

Factors Influencing the Successful Measurement of the Sound Power Absorption Coefficient using Cepstral Techniques

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ABSTRACT

The sound power absorption coefficient of a surface can be measured by insonifying it with a broadband signal. The incident wave and reflected wave are then compared to determine the absorption coefficient. In principle the Cepstral analysis allows one to directly extract the ratio of the reflected wave to the incident wave. The ratio is then transformed into the frequency domain, and the squared magnitude is subtracted from 1 to give the absorption coefficient. This paper explores the factors in the measurement chain that limit the accuracy of the measurement. The results of measurements performed on four surfaces using the cepstral technique are correlated to measurements performed using the impedance tube and presented.

INTRODUCTION

There are two standardized methods to measure the sound power absorption coefficient of road surfaces *in-situ* (International Organization for Standardization, 2002, 2010); the extended surface and the spot method.

The extended surface method is used to measure the average absorption coefficient over a large 3 m² area of the road surface. A large loudspeaker is attached to a rig that is propelled forward by a vehicle. A microphone is placed some distance directly under the loudspeaker above the road. The incident and reflected wave are then compared to measure the absorption coefficient.

The spot method measures the absorption coefficient of a 0.08 m² 'spot' of the road. A loudspeaker is attached to the end of a heavy, rigid tube. The tube has two microphones attached to its side. The tube is then placed on the road surface, and by measuring the transfer function between the two microphones the absorption coefficient can be determined. The system requires that the two microphones are carefully calibrated to ensure that the phase difference between them is accounted for.

This research was focused on the development of an apparatus that could automatically determine the sound power absorption coefficient of a small sample area without the need of handling or calibrating the microphone.

The problem of measuring the absorption coefficient is essentially a problem of separating an incident wave from a reflected wave; if a perfect impulsive sound source was available this would be a trivial effort. Unfortunately, due to physical constraints, generating an impulsive signal is a non-trivial task.

Bolton and Gold (1984) developed the theory of using cepstral techniques to determine the absorption coefficient of materials in an anechoic chamber. Jongens (1993) applied cepstral processing techniques to absorption coefficient measurement in an apparatus similar to that of the impedance tube. Jongens was able to achieve results that correlated

closely with those from measurements performed with an impedance tube.

Jongens (1993) highlighted some issues that influenced measurement accuracy; this paper discusses some of those issues in more detail. The accuracy of the measurement is dependent on the spectrum of the measurement signal as well as the absorption coefficient of the material sample being measured. If either the spectrum of the excitation signal or the absorption coefficient of the material are not smooth the accuracy of the measurement is reduced. This paper also presents the results obtained using the apparatus developed to automatically determine the sound power absorption coefficient - the operator need only attach material to a tube and press a button to perform the measurement.

CEPSTRAL THEORY

The theory of using cepstral processing techniques is introduced by considering the geometry in Figure 1. A loudspeaker is connected to a tube that is L meters in length. At the other end is a material sample the microphone is located l meters from the sample. The tube is used so that one may ignore the effects of acoustic wave geometrical spreading. The loudspeaker emits a signal $s(t)$, which impinges on the sample. The sample modifies the signal and reflects some of the energy back towards the microphone. The signal captured by the microphone, $p(t)$ is

$$p(t) = s(t) + s(t) \otimes h(t - \tau). \quad (1)$$

Here \otimes is the convolution operator, $h(t)$ is the impulse response of the sample, and τ is the time it takes for the sound wave to propagate $2l$ meters.

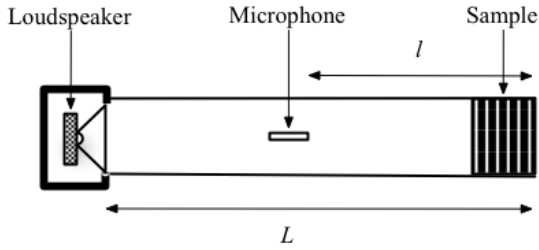


Figure 1. Sketch of measurement geometry used to determine the acoustic absorption coefficient of a material using a loudspeaker and a single microphone.

The Fourier spectral density of $p(t)$ is obtained by taking the square modulus of the Fourier transform of the signal in Equation 1,

$$|P(f)|^2 = |S(f)|^2 [1 + H(f)e^{-2j\pi f\tau}] \cdot [1 + H^*(f)e^{+2j\pi f\tau}]. \quad (2)$$

Here $|P(f)|^2$ is the power spectral density (PSD) of $p(t)$, $|S(f)|^2$ is the PSD of the input signal $s(t)$, $H(f)$ is the frequency response of the sample, and $H^*(f)$ its conjugate.

The logarithm of the total PSD transforms the multiplications into additions, therefore

$$\ln|P(f)|^2 = \ln|S(f)|^2 + \ln|1 + H(f)e^{-2j\pi f\tau}| + \ln|1 + H^*(f)e^{+2j\pi f\tau}|. \quad (3)$$

The power series expansion of the natural logarithm, known as the Mercator series, is

$$\begin{aligned} \ln(1 + x) &= \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^n \\ &= x - x^2/2 + x^3/3 - \dots \end{aligned} \quad (4)$$

This holds so long as $|x| \leq 1$ and $x \neq -1$.

The reflection coefficient, α_r , is equal to the magnitude of the frequency response of the sample, $H(f)$. For passive reflectors $H(f)$ is going to be less than 1, and therefore the Mercator series representation can be used.

The cepstrum of the total pressure, $\tilde{p}(t)$, is found by expanding the logarithmic power spectrum from Equation 3 by using the Mercator series in Equation 4, and then inverse Fourier transforming the result. The resulting cepstrum is

$$\begin{aligned} \tilde{p}(t) &= \tilde{s}(t) + h(t - \tau) - \\ &h(t - \tau) \otimes h(t - \tau)/2 + \dots \\ &+ h(-t - \tau) - \\ &h(-t - \tau) \otimes h(-t - \tau)/2 + \dots \end{aligned} \quad (5)$$

Here $\tilde{p}(t)$ is the total cepstrum, $\tilde{s}(t)$ is the power cepstrum of the direct signal, and $h(t)$ is the impulse response of the sample. The terms where the impulse response of the sample is convoluted with itself a number of times are referred to as *rahmonics*. The cepstrum, illustrated in Figure 2, shows the direct cepstrum, the impulse response, and subsequent *rahmonics*.

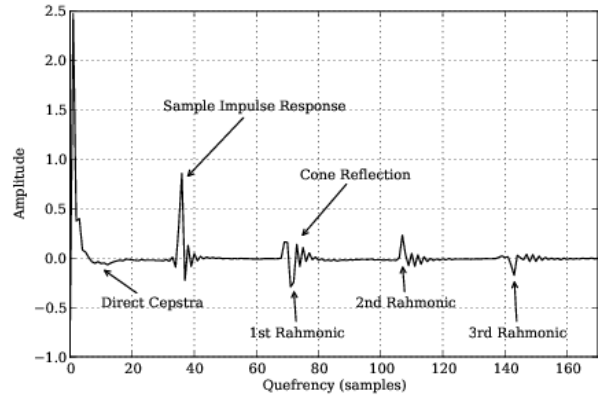


Figure 2. Annotated cepstrum for a reflective sample showing the direct cepstrum, the impulse response of the material, the reflection from the cone, and the *rahmonics*.

It can be seen in Figure 2 that the impulse response of the sample can be *liftered* - *quefrency* equivalent of time domain windowing - directly from the cepstrum, as long as the cepstrum of the direct signal is at negligible values relative to the impulse response of the material at τ seconds. The limit on the length of the *lifter* is determined by the arrival of the first *rahmonic* or the first late reflection - the reflection from the loudspeaker cone - which ever appears first.

The impulse response of the surface under test can be extracted from the cepstrum using a *bandpass lifter*.

From the extracted impulse response the absorption coefficient, $\alpha(f)$, can be determined by Fourier-transforming the impulse response, squaring the result and subtracting it from 1;

$$\alpha(f) = 1 - F\{h(t)\}^2 \quad (6)$$

FACTORS INFLUENCING ACCURACY

Successful extraction of a material's impulse response from the cepstral domain requires three conditions to be met. The first is that the direct cepstrum has fallen to negligible levels before the arrival of the material's impulse response. The second condition is that there is no offset in the *liftered* impulse response. Finally, the impulse response of the material must be shorter than the bandpass lifter length.

The length of the direct cepstrum is determined by the measurement's spectrum shape - any deviation from a flat spectrum increases the direct cepstrum length. The spectrum produced by the system is dominated by the frequency response of the loudspeaker. Coupling the loudspeaker to a tube improves the system's ability to produce low frequencies; as the tube having an impedance loading effect on the loudspeaker. This has the unwanted effect of increasing the length of the loudspeaker's impulse response.

Another issue with the loudspeaker is that above a certain cut off frequency the cone no longer moves as a rigid object. The result of the breakup of the cone is that the loudspeaker introduces a fine structure to the measurement spectrum. The influence of cone breakup can be reduced by placing a glass fibre plug inside the tube, directly in front of the loudspeaker.

The effects on the loudspeaker of the glass fibre plug and impedance loading results in low frequency components of the signal being emphasized. This low frequency emphasis causes the impulse response of the loudspeaker to extend far beyond the measurement period. A high pass filter was used

to reduce the emphasis, and therefore shorten the impulse response.

Any offset in the impulse response introduces an undulation into the measured absorption coefficient. An offset can be introduced into the cepstrum by non-linearities, a DC offset in the measurement system, or the tail of the direct cepstrum. The undulation arises from the frequency response of the bandpass lifter used to extract the impulse response.

Typical samples that will be measured with the cepstral tube include asphalt road surfaces. Porous asphalt samples have an absorption maximum between 500 Hz and 2000 Hz. The impulse response length of an asphalt sample is dependent on the width of the frequency band of high absorption values; a narrow band will have a long impulse response. The length of the tube sets the limit of the frequency resolution of the system.

Accuracy of the low frequency range of the absorption coefficient will be affected by energy truncated by the *bandpass lifter*. Therefore the sample's impulse response should fall to negligible levels before the *bandpass lifter* cut off.

Offset in the Cepstrum

If the offset of the cepstrum in the *liftered* impulse response is not exactly zero then undulations will be superimposed on the absorption coefficient. This offset is a combination of a DC offset in the cepstrum and the tail of the direct cepstrum. It is a result of the non-perfect impulse response of the system. To illustrate the effect that this offset has on the absorption coefficient it is going to be assumed that the offset is constant in the *liftered* impulse response.

The *liftered* impulse response, $h[n]$, with a small DC offset, ϵ , is

$$h'[n - \tau] = (h[n - \tau] + \epsilon) \cdot w[n - \tau]. \quad (7)$$

Here $h[n]$ is the “true” sample impulse response, $w[n]$ the *band pass lifterer*, and τ is the delay before the arrival of the impulse response. The *liftered* frequency response, $H'(f)$, is found by taking the Fourier Transform,

$$H'(f) = [\Re\{H(f)\} + j\Im\{H(f)\} + \epsilon W(f)]e^{-j2\pi f\tau}. \quad (8)$$

Here $\Re\{H(f)\}$ and $\Im\{H(f)\}$ are the real and imaginary components of the “true” frequency response of the material, respectively. $W(f)$ is the frequency response of the *band pass lifter*. Expanding the exponential in terms of sines and cosines the following is obtained

$$H'(f) = [(\epsilon W(f) + \Re\{H(f)\}) \cos(2\pi f\tau) + \Im\{H(f)\} \sin(2\pi f\tau)] + j[\Im\{H(f)\} \cos(2\pi f\tau) - (\epsilon W(f) + \Re\{H(f)\}) \sin(2\pi f\tau)] \quad (9)$$

The reflection coefficient is found by squaring the modulus of Hf ,

$$|H'(f)|^2 = [\Re\{H(f)\}^2 + \Im\{H(f)\}^2] + [\epsilon^2 W^2(f) + 2\epsilon W(f)\Re\{H(f)\}] = |H(f)|^2 + \epsilon^2 W^2(f) + 2\epsilon W(f)\Re\{H(f)\} \quad (10)$$

The resulting reflection coefficient is the sum of three terms;

- the “true” reflection coefficient,
- the square of the DC offset multiplied by the frequency response of the band pass lifter, and
- twice the DC offset multiplied by the product frequency response of the band pass lifter and the real component of the “true” reflection coefficient.

Driver Nonlinearities

Nonlinearities in the loudspeakers driver's response should be noted when measuring the loudspeaker impulse response. This is due to the fact that methods employed to measure the impulse response are under the assumption that the device under test (DUT) is a linear-time-invariant (LTI) system. Depending on the severity of the nonlinearity, it may have a significant influence on the measured response.

The major source of driver non-linearities lie in the electrodynamic motor (Rossi, 1988), and are due to the non-linearity of the spider, the non-uniformity of the radial induction in the air-gap, the variation of self inductance in relation to both the instantaneous position of the voice coil, and the current flowing through it.

Non-linearities due to the spider and radial induction in the air gap are mainly an issue around the loudspeaker resonant frequency. This is where the loudspeaker diaphragm movement is at a maximum. The typical magnitude of the second and third order distortions for these nonlinearities are between 1 and 4 (Rossi, 1988).

Non-linearity due to self inductance becomes decisive in the mid-range frequencies, and second and third order distortions are of the order of between 0.1 and 1.

Figure 3 illustrates the effect that non-linearities have on the cepstrum. The solid line shows the cepstrum of a Maximum Length Sequence (MLS). The dashed line shows cepstrum of the MLS with a 0.1% second order distortion. It can be seen that the distorted signal has a 0.003 offset in the cepstrum.

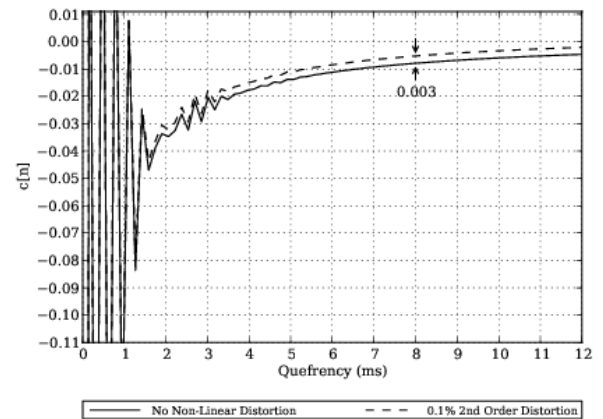


Figure 3. A 0.1% 2nd order distortion results in the direct cepstrum being offset by approximately 0.003.

Loudspeaker Response

The loudspeaker is the weakest link in the measurement chain, therefore dominates the response. There are two significant factors that are introduced by the loudspeaker.

At frequencies below the fundamental resonance of the loudspeaker the loudspeaker is stiffness controlled. If the loudspeaker is more than critically damped, then the response of the loudspeaker reduces by 6 dB / octave with reducing frequency. Provided that the frequency response is approximately flat beyond the sampling frequency the loudspeaker can be thought of as a high pass filter. Therefore the length of the impulse response of the loudspeaker is inversely proportional to the cutoff frequency of the loudspeaker - approximately equal to the fundamental resonance of the loudspeaker.

At frequencies higher than the first rim resonance frequency of the diaphragm occurs, known as the breakup frequency of the cone. Above this frequency bending and longitudinal waves are excited in the cone which introduces fine structure to the frequency response of the loudspeaker.

Coupling of Loudspeaker and Tube

Coupling the loudspeaker to a tube has the effect of loading the impedance of the loudspeaker. Impedance loading of the loudspeaker means that lower frequencies are boosted which reduces the cut off frequency of the loudspeaker's frequency response. The reduced cut-off frequency increases the length of the impulse response of the loudspeaker.

Attaching the tube to the loudspeaker enclosure also introduces sharp edges into the propagation path of the acoustic wave, shown in Figure 4. These sharp edges create edge waves which create a random incidence field in the tube.

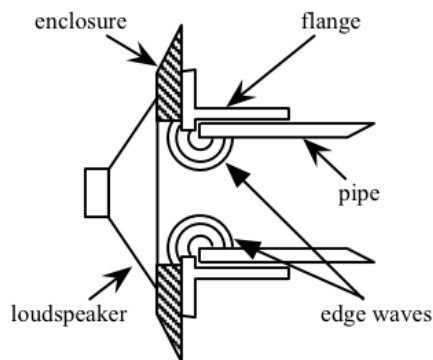


Figure 4. The connection of the loudspeaker to the tube. The loudspeaker is mounted on the inside of the enclosure, a flange is mounted on the outside of the enclosure, and the tube is placed inside the flange. The edge waves generated by the sharp edges are identified.

Measurement Electronics

If only part of the sound generating chain and acquisition chain is measured separately it can be used to improve the cepstrum. If the output of the digital to analog converter is connected directly to the analog to digital converter, then the electrical generator-acquisition chain, p_{ga} , can be captured.

The logarithmic power spectrum of the total system, $P_s(f)$, is obtained by taking the Fourier transform of $p(t)$,

$$\begin{aligned} \ln|P_s(f)|^2 &= \ln|P(f)|^2 \\ &= \ln|X(f)H_{DAC}(f)H_{PA}(f) \\ &\quad H_{LS}(f)H_{MIC}(f)H_{ADC}(f)|^2 \\ &= \ln|X(f)|^2 + \ln|H_{DAC}(f)|^2 \\ &\quad + \ln|H_{PA}(f)|^2 + \ln|H_{LS}(f)|^2 \\ &\quad + \ln|H_{MIC}(f)|^2 + \ln|H_{ADC}(f)|^2. \end{aligned} \tag{11}$$

Here $X(f)$, $H_{DAC}(f)$, $H_{LS}(f)$, $H_{MIC}(f)$, $H_{ADC}(f)$ are the frequency responses of the excitation signal, the digital to analog converter, the loudspeaker, the microphone and the analog to digital converter, respectively. The logarithmic power spectrum of the electrical generator-acquisition chain, $P_{ga}(f)$ is,

$$\ln|P_{ga}(f)|^2 = \ln|X(f)|^2 + \ln|H_{DAC}(f)|^2 + \ln|H_{LS}(f)|^2 + \ln|H_{ADC}(f)|^2 \tag{12}$$

Thus if the electrical generator-acquisition chain is subtracted from the measurement signal, the direct cepstrum becomes dependent on the impulse response of the loudspeaker and microphone alone. It is the equivalent of deconvolving the system's electronics from the microphone signal.

Since the Fourier transform and its inverse transform are linear operators the electrical generator-acquisition chain can be subtracted from the measurement signal in either the logarithmic spectrum domain or in the cepstral domain.

The cepstrum for the captured microphone and signal generating signals are shown in Figure 5. It can be seen that the generators cepstrum is superimposed in the microphone cepstrum.

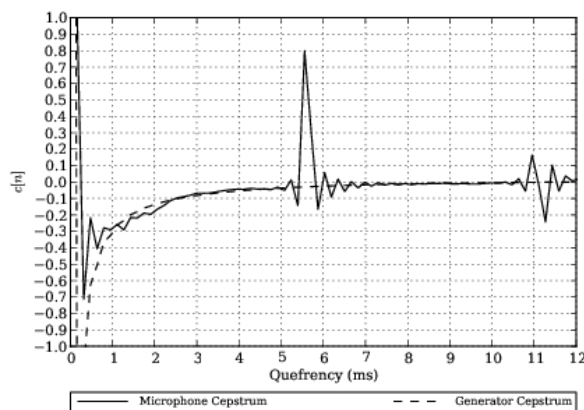


Figure 5. The cepstrum for the captured microphone and generator signals.

Extraction Window

To determine the absorption coefficient of the loudspeaker the impulse response of the sample needs to be extracted from the cepstrum. Discontinuities at the edges of the impulse response introduce undulations into the frequency response of the sample. In order to reduce the discontinuities a window is applied to the impulse response. The purpose of the window is to gently roll the edges of the impulse response to zero at the expense of frequency resolution of the transformed response.

The Sample under Test

Morgan (2003) discusses the accuracy of the MLS technique, used in the Extended Surface Method, relating to the surface being measured. It was found that the less sound energy absorbed by a material the greater the uncertainty. This can be

shown by noting the variance of the absorption coefficient, σ_α^2 , is

$$\begin{aligned} \sigma_\alpha^2 &= \left(\frac{d\alpha}{dH}\right)^2 \sigma_H^2 \\ &= (-2H)^2 \sigma_H^2. \end{aligned} \quad (13)$$

Here α is the absorption coefficient, H is the frequency response of the material, and σ_H^2 is the variance of the frequency response of the material.

If one considers the fractional error of the absorption coefficient, σ_α^2/α , it can be seen that for low levels of absorption the fractional error due to uncertainties in the system becomes significant.

Variance in the determined frequency response of a material is due to a combination of all system uncertainties. This illustrates the importance of minimizing all uncertainties especially for measuring material with little absorption.

EXPERIMENTAL VERIFICATION

We performed measurements in the University laboratory using commercially available equipment as shown in Figure 6:

1. an excitation signal was generated using Python 2.7 on an Apple Macbook Pro (Early 2011) computer;
2. the digital signal was converted to an analogue signal by a Tascam US-122MKII external sound card;
3. a laboratory power amplifier then amplified the signal;
4. the amplified signal was fed back to the sound card through a 50:1 voltage divider, and also connected to the APart-Audio OVO5T loudspeaker;
5. the loudspeaker was enclosed in a special cabinet that connected to a 2 meter PVC tube;
6. halfway along the tube a Bruel & Kjaer Type 4134 microphone connected to a Bruel & Kjaer Type 2619 preamplifier was centered inside the tube with a centering device. The preamplifier was powered by a Bruel & Kjaer Type 2803 power supply;
7. the material sample under test was placed inside a thick steel sample holder mounted to the end of the tube;
8. the captured microphone signal was converted to a digital signal with the sound card;
9. the digital signal was then processed in the computer.

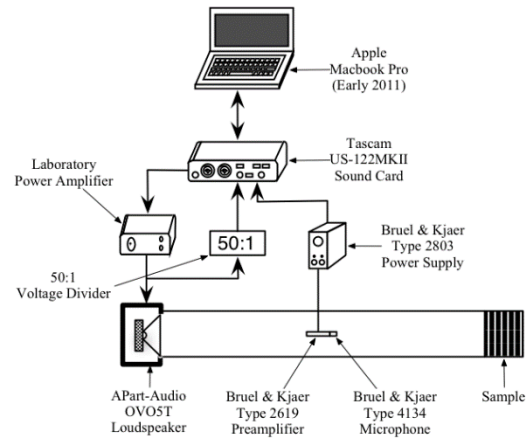


Figure 6. Apparatus used to determine the absorption coefficient using the cepstral technique.

The following measurement parameters were chosen:

- Inverse Repeat Sequence Length: 32766 samples (14 taps used to generate the signal);
- measurement bandwidth: 3.5 kHz;
- measurement sample rate: the signal was captured with a sampling frequency of 44.1 kHz, and then decimated to 7.35 kHz;
- high pass filter: 2nd order digital Butterworth with cut off frequency of 1600 Hz;
- low pass filter: 8th order digital Butterworth with cut off frequency of 3500 Hz;
- tube dimensions: 2 meter long, 110 mm diameter tube;
- bandpass filter: 4.7 ms one-sided Tukey window with 1 ms taper starting at 5.3 ms.

Numerous materials were tested using the cepstral tube and correlated with the measurements performed with a Bruel & Kjaer Type 4001 impedance tube. The results of four of these are presented.

The thick steel backing of the sample holder was set flush with the front of the sample holder. The sample holder was then used as a reflective sample to highlight any inaccuracies in the measurements.

The cepstrum of the measurement is shown in Figure 7, as well as the window used to lift the impulse response of the surface.

The impulse response of the surface was then Fourier-transformed, squared, and subtracted from 1, in accordance with Equation 6. The resulting absorption coefficient, and the results obtained with the impedance tube, are plotted in Figure 8.

The agreement between the cepstral technique and the impedance tube are close over the frequency range of 200 Hz to 2000 Hz. A fluctuation due to the bandpass filter frequency response can be seen.

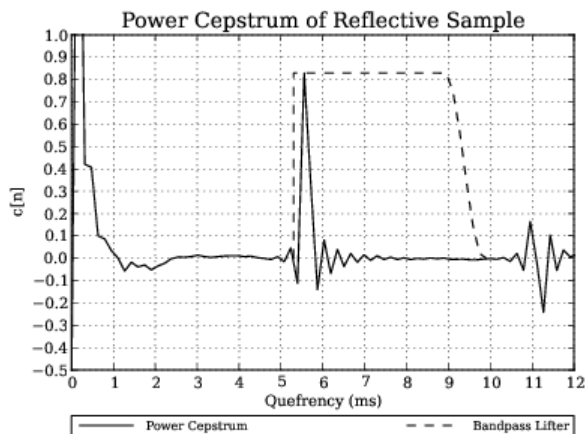


Figure 7. Power cepstrum for the sample holder which was used as a reflective sample.

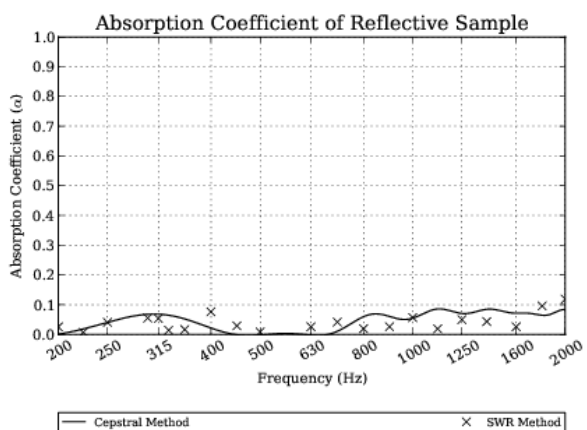


Figure 8. Absorption coefficient for the sample holder which was used as a reflective sample.

The next measurement was performed on a 60 mm thick glass fibre sample. The sample was selected as it is typical of porous absorbers and has a smooth transfer function.

The power cepstrum of the measurement is shown in Figure 9 and the impulse response of the sample can be seen in the windowed section. The impulse response of the sample falls to negligible levels in approximately 3 ms.

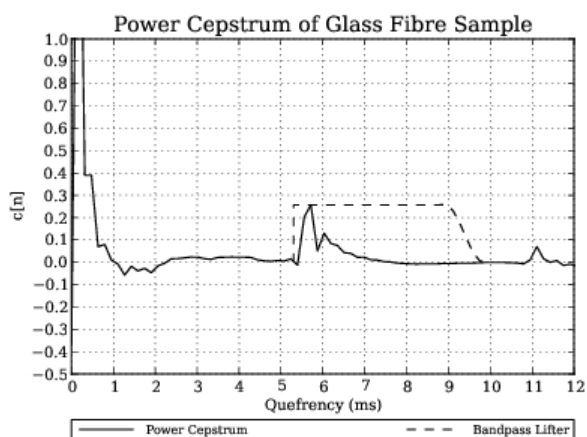


Figure 9. Power cepstrum for 60 mm of glass fibre.

The absorption coefficient is shown in Figure 10. There is a very close agreement of the cepstral tube results and those obtained with the impedance tube.

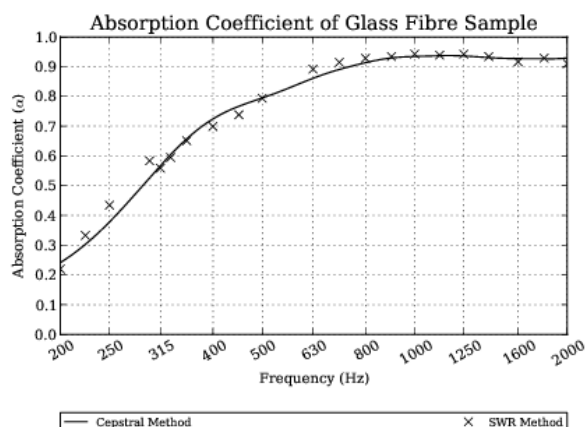


Figure 10. Absorption coefficient for 60mm of glass fibre.

The third sample was of a thick porous asphalt road surface called Twinlay as it contains two porous layers of different sized stone aggregate. It displays two absorption maxima in the 500 Hz to 2000 Hz frequency range with an absorption bandwidth of approximately 200 Hz.

The power cepstrum of the measurement is shown in Figure 11. It can be seen that there is energy in the tail of the surface's impulse response that extends beyond the length of the bandpass lifter.

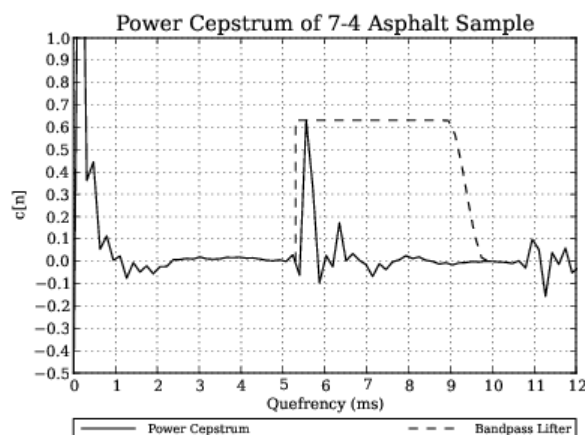


Figure 11. Power cepstrum for Twinlay asphalt sample.

The resulting absorption coefficient is plotted in Figure 12. There is a fairly good agreement with the cepstral tube measurements and the impedance tube measurements. The accuracy of the low frequency portion of the absorption coefficient, between 200 Hz and 350 Hz, has been reduced because the impulse response was truncated. A fluctuation can be noted, particularly in the low frequency range. The fluctuation is due to the small offset observed in the power cepstrum.

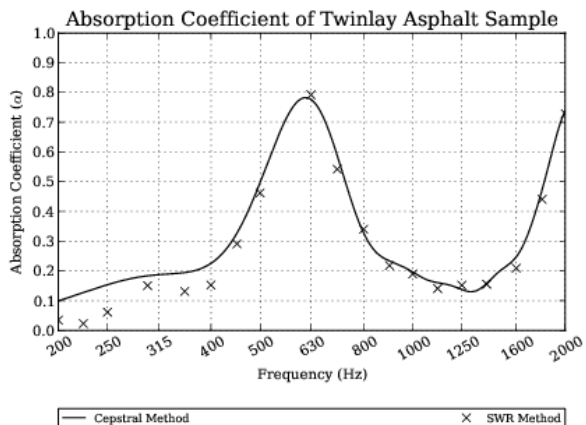


Figure 12. Absorption coefficient for Twinlay asphalt sample.

Figure 12 shows that the resolution obtained is sufficient to measure the absorption coefficient maximum of the TWINLay sample. Figure 13 shows the impulse response of a Helmholtz resonator with a narrow band of absorption. The bandwidth of the absorption peak is less than the system is able to accurately measure. It can be seen that there is still energy in the tail outside the *bandpass lifter*. Figure 14 shows the absorption coefficient curve of the measurement. The energy that is truncated by the *bandpass lifter* results in a poor correlation with the impedance tube measurement.

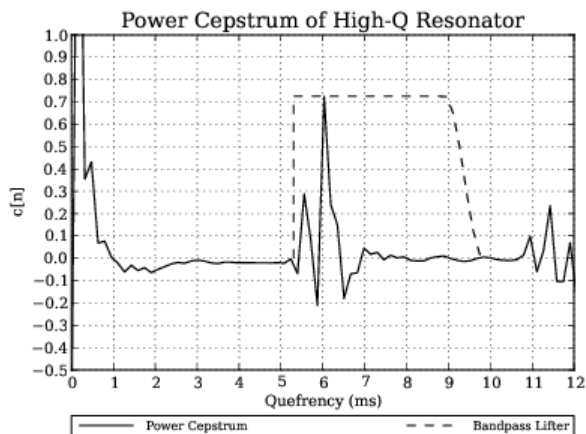


Figure 13. Power cepstrum for high-q resonator sample.

CONCLUSIONS

We showed how cepstral techniques can be used to determine the sound power absorption coefficient using a cepstral tube.

Factors that influence measurement accuracy were identified. It was shown that the system electronics influence on results can be reduced using cepstral subtraction.

For samples where the absorption coefficient varied slowly with frequency, as seen in the reflective and glass fibre sample, a close correlation was found between the cepstral tube measurement and the impedance measurements. The average difference between measurements was less than 0.03.

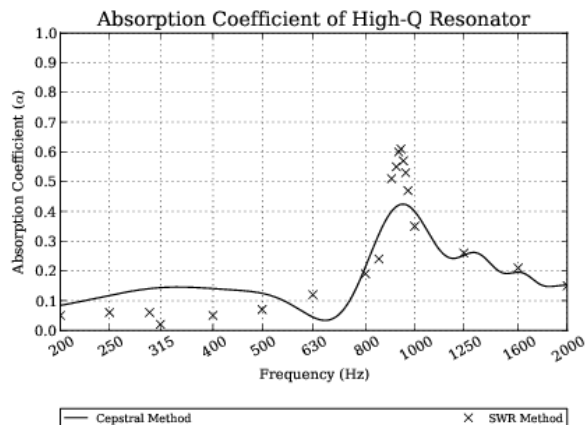


Figure 14. Absorption coefficient for pegboard resonator.

Where the absorption coefficient was more complicated, as shown in the Twinlay asphalt sample and High-Q resonator, the accuracy of the cepstral tube was reduced. This is due to the low frequency energy present in the sample's impulse response that is cut off by the bandpass lifter. This was shown in Figure 13. However a close correlation was found between the cepstral tube and impedance tube measurements in the frequency range that is of interest when measuring porous asphalt samples. The average difference between the two sets of results was less than 0.03 over the frequency range of 500 Hz to 2000 Hz.

A fluctuation present in all the absorption coefficient results was due to a small varying offset in the liftered impulse response. It is thought that this offset was due to a limitation of the low-cost hardware; further measurements need to be performed with high quality equipment to confirm this.

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