Modal Analysis of Structures with Uncertainties using Polynomial Chaos Expansion

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ABSTRACT

Engineering structures are complex and structural uncertainties are generated from variability in the material or geometric properties, or in the manufacturing and assembly process. Such variations generate differences in the dynamic responses of the structures across an ensemble of nominally identical systems, for example, vehicles off the production line. This work describes the theory and application of polynomial chaos expansion to examine the natural frequencies and modeshapes of structures with uncertainties. The modal statistics of a two degree-of-freedom mass-spring chain is examined. The structural uncertainty across an ensemble of nominally identical mass-spring chains is generated using mass and stiffness perturbations. The parameter uncertainty is constructed using log-normal and uniform distributions. Results obtained using the polynomial chaos expansion method are compared with Monte Carlo simulations.

INTRODUCTION

Typical engineering structures include bridges, buildings, offshore structures, ships, vehicles and aerospace structures. These structures generally possess randomness due to variability in their geometric or material parameters. The difficulty in attempting to predict the dynamic responses of engineering structures with uncertainties is immense, due to the large amount of physical variables which might be uncertain and the lack of data regarding the statistical distribution of these variables. Furthermore, for an ensemble of nominally identical structures such as vehicles off the production line, variability in the dynamic responses of each ensemble member occurs because of uncertainties in the manufacturing and assembling process, for example, due to spot welding. This phenomenon has been demonstrated by measuring structural-acoustic transfer functions in 98 nominally identical automotive vehicles and observing significant differences in the interior noise levels (Kompella and Bernhard, 1993).

Generally the first step to predict the dynamic responses of a structure is to determine its natural frequencies and modeshapes. Hence, modal analyses can be combined with parameter uncertainties to examine their influence on the dynamic characteristics. Models of uncertainty are usually based on either a parametric or non-parametric description of uncertainty, or sometimes, on a combination of both. A parametric description of uncertainty means that the parameters of the dynamic system are taken to be uncertain variables that can be described statistically. Uncertainty is then propagated through the equations of motions using various techniques, including the perturbation method (Adhikari and Manohar, 1999), interval analysis (Moens and Vandepitte, 2005) and fuzzy theory (Soize, 1993). The inherent limitation of a parametric method is that for acceptable accuracy, the analytical model can be quite complex resulting in significant computational cost.

A non-parametric analysis of uncertainty assumes that regardless of their detailed nature, the uncertainties in the system can be described using a ‘universal uncertainty’ model. Recently, random matrix theory and the polynomial chaos expansion method have been implemented to investigate the dynamic characteristics of structures with uncertainty. To apply random matrix theory, the system should be sufficiently random for statistical overlap to occur (Kessissoglou and Lucas, 2009), and as such is more suited to the study of high frequency dynamics involving the higher order modes. The polynomial chaos expansion was first introduced as the homogeneous chaos (Wiener, 1938).

The polynomial chaos expansion method represents the uncertain variables by orthogonal polynomials of standard random variables and can be applied to examine low frequency dynamics. An uncertainty model using polynomial chaos expansion theory was first applied to solid mechanics using a combined finite element and Hermite chaos method (Ghanem and Spanos, 1991). Since then the polynomial chaos expansion method has been widely implemented in many disciplines, including solid mechanics (Ghanem, 1999), fluid mechanics (Rupert and Miller, 2007), thermodynamics (Lin and Karniadakis, 2006), structural dynamics and random vibrations (Sepahvand et al., 2007) and fluid-structure interaction problems (Witteveen et al., 2007).

This paper examines application of the polynomial chaos expansion method to investigate the modal analyses of structures with uncertainties. The natural frequencies and modeshapes of a two degree-of-freedom mass-spring chain randomized by mass and stiffness perturbations are studied. Parameter uncertainty was constructed using log-normal and uniform distributions. For each mass-spring chain, uncertainty across an ensemble of nominally identical mass-spring chains was generated in order to observe the probability distribution of the modal statistics. The modal results predicted using the polynomial chaos expansion method are compared with Monte Carlo simulations.

NUMERICAL MODEL

Polynomial chaos expansion theory

The basic methodology using polynomial chaos (PC) expansion involves replacing the stochastic system equations with deterministic equations. The solutions of the deterministic equations then approximate the solutions of the stochastic system equations. The first step in applying the polynomial chaos expansion theory is to project the uncertain variables onto a stochastic space spanned by a set of mutually orthogonal polynomials Ψ_i, which are functions of a multi-dimensional random variable $\boldsymbol{\xi} = \{\xi_1, \xi_2, \ldots, \xi_n\}$. Every random variable has a corresponding random space $\xi_i \in \Omega_i$ $(i = 1,2,\ldots,n)$. Then the uncertain variable χ can be expressed as (Xiu and Karniadakis, 2002)

$$\chi = \sum_{i=0}^{\infty} x_i \Psi_i(\boldsymbol{\xi}) \tag{1}$$

where x_i are the deterministic coefficients. The random base functions Ψ_i are a set of multi-dimensional polynomials in terms of $\boldsymbol{\xi}$ with the orthogonal relation of

$$\mathbf{E}(\Psi_i, \Psi_j) = \delta_{ij}\mathbf{E}(\Psi_i^2) \tag{2}$$

where δ_{ij} is the Kronecker delta and $\mathbf{E}$ represents the expected value in the probability space. Selection of the random base function Ψ_i depends on the probability density function of random variables (Xiu and Karniadakis, 2002).

According to the orthogonal feature, the unknown coefficients x_i can be determined by stochastic Galerkin projection (Sepahvand et al., 2010)

$$x_k = \frac{1}{\mathbf{E}(\Psi_k^2)} \int_{\Omega} \chi \Psi_k(\boldsymbol{\xi}) d\mu(\boldsymbol{\xi}), \; k = 0,1,2,\ldots \tag{3}$$

where $d\mu(\boldsymbol{\xi})$ is the probability measure in the random space Ω. If the random variables ξ_i are continuous and mutually independent, then $d\mu(\boldsymbol{\xi})$ can be expressed as

$$d\mu(\boldsymbol{\xi}) = \rho_1(\xi_1)\rho_2(\xi_2) \ldots \rho_n(\xi_n) d\xi_1 d\xi_2 \ldots d\xi_n \tag{4}$$

where $\rho(\xi)$ is the probability density function of the random variable.

Modal analysis using polynomial chaos expansion theory

The equation of motion of the undamped mass-spring chain is given by

$$(\mathbf{K} - \omega^2\mathbf{M})\mathbf{X} = 0 \tag{5}$$

where $\mathbf{K}$ and $\mathbf{M}$ are the global stiffness and mass matrices, respectively. For an n degree-of-freedom system and assuming $\omega^2 = \lambda$, we have

$$\mathbf{K}\mathbf{X}_i - \lambda_i \mathbf{M}\mathbf{X}_i = 0, \; i = 1,2,\ldots,n \tag{6}$$

with the normalization condition

$$\mathbf{X}_i^T\mathbf{X}_i = 1, \; i = 1,2,\ldots,n \tag{7}$$

For the mass-spring systems with uncertain parameters, the eigenvalue problem can be expressed by

$$\mathbf{K}(\boldsymbol{\xi_k})\mathbf{X}_i(\boldsymbol{\xi}) - \lambda_i(\boldsymbol{\xi})\mathbf{M}(\boldsymbol{\xi_m})\mathbf{X}_i(\boldsymbol{\xi}) = 0, \; i = 1,2,\ldots,n \tag{8}$$

$$\mathbf{X}_i^T(\boldsymbol{\xi})\mathbf{X}_i(\boldsymbol{\xi}) = 1, \; i = 1,2,\ldots,n \tag{9}$$

where $\boldsymbol{\xi_k} \in \Omega_k$, $\boldsymbol{\xi_m} \in \Omega_m$, $\boldsymbol{\xi} \in \Omega_k \otimes \Omega_m$.

In this work, the uncertain parameters are represented by the truncated PC expansions with a limited number of polynomials:

$$\mathbf{K}(\boldsymbol{\xi_k}) = \sum_{p=0}^{N_k} \mathbf{K}_p \Psi_{k,p}(\boldsymbol{\xi_k}) \tag{10}$$

$$\mathbf{M}(\boldsymbol{\xi_m}) = \sum_{q=0}^{N_m} \mathbf{M}_q \Psi_{m,q}(\boldsymbol{\xi_m}) \tag{11}$$

$$\lambda_i(\boldsymbol{\xi}) = \sum_{r=0}^{N_\lambda} \lambda_{i,r} \Psi_r(\boldsymbol{\xi}), \; i = 1,2,\ldots,n \tag{12}$$

$$\mathbf{X}_i(\boldsymbol{\xi}) = \sum_{s=0}^{N_X} \mathbf{X}_{i,s} \Psi_s(\boldsymbol{\xi}), \; i = 1,2,\ldots,n \tag{13}$$

where N_k, N_m, N_λ, N_X are respectively the number of polynomials to represent the stiffness matrix, mass matrix, natural frequencies and modeshapes.

Substituting the expansion equations given by Eqs. (10)-(13) into Eqs. (8) and (9), multiplying by a random base tion $\Psi_t(\boldsymbol{\xi})$ and then using Galerkin projection results in:

$$\begin{aligned}
&\sum_{p=0}^{N_k}\sum_{s=0}^{N_X} \mathbf{K}_p \mathbf{X}_{i,s} \mathbf{E}[\Psi_{k,p}(\boldsymbol{\xi_k})\Psi_s(\boldsymbol{\xi})\,\Psi_t(\boldsymbol{\xi})] - \\
&\sum_{r=0}^{N_\lambda}\sum_{q=0}^{N_m}\sum_{s=0}^{N_X} \lambda_{i,r}\mathbf{M}_q\mathbf{X}_{i,s}\mathbf{E}[\Psi_r(\boldsymbol{\xi})\Psi_{m,q}(\boldsymbol{\xi_m})\Psi_s(\boldsymbol{\xi})\,\Psi_t(\boldsymbol{\xi})] \\
&= 0, \;\; t = 0,1,2,\ldots,N_t
\end{aligned} \tag{14}$$

$$\begin{aligned}
&\sum_{s_1=0}^{N_X}\sum_{s_2=0}^{N_X} \mathbf{X}_{i,s_1}^T \mathbf{X}_{i,s_2} \mathbf{E}[\Psi_{s_1}(\boldsymbol{\xi})\Psi_{s_2}(\boldsymbol{\xi})\,\Psi_t(\boldsymbol{\xi})] - \mathbf{E}[\Psi_t(\boldsymbol{\xi})] \\
&= 0, \;\; t = 0,1,2,\ldots,N_t
\end{aligned} \tag{15}$$

where N_t is the number of base polynomials.

In this paper, to simplify further analysis we assume that $N_\lambda = N_X = N_t = w$. Thus Eqs. (14) and (15) consist of $(n+1)\times(w+1)$ separate equations. The deterministic coefficients of the polynomial chaos expansion for the i^{th} natural frequency are given by $[\lambda_{i0}, \lambda_{i1}, \ldots, \lambda_{iw}]^T$.The deterministic coefficients of the polynomial chaos expansion for the i^{th} modeshape are given by $[\mathbf{X}_{i0}, \mathbf{X}_{i1}, \ldots, \mathbf{X}_{iw}]^T$. The deterministic coefficients are solved for each mode, which consists of $(n+1)\times(w+1)$ elements.

To solve the system of nonlinear equations, the Newton-Ralphson algorithm is adopted (Ghanem and Ghosh, 2007). To start the iterative process, the natural frequency and modeshapes of the nominal mass-spring chain are set as the initial estimate.

TWO DEGREE-OF-FREEDOM MASS-SPRING CHAIN WITH UNCERTAIN STIFFNESS

Using the PC expansion method, modal analysis of a two degree-of-freedom mass-spring system shown in Fig. 1 with variability in its stiffness is initially examined. The nominal values of the parameters for the mass-spring chain are listed in Tab. 1. The nominal natural frequencies are 1 rad/s and 2.34 rad/s, with corresponding modeshapes [0.71 0.71] and [0.89 -0.45], respectively.

The uncertain stiffness of each spring is assumed to follow log-normal and uniform distributions, which are represented by the Hermite and Legendre polynomials (Sepahvand et al., 2010). The probability density function, $\rho(x)$, mean and variance value of a log-normal distribution are given by Eqs. (16)-(18).

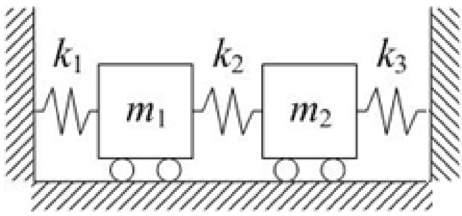


Figure 1. Two DOF mass-spring chain

Table 1. Parameters of the mass-spring chain

Parameter	Value
m_1	1 kg
m_2	2 kg
k_1	1 N/m
k_2	3 N/m
k_3	2 N/m

$$\rho(x) = \frac{1}{x\varepsilon\sqrt{2\pi}} e^{\frac{-(\ln x-\mu)^2}{2\varepsilon^2}}, \quad x \geq 0 \tag{16}$$

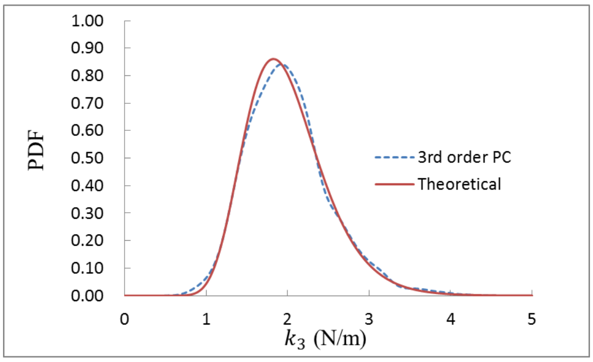
$$\text{mean}(x) = e^{(\mu+\varepsilon^2/2)} \tag{17}$$

$$\text{var}(x) = e^{(2\mu+2\varepsilon^2)} - e^{(2\mu+\varepsilon^2)} \tag{18}$$

The uniform distribution can be represented exactly by the first-order Legendre polynomial expansion. In this paper, the log-normal distribution is represented by a third-order Hermite polynomial expansion. Good agreement between the probability density functions (PDFs) obtained using the exact theoretical solution and the 3rd order polynomial chaos expansion is achieved, as shown in Figure 2.

Uncertain stiffness by the same base random variable

In this case, the uncertain stiffness are assumed to follow log-normal distribution with different mean and variance values, which are listed in Tab. 2.



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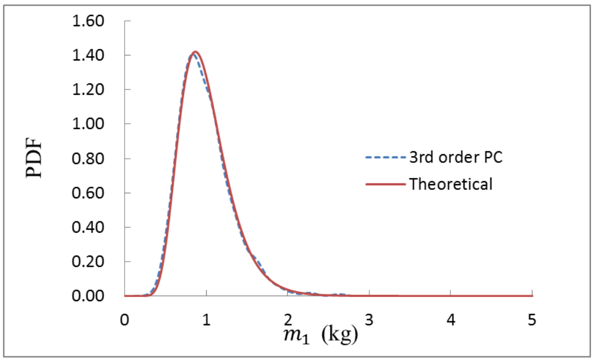

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Figure 2. Probability density function of uncertain parameter (a) k_3, mean = 2 N/m, var = 0.25 N^2/m^2 (b) m_1, mean = 1 kg, var=0.1 kg^2

Table 2. Parameters of the uncertain stiffness

Parameter	Mean	Variance
k_1	1 N/m	0.5 N^2/m^2
k_2	3 N/m	0.36 N^2/m^2
k_3	2 N/m	0.25 N^2/m^2

The log-normal distributions for the three springs are represented by the same base random variable ξ_1. Thus the system base random vector becomes $\boldsymbol{\xi} = \{\xi_1\}$. The stiffness can be expressed by a Hermite polynomial chaos expansion as (Sepahvand et al., 2010):

$$K_1(\xi_1) = \sum_{p=0}^{3} k_{1_p} H_p(\xi_1) \tag{19}$$

$$K_2(\xi_1) = \sum_{p=0}^{3} k_{2_p} H_p(\xi_1) \tag{20}$$

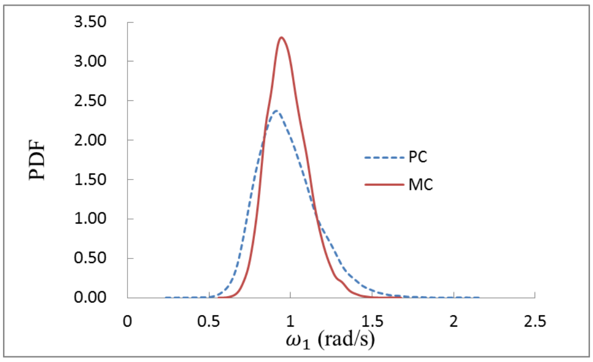
$$K_3(\xi_1) = \sum_{p=0}^{3} k_{3_p} H_p(\xi_1) \tag{21}$$

The natural frequencies and modeshapes can be expressed using the third-order Hermite polynomial chaos expansion as

$$\lambda_i(\xi_1) = \sum_{r=0}^{3} \lambda_{i,r} \Psi_r(\xi_1), i = 1,2 \tag{22}$$

$$\mathbf{X}_i(\xi_1) = \sum_{s=0}^{3} \mathbf{X}_{i,s} \Psi_s(\xi_1), i = 1,2 \tag{23}$$

The PDFs of the natural frequencies from polynomial chaos (PC) expansion theory and Monte Carlo (MC) simulations are shown in Fig. 3.



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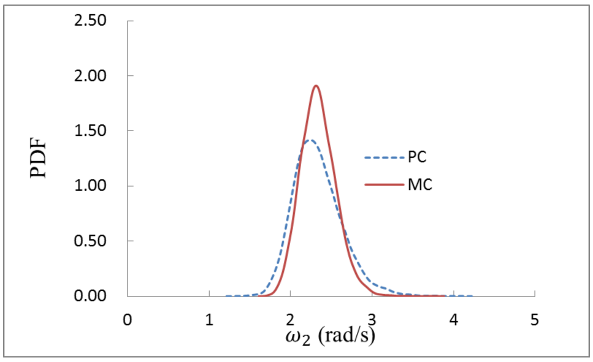

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Figure 3. Probability density function of the natural frequencies ω_1, ω_2 of the mass-spring chain with uncertain stiffness represented by the same base random variable

The Monte Carlo results were obtained from sampling 10000 sets of natural frequency data from the mass-spring chain, whose stiffness parameters were generated from the log-normal distributions specified above. The natural frequencies of the mass-spring chain were obtained in Matlab by solving $(\mathbf{K}-\omega^2\mathbf{M})\mathbf{X}=0$.

There are obvious differences between the PC and MC results. Hence, representing the three different stiffness by the same base variable is not accurate enough. In the MC simulations, the three stiffnesses are defined as mutually independent. In the PC expansion, the three different stiffness are approximated by the same base variable, which are not mutually independent and results great differences from MC results.

Uncertain stiffness by different base random variables

Uncertain stiffness using uniform and log-normal distributions

In this case, the uncertain stiffness k_1, k_3 are assumed to follow a log-normal distribution with different mean and variance values, which are listed in Tab. 3.

Table 3. Parameters of the uncertain stiffness

Parameter	Mean	Variance
k_1	1 N/m	0.5 N^2/m^2
k_3	2 N/m	0.25 N^2/m^2

The uncertain stiffness k_2 is assumed to follow a uniform distribution $U(2,4)$ with a mean value of 3 N/m. The two log-normal distribution are represented by the same base random variable ξ_1. The uniform distribution is represented by the base random variable ξ_2. Thus the system base random vector becomes $\boldsymbol{\xi}=\{\xi_1,\xi_2\}$. The stiffness can be expressed using the PC expansion as

$$K_1(\xi_1)=\sum_{p=0}^{3} k_{1_p} H_p(\xi_1) \qquad (24)$$

$$K_2(\xi_2)=\sum_{p=0}^{1} k_{2_p} L_p(\xi_2) \qquad (25)$$

$$K_3(\xi_1)=\sum_{p=0}^{3} k_{3_p} H_p(\xi_1) \qquad (26)$$

The natural frequencies and modeshapes can be expressed by a third-order PC expansion as

$$\lambda_i(\xi_1,\xi_2)=\sum_{\boldsymbol{r=0}}^{\boldsymbol{3}} \lambda_{\boldsymbol{i,r}}\Psi_r(\xi_1,\xi_2)\,, i=1,2 \qquad (27)$$

$$\mathbf{X}_{\boldsymbol{i}}(\xi_1,\xi_2)=\sum_{\boldsymbol{s=0}}^{\boldsymbol{3}} \mathbf{X}_{\boldsymbol{i,s}}\Psi_s(\xi_1,\xi_2)\,, i=1,2 \qquad (28)$$

To simplify the analysis, the interaction polynomial terms in $\Psi(\xi_1,\xi_2)$, which in this case is the product of Legendre and Hermite polynomials, are ignored. The PDF of the natural frequencies from the PC and MC simulations are shown in Fig. 4. The PDF of the first natural frequency is not well predicted by the PC theory, which is attributed to representing k_1, k_3 by the same base random variable and ignoring the interaction polynomial terms.

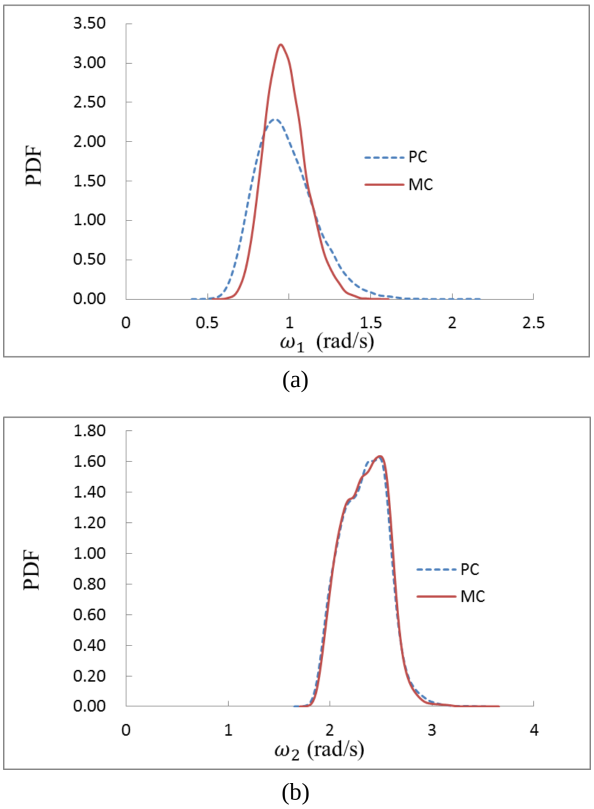


Figure 4. Probability density function of the natural frequencies ω_1, ω_2 of the mass-spring chain with uncertain stiffness using uniform and log-normal distributions

Uncertain stiffness using different log-normal distributions

In this case, the uncertain stiffnesses k_1, k_2, k_3 are assumed to follow log-normal distributions with different mean and variance values, which are listed in Tab. 2. The three log-normal distributions are represented by three different base random variables ξ_1,ξ_2,ξ_3. Thus the system base random vector becomes $\boldsymbol{\xi}=\{\xi_1,\xi_2,\xi_3\}$. The stiffness can be expressed using the Hermite PC expansion as

$$K_1(\xi_1)=\sum_{p=0}^{3} k_{1_p} H_p(\xi_1) \qquad (29)$$

$$K_2(\xi_2)=\sum_{p=0}^{3} k_{2_p} H_p(\xi_2) \qquad (30)$$

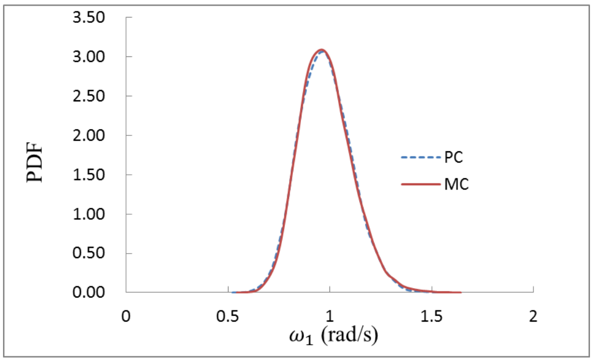
$$K_3(\xi_3)=\sum_{p=0}^{3} k_{3_p} H_p(\xi_3) \qquad (31)$$

The natural frequencies and modeshapes can be expressed by a third-order PC expansion as

$$\lambda_i(\xi_1,\xi_2,\xi_3)=\sum_{\boldsymbol{r=0}}^{\boldsymbol{3}} \lambda_{\boldsymbol{i,r}}\Psi_r(\xi_1,\xi_2,\xi_3)\,, i=1,2 \qquad (32)$$

$$\mathbf{X}_{\boldsymbol{i}}(\xi_1,\xi_2,\xi_3)=\sum_{\boldsymbol{s=0}}^{\boldsymbol{3}} \mathbf{X}_{\boldsymbol{i,s}}\Psi_s(\xi_1,\xi_2,\xi_3)\,,\ i=1,2 \qquad (33)$$

In this case the interaction polynomial terms in $\Psi(\xi_1,\xi_2,\xi_3)$ are ignored. The PDFs of the natural frequencies and modeshapes from PC and MC simulation are presented in Figs. 5 and 6, respectively.



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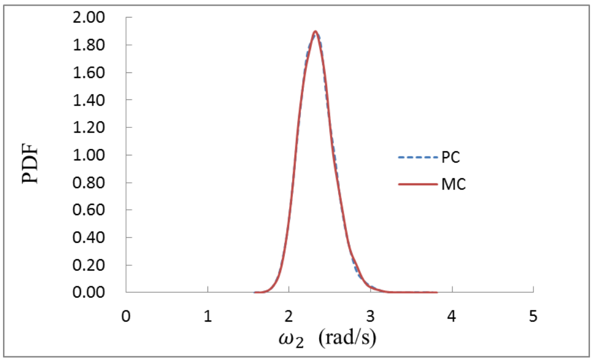

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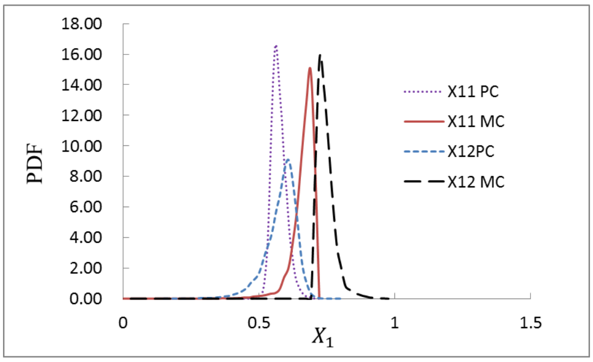
Figure 5. Probability density function of the natural frequencies ω_1, ω_2 of the mass-spring chain with uncertain stiffness using different log-normal distributions



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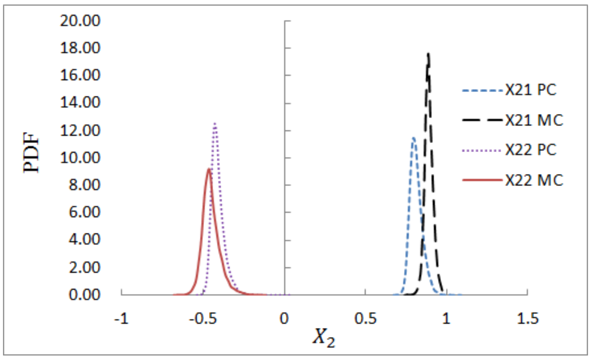

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Figure 6. Probability density function of the modeshapes X_1, X_2 of the mass-spring chain with uncertain stiffness using different log-normal distributions

The PDFs of both natural frequencies are well predicted by the third-order PC expansion with different base random variables. In the modeshapes analysis, the PC expansion results are not accurate enough. Greater accuracy could be achieved by including the higher order PC expansion and the interaction polynomial terms, which would result in a significantly higher computational cost.

TWO DEGREE-OF-FREEDOM MASS-SPRING CHAIN WITH UNCERTAIN MASSES

The uncertain masses in the mass-spring chain are assumed to follow log-normal and uniform distributions. The PDFs of the natural frequencies and modeshapes predicted using the PC theory are compared with MC simulations.

Uncertain masses using uniform and log-normal distributions

In this case, the uncertain mass m_1 is assumed to follow a uniform distribution $U(0.5,1.5)$ with a mean value of 1 kg. The uncertain mass m_2 is assumed to follow a log-normal distribution with mean and variance values listed in Tab. 4.

Table 4. Parameters of the uncertain mass

Parameter	Mean	Variance
m_2	2 kg	0.25 kg^2

The uniform distribution is represented by the base random variable ξ_1.The log-normal distribution is represented by the base random variable ξ_2. Thus the system base random vector becomes $\boldsymbol{\xi} = \{\xi_1, \xi_2\}$. The masses can be expressed by the PC expansion as

$$m_1(\xi_1) = \sum_{p=0}^{1} m_{1_p} L_p(\xi_1) \tag{34}$$

$$m_2(\xi_2) = \sum_{p=0}^{3} m_{2_p} H_p(\xi_2) \tag{35}$$

The natural frequencies and modeshapes can be expressed by a third-order PC expansion as

$$\lambda_i(\xi_1, \xi_2) = \sum_{r=0}^{3} \lambda_{i,r} \Psi_r(\xi_1, \xi_2), i = 1,2 \tag{36}$$

$$\mathbf{X}_i(\xi_1, \xi_2) = \sum_{s=0}^{3} \mathbf{X}_{i,s} \Psi_s(\xi_1, \xi_2), \; i = 1,2 \tag{37}$$

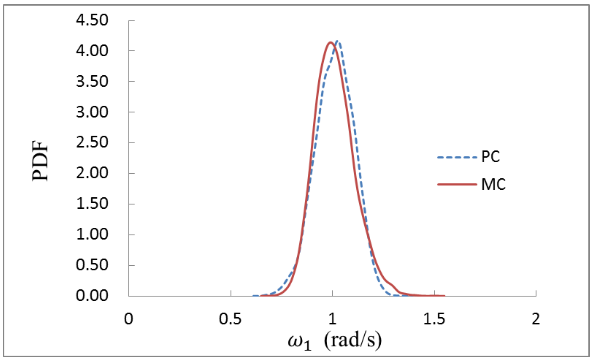
To simplify the analysis, in this case the interaction polynomial terms in $\Psi(\xi_1, \xi_2)$ are ignored. The PDFs of the natural frequencies from the PC and MC simulations are shown in Fig. 7. The PDF of the second natural frequency is not well predicted by the PC theory, which is attributed to ignoring the interaction polynomial terms.

Uncertain masses using different log-normal distributions

In this case, the uncertain masses m_1, m_2 are assumed to follow log-normal distributions with different mean and variance values, which are listed in Tab. 5.

Table 5. Parameters of the uncertain masses

Parameter	Mean	Variance
m_1	1 kg	0.1 kg^2
m_2	2 kg	0.25 kg^2



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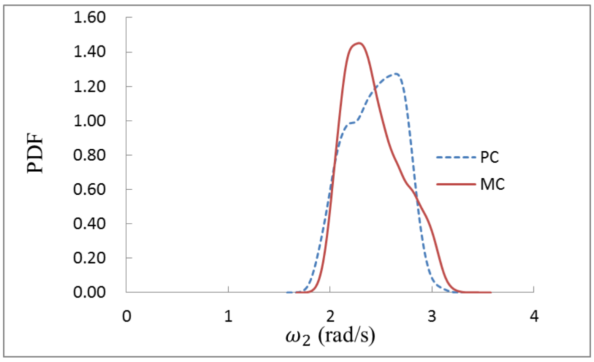

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Figure 7. Probability density function of the natural frequencies ω_1, ω_2 of the mass-spring chain with uncertain masses using uniform and log-normal distributions

The two log-normal distribution are represented by two different base random variables ξ_1, ξ_2. Thus the system base random vector is $\boldsymbol{\xi} = \{\xi_1, \xi_2\}$. The masses can be expressed by the Hermite PC expansion as

$$m_1(\xi_1) = \sum_{p=0}^{3} m_{1_p} H_p(\xi_1) \tag{38}$$

$$m_2(\xi_2) = \sum_{p=0}^{3} m_{2_p} H_p(\xi_2) \tag{39}$$

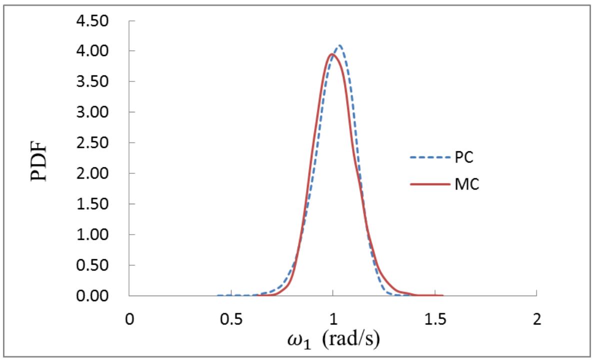
The natural frequencies and modeshapes can be expressed by a third-order PC expansion as

$$\lambda_i(\xi_1, \xi_2) = \sum_{r=0}^{3} \lambda_{i,r} \Psi_r(\xi_1, \xi_2),\ i = 1,2 \tag{40}$$

$$\mathbf{X}_i(\xi_1, \xi_2) = \sum_{s=0}^{3} \mathbf{X}_{i,s} \Psi_s(\xi_1, \xi_2),\ i = 1,2 \tag{41}$$

In this case the interaction polynomial terms in $\Psi(\xi_1, \xi_2, \xi_3)$ are ignored. The PDFs of the natural frequencies and modeshapes from the PC and MC simulations are shown in Figs. 8 and 9, respectively.

The PDFs of both natural frequencies are well predicted by the third-order PC expansion with different base random variables. However for the modeshapes results, the differences between PC and MC are obvious. To improve the accuracy of the PC expansion theory in the modeshape analysis, one extra interaction polynomial term $\xi_1\xi_2$ is taken into account. The PDFs of the modeshapes from the PC simulation with one interaction term and from the MC simulations are shown in Fig. 10.



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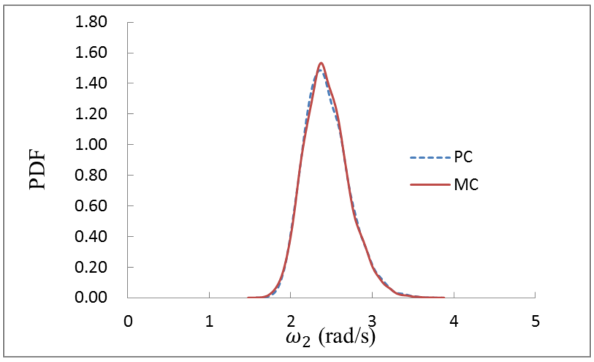

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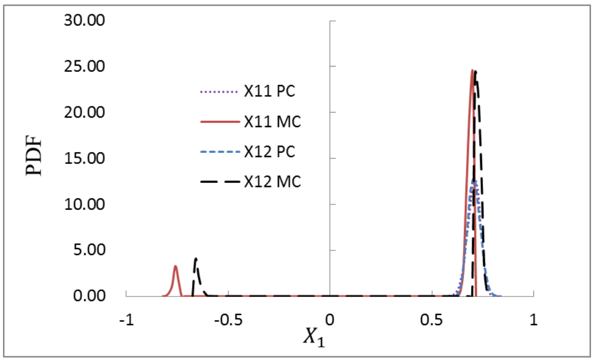
Figure 8. Probability density function of the natural frequencies ω_1, ω_2 of the mass-spring chain with uncertain masses using different log-normal distributions



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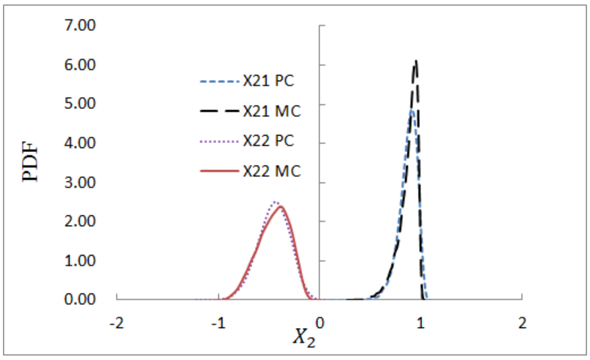

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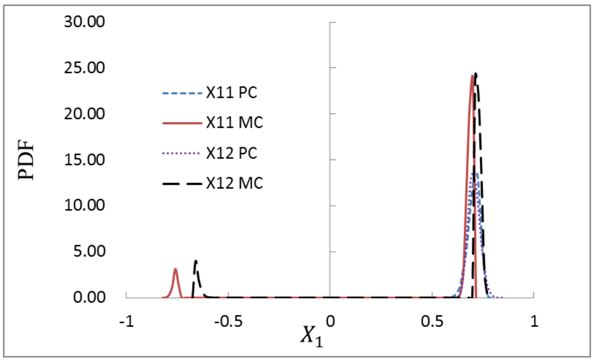
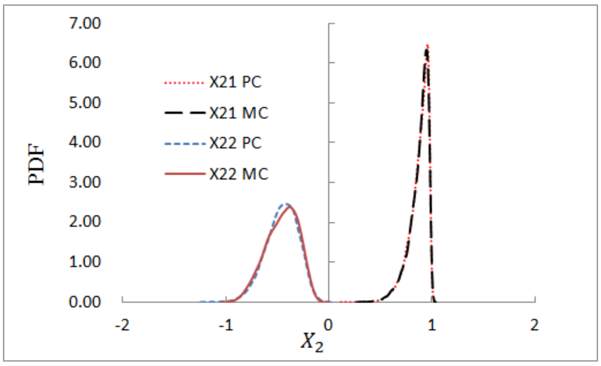
Figure 9. Probability density function of the modeshapes X_1, X_2 of the mass-spring chain with uncertain masses using different log-normal distributions



(a)



(b)

Figure 10. Probability density function of modeshapes X_1, X_2 with uncertain masses predicted using polynomial chaos theory with one interaction term and from Monte Carlo simulations

When one extra interaction term is involved, the PC expansion theory can approximate the PDF of the modeshapes more accurately, especially for the second modeshape as shown above. However, the computational cost for the PC simulations increases exponentially when the interaction term is included, and takes more time than the MC simulations.

CONCLUSIONS

This paper examines the modal statistics of a 2 degree-of-freedom mass-spring chain with several uncertain parameters using the polynomial chaos expansion method. The uncertain masses and stiffness are constructed using uniform and log-normal distributions and represented by different base random variables. As the number of base random variables increases, the natural frequency results can be well approximated by polynomial chaos expansion. To improve the modeshape analyses using polynomial chaos, the interaction polynomial terms should be taken into account. However, more base random variables and interation polynomial terms result in much higher computational cost, and becomes significantly slower compared with Monte Carlo simulations. Thus the polynomial chaos expansion method is not suitable for complex structures with many uncertain variables.

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