, and excitation locations, in SI units

Steel shell	$E = 1.95\times10^{11}$, $\sigma = 0.29$, $\rho_s = 7700$, $a = 1$, $h = 0.01$ (thin), $h = 0.1$ (thick), $2L = 2$ (length of short shell), $2L = 10$ (length of long shell)
Exterior fluid	$\rho_e = 1000$, $c_e = 1500$ (water) $\rho_e = 1.21$, $c_e = 343$ (air)
Interior fluid	$\rho_i = 1.21$, $c_i = 343$ (air)
Monopole source	$P_0 = 1$, $(r,\phi,z) = (2a/3,0,0)$ (centre), $(r,\phi,z) = (2a/3,0,0.9)$ (end for short shell), $(r,\phi,z) = (2a/3,0,4.5)$ (end for long shell)
Maximum mode numbers used for theoretical method	axial modes: 9, circumferential modes:10, acoustical modes: 10 (short shell), 20 (long shell)

In Table 1, the monopole source is slightly shifted from the central point to $(r,\phi,z) = (2a/3,0,0)$ in order to excite both axisymmetric and non-axisymmetric modes. This source location is named the central excitation in this paper. Results for multiple excitations are not shown here to conserve space. Damping in the shell wall is included by using a complex representation of the Young's modulus $E = E(1-i\eta)$ where η is the loss factor and has a value of 0.02. The sound pressure was calculated at 1000 m with $\theta = 90°$ and $\phi = 0°$, and normalised to 1 m range by adding 60 dB. The dB reference level is 1 μPa.

Air-loaded Shell

For an initial comparison, a simple example using a light exterior fluid is presented. In this case, a short shell with air-loaded exterior was excited by a monopole source. The far-field radiated pressure for the analytical method was calculated from Equation (11) using the density and sound speed of air. The Sysnoise method described in the previous section was used to compare with the analytical method.

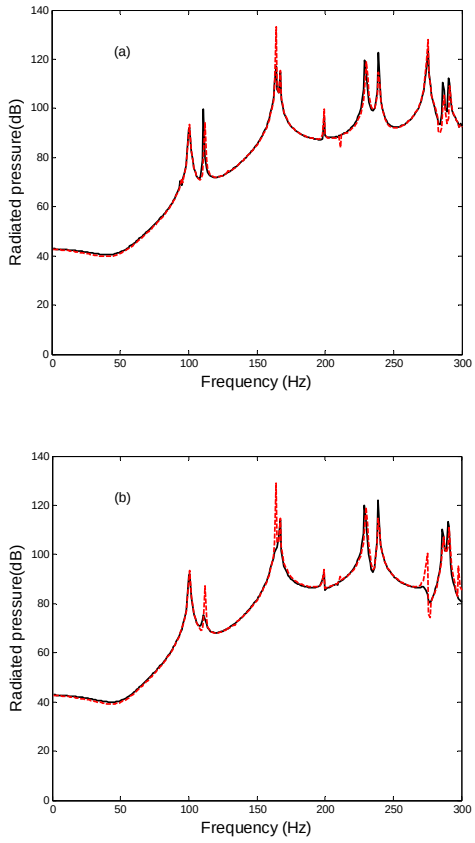

Figure 2. Far-field radiated pressure from an air-loaded short and thin shell due to a monopole source excitation: (a) with central excitation; (b) with end excitation. ─, from the analytical method; - - -, from the Sysnoise numerical method.

Figure 2 shows comparisons of the radiated pressure from the analytical method and Sysnoise method for two excitation locations in the short and thin cylindrical shell. The sharp peaks occur at frequencies which correspond to the natural frequencies of the interior enclosure with rigid boundaries, while some less significant peaks correspond to the resonances of acoustically efficient shell modes. Figure 3 shows the results for a thick shell. Results shown in Figures 2 and 3 indicate excellent agreement between the analytical and numerical calculations.

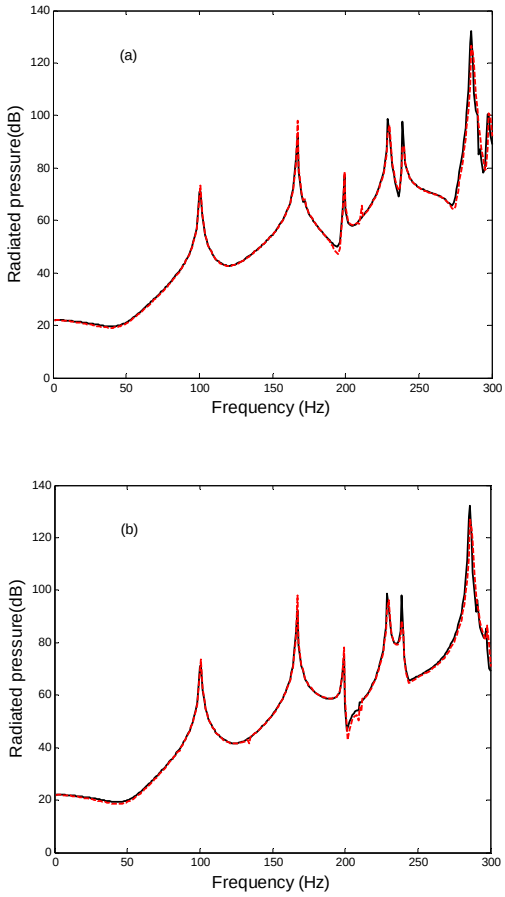

Figure 3. Far-field radiated pressure from an air-loaded short and thick shell due to a monopole source excitation: (a) with central excitation; (b) with end excitation. ─, from the analytical method; - - -, from the Sysnoise numerical method.

Water-loaded Shell

In this case, the fully coupled FE/BE method described previously was used instead of the Sysnoise method in order to demonstrate the effect of water loading. Note that the boundary conditions used in the analytical and numerical methods differ slightly as described in the preceding section. Specifically, the fully coupled FE/BE model had constraints on the three translational directions, whereas the analytical model had constraints only in the radial and circumerical directions and no constraint in the axial direction. In the following analysis, results are presented for thin shells only.

Figure 4 shows the comparisons of radiated pressure from the analytical and numerical methods for two excitation locations. Good agreement for both excitation locations can be seen up to approximately 120 Hz, beyond which, there are some differences in both amplitudes and resonant modes. The differences are believed to be mainly due to the different boundary conditions in the analytical and fully coupled methods. This postulation is based on the excellent agreement between the analytical and Sysnoise methods in air using the same boundary conditions, as mentioned in the preceding section.

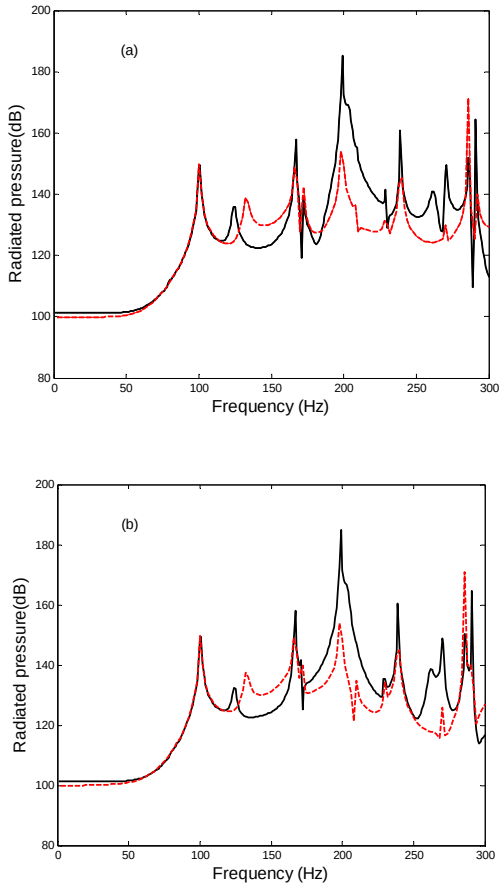

Figure 4. Far-field radiated pressure from a water-loaded short and thin shell due to a monopole source excitation: (a) with central excitation; (b) with end excitation. ─, from the analytical method; - - -, from the fully coupled FE/BE method.

To reduce the effect of boundary conditions on radiated pressure, longer cylindrical shells were used in both methods. Figure 5 shows the corresponding results for 10 m long cylindrical shells. Results indicate that significant improvement is achieved for both source locations over the whole frequency range. The discrepancy between the analytical and coupled FE/BE methods may be further reduced if a finer frequency step is used in the numerical method. Comparing Figure 4 with Figure 5, it can be seen that increasing the length of the shell does not significantly change the amplitude of the radiated pressure but results in lower natural frequencies. Although some shell frequencies decrease with lengthening of the shell, the most significant resonant peaks in the figures are internal acoustic modes which decrease significantly as internal cavity length increases.

The typical effect of water loading on the shell vibration is to reduce the shell resonance frequencies by mass loading. Vibrational levels do increase as a result but only of the order of 20 dB when changing from exterior air to water. However, the much higher specific acoustic impedance of water is a better match to the shell impedance so the radiated pressure levels are much higher in water than in air. For example, the pressure radiated from a piston moving with a given velocity is proportional to specific acoustic impedance. It should also be noted that a given pressure level of a plane wave in water represents a much lower sound intensity because of the much higher specific acoustic impedance.

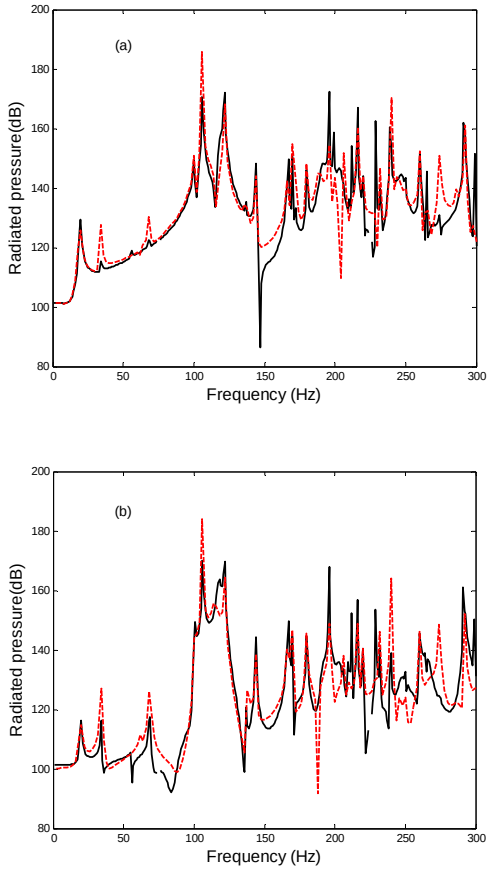

Figure 5. Far-field radiated pressure from a water-loaded long and thin shell due to a monopole source excitation: (a) with central excitation; (b) with end excitation. ─, from the analytical method; - - -, from the fully coupled FE/BE method.

DISCUSSIONS AND FUTURE WORK

This paper presents a number of approaches, as an initial study, for the modelling of sound radiation from a cylindrical shell due to airborne noise. The Sysnoise method for the fluid-loaded cylinder uses a two step method to determine the radiated noise as an exterior–interior coupled solution could not be directly obtained. It involves the evaluation of the modes of the externally fluid-loaded cylinder and subsequently uses these modes to solve the interior acoustic problem. The approach works well when the exterior fluid is light, but has failed to agree with analytical and fully coupled FE/BE methods when the fluid is water. Work is underway to provide an explanation for this failure.

The fully coupled FE/BE method could be used for both air and water loading as it provides a coupled exterior-interior solution to the radiation problem. However, the FE/BE method used here can only deal with a limited set of boundary conditions, such as zero translational motion at the ends of the cylinder. Further work is required to model other boundary conditions including shear-diaphragm and free-at-both-ends. As the length to diameter ratio of the cylindrical shell is increased, the influence of boundary conditions on sound radiation is reduced, as demonstrated in the present study.

For the cylinder in air, the boundary conditions in the analytical and Sysnoise models are identical and excellent agreement is observed. Hence, for the cylinder in water, the effect of different boundary conditions is believed to be a contributor to the discrepancies between the analytical and fully coupled FE/BE methods. This is further supported by results of the long cylindrical shell which show significant improvements between these two methods. Since an accurate representation of the shear diaphragm is not implemented in the current coupled FE/BE method, it is suggested that the analytical model to be modified such that identical boundary conditions could be used for comparison.

Another reason for the differences may be due to the thin-shell theory used in the analytical model. Comparing other thin-shell theories such as Flügge (1973) and Pan and Hansen (1997), the current shell equation (Donnell formulation) has fewer terms, which may underestimate the effect of bending. More accurate shell equations could be used in the future. It should be noted that the bare shell resonance frequencies of both the Sysnoise and analytical models were in close agreement, but that there were slight discrepancies when in water. It is not currently known if the fully coupled FE/BE shell modes for the cylinder in water accurately match the analytical model as it does not use a modal approach. It is therefore possible that some differences are also due to the manner in which water loading is applied.

It is interesting to note that the simple analytical method provides good agreement of sound radiation when compared with the numerical methods. The analytical method has particular advantages over the numerical methods on computational time (typically less than one minute) and the flexibility on the selection of parameters such as damping and structural as well as acoustical modes. This approach is most suitable for concept studies where a rapid assessment on a simplified initial design is required. Detailed studies for more complex structures can be conducted by a numerical approach based on the methods described in the preceding sections.

CONCLUSIONS

An analytical method has been developed for predicting far-field sound radiation from a water-loaded finite cylindrical shell, excited by multiple interior monopole sources. The results of the radiated sound from the shell excited by the monopole source show the dominance of sharp peaks due to interior resonances.

The analytical method has been applied to an air-loaded cylindrical shell and compared with a Sysnoise FE/BE numerical approach. Excellent agreement is obtained between the two methods. A water-loaded numerical solution is then obtained using fully coupled FE/BE code. Good agreement between the analytical and the fully coupled FE/BE method is achieved over the whole frequency range except for a few points at high frequencies, which is most likely due to computational errors, slight differences between the end-plate boundary conditions, and the chosen thin shell theory.

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Nomenclature

a	radius of cylindrical shell
c	sound speed
$\mathbf{C}_{e,fs}$	structural-acoustic matrix due to exterior fluid
$\mathbf{C}_{i,fs}$	structural-acoustic matrix due to interior fluid
$\mathbf{C}_{e,sf}$	$=\mathbf{C}_{e,fs}{}^{T}\Theta$
$\mathbf{C}_{i,sf}$	$=\mathbf{C}_{i,fs}{}^{T}\Theta$
E	Young's modulus
E_1	$=Eh/(1-\sigma^2)$
e_n	$=1$ for $n=0$ and $e_n=2$ for $n>0$
e_q	$=1$ for $q=0$ and $e_q=2$ for $q>0$
F	excitation stress
F_{mn}	modal force
$\mathbf{f_s}$	nodal structural forces
$\mathbf{G}$	frequency dependent BE influence matrix
h	thickness of shell
$\mathbf{H}$	frequency dependent BE influence matrix
H_n	Hankel function of order n
i	$=\sqrt{-1}$ (complex unit)
j	number of monopole sources
J_n	Bessel function of order n
k	ω/c (wave number)
$\mathbf{K}$	FE stiffness matrix
L	half length of cylindrical shell
m	axial mode number
$\mathbf{M}$	FE mass matrix
n	circumferential mode number
p_G	Green's function
p_i	interior pressure
p_r	far-field radiated pressure
p_{ref}	reflected pressure (scattering term)
P_0	complex monopole source amplitude
$\mathbf{P}$	vector of nodal values for pressure
$\mathbf{P}_{e,inc}$	vector of incident pressure due to interior fluid
$\mathbf{P}_{i,inc}$	vector of incident pressure due to exterior fluid
q	acoustic mode number
s	index number
t	Time
$\mathbf{u}$	vector of nodal values for displacement
U_{mn}	spectral axial displacement
V_{mn}	spectral circumferential displacement
W_{mn}	spectral radial displacement
W	radial displacement
α_m	$m\pi/2L$
β^2	$h^2/12a^2$
δ	Delta function
γ	$=(k_e^2-\alpha_m^2)^{1/2}$ with $\mathrm{Im}(\gamma)\geq 0$
γ_q	$=(k_i^2-q^2\pi^2/4L^2)^{1/2}$
η	loss factor, 0.02 when set non-zero

ρ	mass density
ρ_s	structural density
σ	Poisson's ratio
ω	circular frequency
Θ	boundary mass matrix
(x, y, z)	Cartesian coordinates
(r, ϕ, z)	cylindrical coordinates
(R, θ, ϕ)	spherical coordinates

Superscripts

'	differentiation with respect to its argument
T	transpose of a matrix

Subscripts

e	exterior fluid
i	interior fluid
0	source location