Shallow water sound propagation over a layered calcarenite seafloor: an exercise in benchmarking various models

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ABSTRACT

The sediment deficient western and southern Australian coastal seafloors consist of semi-compact layered calcarenite. Although there is a large body of work investigating and modelling the characteristics of underwater sound propagation over a layered elastic seafloor, there are still difficulties associated with modelling complex shallow water environments like the Australian shelf. For a range independent marine environment with a calcarenite bottom, the acoustic propagation characteristics have been previously modelled using numerical methods based on wavenumber integration theory. This work investigates the ability of other sound propagation models to accurately predict the acoustic field over calcarenite seafloors, specifically models based on normal mode theory. The benefits and drawbacks of this alternative modelling approach are discussed.

INTRODUCTION

In underwater acoustics a shallow water environment is typically characterised by a water depth on the order of 20 m to 100 m. The seafloor and sea surface form an acoustic waveguide for underwater sound propagation in the water column. The structure and material properties of the seafloor become important when modelling sound propagation in shallow water environments, even more so when comparing real data to model results.

The coastal marine areas of southern, western, and north western Australia are characterised by an ocean bottom composed of well-cemented or semi-cemented limestone rock called calcarenite (Duncan et al., 2009, Collins, 1988, James et al., 1994). It has been shown by Duncan et al. (2009), that when modelling the transmission loss of sound propagating over calcarenite seafloors, compressional wave to shear wave conversion in the solid seafloor in part determine how much acoustic energy is trapped and can propagate in the ocean interior. Sound propagation models must account for this conversion in order to model the sound field in the water column.

To model low frequency (10 Hz – 500 Hz) sound propagation in the ocean, the water column can be treated as a layered medium with variable acoustic properties and the ocean bottom as a layered elastic medium (Ewing et al., 1957). Sound propagation over the Australian shelf has been modelled using various geoacoustic parameters for the ocean bottom (Parsons and Duncan, 2011). Layered ocean bottoms with combinations of sand layers, calcarenite layers (Duncan et al., 2009), and sedimentary basements (Duncan et al., 2013) have been used to investigate the influence of the ocean bottom on the water column sound field.

Sound propagation models based on wavenumber integration theory are able to accurately model the sound field in layered acoustic and/or elastic media (Jensen et al., 1994). Thus, wavenumber integration models such as SCOOTER (Porter, 2010) have been favoured when modelling sound propagation over calcarenite bottoms. However, wavenumber integration sound propagation models are limited to modelling range independent environments and incur large computational cost at high frequencies or long ranges.

Sound propagation models based on the parabolic equation (PE) method are also capable of modelling acoustic interaction with an elastic bottom (Collins, 1993, Collins, 1989). PE models were developed to model range dependent (Jensen and Ferla, 1990) environments and have been used extensively for that purpose. However, when used to model sound propagation over layered calcarenite bottoms, some PE models can be prone to numerical instabilities as discussed by Duncan et al. (2009). A newly developed seismoacoustic PE model has performed better when modelling laterally varying elastic layers (Collis et al., 2008), however further benchmarking is required and accurate results can come at high computational cost.

In this paper we investigate the potential to use other available numerical propagation models to model range independent sound propagation in oceans with calcarenite bottoms. This work was carried out as a first step to developing a new approach to modelling range dependent propagation in these environments. We compare the accuracy and low frequency performance of two models based on normal mode theory against a reference model. The models, ORCA (Westwood et al., 1996) and KRAKENC (Porter, 2010) are candidates for this comparison and SCOOTER was used to compute reference solutions.

SOUND PROPAGATION CHARACTERISTICS

Calcarenite is characterised by a shear wave speed lower than the sound speed of water (Duncan et al., 2009). For underwater sound propagation, the low shear wave speed in a calcarenite seafloor acts as, and is analogues to, a form of attenuation for acoustic waves (Brekhovskikh, 1960). For sound propagation in the ocean where calcarenite is present, this energy loss mechanism is significant even at low grazing angles where good propagation is be expected (Duncan et al., 2009). This mechanism is schematically depicted with a ray diagram in Figure 1.

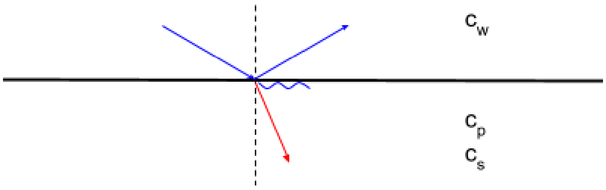

Figure 1. Shear wave conversion as an attenuation mechanism for an acoustic wave incident on a calcarenite halfspace. The blue arrows represent incident and reflected compressional wave rays in the ocean and the blue wiggle represents an evanescent compressional wave in the bottom. The red arrow represents the transmitted shear wave in the bottom. The symbols, c_w, c_p, and c_s respectively represent different wave speeds for the compressional wave in water, the compressional wave in the halfspace, and the shear wave in the halfspace.

Some regions contain a layer of well-cemented calcarenite cap rock at the seafloor (Duncan et al., 2013). The presence of a cap layer increases the shear wave loss mechansim for incident acoustic waves (Duncan and Gavrilov, 2012). The influence of this cap rock layer is expected at wavelengths comparable to or smaller than the layer thickness but modelling results predict losses to occur even for compressional wavelengths larger that the layer thickness (Duncan and Gavrilov, 2012).

NORMAL MODES AND WAVENUMBER INTEGRATION SOUND PROPAGATION MODEL COMPARISON

For a layered calcarenite ocean bottom, a representative geoacustic model has been considered by (Duncan et al., 2013). A slightly modified senario is presented in Table 1. The acoustic waveguide is made up of three layers and a halfspace, each layer contains constant geoacustic parameters: h is the layer thickness, ρ is the layer density, c_p is the compressional wave speed, α_p is the compressional wave attenuation, c_s is the shear wave speed, and α_s is the shear wave attenuation.

Table 1. The representative geoacoustic parameters for a layered ocean bottom with a well-cemented calcarenite cap, semi-cemented calcarenite, and an underlying sedimentary basement.

	Ocean Water Column	Calcarenite Cap-rock	Sandy Calcarenite Semi-cemented	Sedimentary Basement
h (m)	110	1	1000	∞
ρ (g/cm^3)	1.03	2.2	1.9	3.0
c_p (m/s)	1500	2600	2100	3800
α_p (dB/λ)	0.0	0.52	0.25	0.38
c_s (m/s)	0.0	1200	550	1900
α_s (dB/λ)	0.0	0.48	0.14	0.38

The sound propagation models SCOOTER, KRAKENC and ORCA were run with the geoacoustic model presented above. Transmission loss was calculated at 1 Hz increments from 5 Hz to 100 Hz using each sound propagation model. This frequency range was chosen to assess each model's ability to predict broadband low frequency energy. Figure 2 displays the results from this analysis.

For a complex layered elastic seafloor, KRAKENC was not able to reproduce the same transmission loss pattern as SCOOTER. ORCA however, preformed much better. There are a few subtle differences in the fine pattern of the transmission loss (Figure 2) when comparing SCOOTER to ORCA. More importantly, the magnitudes of low transmission loss peaks, w1hich correspond to energy propagating in the ocean interior, are consistent between SCOOTER and ORCA.

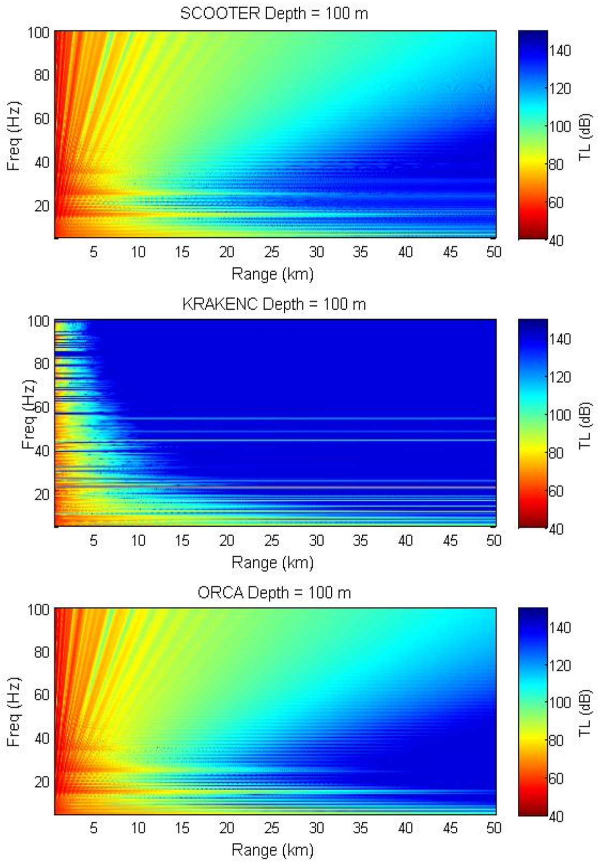

Figure 2. Broadband transmission loss comparison between SCOOTER, KRAKENC, and ORCA. The transmission loss was calculated at a depth of 100 m and the source depth was 15 m.

To further test the robustness of ORCA, a depth dependent sound speed profile was inserted above the layered calcarenite bottom. A representative shallow water profile was chosen to test ORCA in a semi-realistic modelling scenario. The profile consisted of a small positive gradient, 0.028 ms^{-1}/m, from 0 m to 35 m depth to mimic a mixed surface layer (Medwin, 1975). This slightly exaggerated gradient is 1.75 times larger than the mixed layer gradient discussed by Medwin (1975) but was chosen to test each model's performance in complex propagation conditions with semi-realistic values. As the depth increases, the gradient becomes negative with a magnitude of 0.15 ms^{-1}/m. This was chosen to model a downward refracting profile due to temperature decreasing with depth (Duncan and Parsons, 2011). Figure 3 shows the sound speed profile and a transmission loss comparison between ORCA and SCOOTER at 35 Hz.

At a single frequency ORCA performs well. In the right panel of Figure 3, there is some discrepancy in the magnitude of the transmission loss at longer ranges between ORCA and SCOOTER. At 11 km the difference between the peaks is approximately 2 dB. However, the phase of each curve is very consistent with range, with the peaks and nulls of the transmission loss occurring at the same places.

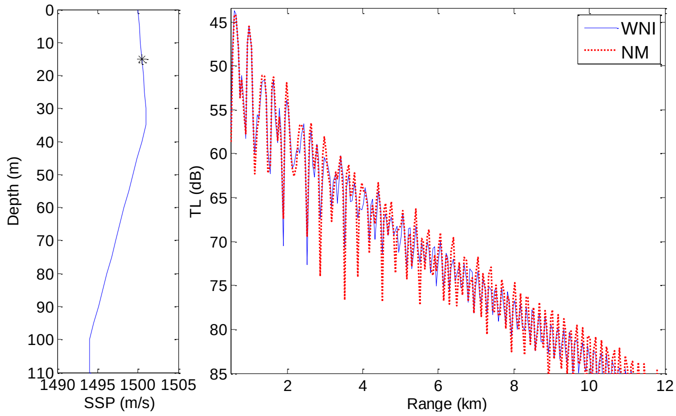

Figure 3. *Left:* Depth dependent sound speed profile, with a source at 15 m. *Right:* Transmission loss at a frequency of 35 Hz for a receiver depth of 20 m calculated by ORCA (red)and compared to the reference solution from SCOOTER (blue).

The locations of the modal eigenvalues found by ORCA are plotted in the complex plane (Figure 4). The imaginary part of the complex wavenumber relates to the amount of attenuation for each mode (Brekhovskikh, 1960). The modes that represent good propagation in the ocean interior lie between k_w and $k_{calcarenite}$, but still have attenuation due to shear wave losses and material absorption.

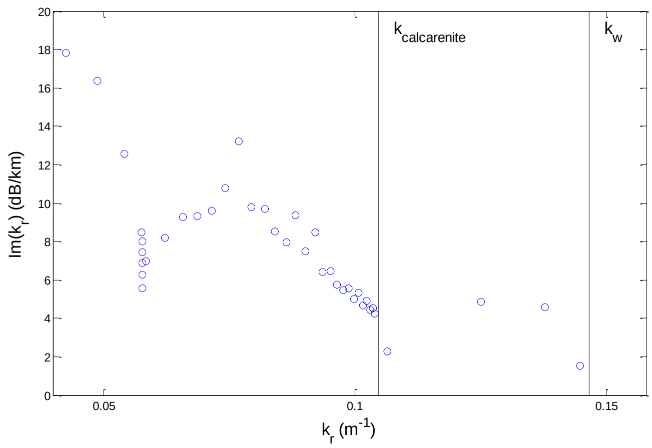

Figure 4. Modal eigenvalues found by ORCA plotted in the complex plane for the depth dependent sound speed profile over calcarenite at 35 Hz. Propagating modes in the water lie between k_w and $k_{calcarenite}$.

Finally, the environment above was tested over a broad band of low frequencies. Again the transmission loss was calculated over frequencies ranging from 5 Hz to 100 Hz at increments of 1 Hz. To better observe some of the fine scale transmission loss differences between SCOOTER and ORCA, a slice of the transmission loss is shown in Figure 5.

Again the main, and most important peaks in the broadband transmission loss are consistent. The curves also converge with increasing frequency.

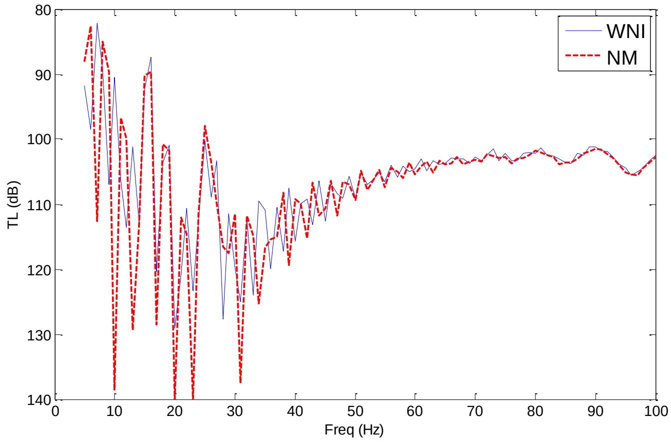

Figure 5. Broadband transmission loss (5 Hz to 100 Hz) results from SCOOTER (blue) and ORCA (red) at a depth of 100 m and a range of 20 km.

DISCUSSION

Referring to the comparison between SCOOTER and KRAKENC results in Figure 2, KRAKENC fails to predict the same transmission loss as the reference solution. KRAKENC numerically calculates ocean normal modes based on a finite-difference approximation to the mode functions which yields a matrix eigenvalue problem (Porter and Reiss, 1984). The modal wavenumbers are then found by using a complex root-finder based on the secant method to find the roots of the matrix problem (Porter and Reiss, 1985). However, KRAKENC's root finder does not find the correct modes to predict broadband transmission loss over a layered calcarenite bottom (Figure 2).

The eigenvalue criterion for ORCA is based on a different approach to solving the modal problem. For gradients in the acoustic sound speed, ORCA calculates modes based on analytic Airy functions and uses exponential functions for isovelocity layers (Westwood et al., 1996). For layered media, matching interface boundary conditions connects the analytic function in each layer, solving the depth dependent part of the acoustic field. Modal eigenvalues are found from a phase function criterion based on upward and downward looking reflection coefficients from a reference depth (Tindle and Chapman, 1994). The result is that many modes are found (Figure 4), including modes propagating in the bottom (leaky modes) which lie to the left of $k_{calcarenite}$ as seen in Figure 4.

The differences in the transmission loss magnitude between SCOOTER and ORCA of 2-3 dB are negligible when considering that each model is based on a different numerical method. Large deviations between the two models occur at high transmission loss where numerical rounding errors are common. Moreover in real data, uncertainty in measured transmission loss is typically dominated by uncertainty in environmental parameters; this uncertainty can be greater than the 2-3 dB difference (Gerstoft et al., 2006). Thus, without comparing models to real data with well constrained geoacoustic parameters, it is not immediately apparent which model is correct. What is important is that both models, SCOOTER and ORCA, predict transmission loss that is consistent with each other for this geoacoustic environment.

The ability of ORCA to replicate the same transmission loss as SCOOTER implies that ORCA may likely be used as a normal mode benchmark sound propagation model for shallow water calcarenite bottom environments. The multi-frequency performance was found to consistently predict the low loss peaks that are expected for a calcarenite bottom. Accurate prediction of these peaks around the critical angle is important, because they are narrow frequency bands where good propagation in the water column is expected (Duncan and Gavrilov, 2012). A more rigorous benchmarking exercise would consider a variety of bottom parameters and is part of ongoing work.

CONCLUSIONS

Underwater sound propagation over a multilayered calcarenite bottom has been modelled using normal mode theory. The resulting transmission loss from the normal mode model, ORCA, was found to be consistent with the benchmark wavenumber integration model, SCOOTER. The other model tested, KRAKENC was found to be inconsistent with SCOOTER. ORCA's performance over a broad band of low frequencies was assessed and found to be robust. Some further benchmarking work is required to determine the cause of the amplitude difference between models. The capability of using normal mode theory to model sound propagation over a layered calcarenite bottom is significant because some normal mode models, namely ORCA, may now be considered as a second option to model this environment. Moreover, there is potential for normal mode models to be modified and extended to calculate the sound field in a range dependent environment with a calcarenite bottom, which is the ultimate aim of this research.

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