Analytical models of acoustic scattering by an infinite air-filled steel cylindrical shell in water under plane-wave excitation

Marshall V. Hall

9 Moya Crescent, Kingsgrove NSW, Australia

ABSTRACT

When a submerged fluid-filled cylindrical shell is ensonified, both structural vibrations (affected by the internal fluid) and standing waves in the internal fluid (caused by the shell) are excited and some of their eigenfrequencies may be comparable. Results obtained using three analytical models for different aspects of scattering by an infinite shell are presented. The first model produces a spectrum of scattering due to an incident orthogonal plane-wave and exhibits all resonances. The second model predicts approximate eigenfrequencies of the structural vibrations. The third model predicts approximate eigenfrequencies of the internal standing waves. For a steel shell submerged in water, changing the shell interior from vacuum to air creates fluid modes but causes negligible difference to the structural modes. For a shell of radius 3.25 m, the spectrum from 0 to 5000 Hz contains 18 structural resonances and approximately 3600 fluid resonances. The bandwidths of the structural resonances decrease from around 100 Hz to 3 Hz as frequency increases, while those of the fluid modes are around 1 cHz.

1 INTRODUCTION

Active control of a noise source may be achieved by surrounding it with an array of appropriate sound sources whose summed radiations will destructively interfere with the noise. For a given number of control sources (set by cost and available space), the problem is to determine their optimum positions and the amplitudes and phases of their frequency components. At present the main application of active noise control is reducing radiation from machinery. A potential future application could be to reduce the scattered radiation from a submerged target (mine or submarine) ensonified by an active sonar. In the theoretical scenario that this paper addresses, the target is an infinitely long cylindrical shell set into vibration by plane waves from a distant sonar. In order to define the signals required of the control sources, it is necessary to have a model of the radiated sound pressure as a function of (orthogonal) range from the shell, and also of azimuth around the shell axis. The purpose of this paper is to explain how this model is generated; it does not claim to present a solution to a realistic scenario.

In order to obtain a model of the radiated sound, equations of motion for the cylindrical shell that predict the amplitude of radial displacement for a given external stimulus are needed. The co-ordinate system selected is axial distance z, azimuth φ, and radial distance r. The shell radius (a) is defined as the average of the internal and external radii. In a general scenario, there are three equations, one for the acceleration of each of the three components of displacement: axial $u = \Delta z$, circumferential $v = a\,\Delta\varphi$, and radial $w = \Delta r$. The equation for the acceleration of any one component contains all three components of displacement and spatial derivatives of them. If the shell is submerged in and/or contains a fluid, then there will also be terms proportional to radial velocity ($\dot{w}$). In the scenario addressed in this paper there is no axial vibration and thus there are two equations, one each for accelerations in the radial and azimuthal directions ($\ddot{w}$ and $\ddot{v}$).

A catalogue of equations of motion for thin cylindrical shells (Leissa, 1993: 31-37) includes the simple Membrane equations of motion that neglect wall thickness, the “Donnell-Mushtari” equations of motion (henceforth “DM”) that add one extra term that depends on wall thickness (in the equation for radial acceleration), and the “Flugge” equations of motion that add seven further such terms spread over the three component equations (if there is no axial vibration then there is only one such term, and it also occurs in the equation for radial acceleration). This paper will employ both the DM and Flugge equations of motion and compare results therefrom. The development of the model through to predicting a scattering spectrum will follow Junger and Feit (1993) [henceforth “JF”]. Since they used DM only, their expressions have been modified where necessary to be compatible with Flugge. The Flugge equations of motion have been used in many papers on radiation from cylindrical shells, of which Eggler et al (2019) is a recent example.

All scattering spectra have been computed exactly, although subject to the accuracy of the assumed equations of motion. The scattered pressure at any frequency is obtained using JF's Eq. (11.61), in which the denominator is the sum of the modal structural impedance [Eq. (7.92)] and the specific acoustic impedance [Eq. (6.42b) with the axial wavenumber set to zero]. Since JF did not consider an internal fluid in their chapters on radiation and scattering, the simultaneous effects of vibrations in the external and internal media on the shell have been computed using the method described by de Jong (1994:17) and Hall (2015). This uses a "net" fluid loading (or specific acoustic impedance) given by the difference between the external and internal loadings (or impedances). Providing it is so amended, JF's Eq. (11.61) should yield all the resonances observable in a spectrum.

The only analytical expression JF provides for eigenfrequencies of a submerged (empty) shell is their Eq. (9.24). For an infinite shell this simplifies to a function of the mode number, the ratio of wall thickness (h) to shell radius, and the ratio of the external medium density (ρ_e) to the shell material density (ρ_s). This approximate expression is obtained by using the low-frequency (LF) assumption $k_e a \ll 1$, where $k_e = 2\pi f / c_e$ in which f is frequency, and c_e is sound-speed in the external medium. As will be seen for the scenarios to be examined, the significant resonances are at high frequencies where the opposite applies ($k_e a > 1$). Thus, the location of a resonance in a spectrum in conjunction with Eq. (9.24) may not identify its mode number (which determines its directivity) or whether its origin is a fluid or structural vibration. To do either, it is necessary to have a model of all eigenfrequencies, so that the mode number and origin of each observed resonance may be determined. JF has a chapter on high-frequency (HF) vibration, but that chapter does not contain an expression for eigenfrequency. The aim of this paper is to discuss new eigenfrequency models sufficiently accurate that there will be a one-to-one correspondence between observed resonances and the various structural and fluid eigenfrequencies in a given scenario.

2 SCENARIOS

The eight scenarios considered (to varying degrees) are listed in Table 1. For all scenarios, the external medium is water and the shell material is steel (both lossless and lossy). Both vacuum and air interiors are considered (odd and even scenario numbers). A plane wave is incident at an azimuth of 180° (to the left in a conventional elevation perspective). The total pressures are calculated at a distance of 6.5 m (two shell radii) from the shell axis, in the forward direction (azimuth 0°) away from the source (scenarios 1 - 4), and in the backward direction (azimuth 180°) toward the source (scenarios 5 – 8). This short distance is selected since a potential application of the analysis is to control scattered pressure by surrounding the cylinder by a number of nearby control sources.

Table 1: The eight scenarios examined

Scenario	Direction	Loss in steel	Interior	Scenario	Direction	Loss in steel	Interior
1	Forward	Lossless	Vacuum	5	Backward	Lossless	Vacuum
2	Forward	Lossless	Air	6	Backward	Lossless	Air
3	Forward	Lossy	Vacuum	7	Backward	Lossy	Vacuum
4	Forward	Lossy	Air	8	Backward	Lossy	Air

The dimensions and elastic properties of the shell, which are copied from a description of a virtual submersible (Zhang & Kessissoglou, 2012), are listed in Table 2. The wall thickness is 1.2% of the radius. The shell's length is infinite. The analysis will cover the frequency band from 1 mHz to 5000 Hz. The acoustic properties assumed for the interior and exterior media are listed in Table 3.

Table 2: Dimensions and elastic properties of the cylindrical steel shell

Parameter	Value
Radius, *a* (m)	3.25
Wall thickness, *h* (m)	0.04
Young modulus, Y (Pa)	210 e9
Loss factor, *1/Q*	0.00 and 0.02
Density, ρ_s (kg/m^3)	7800
Poisson ratio, ν	0.3

Table 3: Acoustic properties of the interior (air) and exterior (seawater)

Parameter	Value
Interior sound speed, c_i (m/s)	340
Interior density, ρ_i (kg/m³)	1.225
Exterior sound speed, c_e (m/s)	1520
Exterior density, ρ_e (kg/m³)	1025

3 ANALYTICAL MODELS OF STRUCTURAL VIBRATION MODES

An important feature of the acoustic properties of a cylindrical shell is its "ring frequency", given by

$$f_r = c_p/2\pi a \tag{1}$$

where $c_p = \sqrt{Y/\rho_s\,(1-\nu^2)}$ is the shell material's plate velocity. For the properties given in Table 2, the plate velocity is 5439 m/s and the shell's ring frequency is 266 Hz. In obtaining eigenfrequencies for all cases, the derivation will use the nondimensional frequency Ω, defined by

$$\Omega = 2\pi f\; a/c_p \tag{2}$$

At the ring frequency, $\Omega = 1$.

3.1 Empty shell in a vacuum

In this case, the eigenvalue equation is a quadratic equation in Ω^2 (JF:219):

$$A\,\Omega^4 + C_n\,\Omega^2 + E_n = 0. \tag{3}$$

For the Donnell-Mushtari equations of shell motion, the coefficients are given by (JF:219):

$$A = 1,\;\; C_n = -(1 + n^2 + \beta^2 n^4),\;\; E_n = \beta^2 n^6 \quad \text{(DM)} \tag{4}$$

where $\beta^2 = h^2/12a^2$ is the "wall thickness parameter" (JF:217). For the radius and thickness shown in Table 2, $\beta^2 = 1.26\; \times 10^{-5}$.

The corresponding Flugge quadratic equation can be derived by inspecting Leissa (1993:38-39). According to Leissa, Biezeno and Grammel (1962) began with the Flugge equations of motion but ended with a different quadratic equation, due to retaining more terms in β^2. This equation will be referred to as "Flugge-Biezeno-Grammel" (henceforth FBG). As may be reverse-engineered from Leissa (1993:39), the coefficients for Eq. (3) are:

$$A = 1,\;\; C_n = -(1 + n^2 + \beta^2(n^2-1)^2),\;\; E_n = \beta^2 n^2 (n^2-1)^2 \quad \text{(FBG)} \tag{5}$$

Each of the two occurrences of n^4 in Eq. (4) is replaced by $(n^2-1)^2$ in Eq. (5). Equation (3) is easily solved:

$$\Omega_n^2 = \left(-C_n\; \pm \sqrt{C_n^2 - 4\,A\,E_n}\right)/2A \tag{6}$$

There are two families of modes, which are denoted here by [-] and [+], the signs before the surd in the solution of the quadratic equation. Mode 0 is a special case for both DM and FBG because the constant $E_0 = 0$ whence $\Omega_0\,[-] = 0$. For DM, $C_0 = -1$ whence $\Omega_0[+] = 1$. For FBG, $C_0 = -(1+\beta^2)$ whence $\Omega_0[+] = \sqrt{1+\beta^2}$. Mode 1 is a special case for FBG because $E_1 = 0$ whence $\Omega_1\,[-] = 0$. Since $C_1 = -2$, $\Omega_1\,[+] = \sqrt{2}$.

3.1.1 [+] family:

Although computed to greater precision, frequencies that exceed 1 Hz are reported here to the nearest integer, for clarity. For the steel shell in a vacuum, the eigenfrequencies of modes 0 to 18 increase from 266 Hz (the ring frequency) to 4802 Hz. Thus,19 modes are required to (nearly) reach the spectrum's maximum frequency of 5000 Hz. For either the DM or FBG equations of motion, the eigenfrequency of mode 0 is the ring frequency. Mode 0 is the axisymmetric (azimuth-independent) mode. To five significant figures (at least) the DM and FBG eigenfrequencies are the same.

3.1.2 [-] family:

The eigenfrequencies of modes 0 to 18 increase from 0.0 Hz to 306 Hz (DM) and 305 Hz (FBG). On comparing DM and FBG, the eigenfrequencies differ significantly for the low-order modes, but converge as the mode number increases. For both DM and FBG, the eigenfrequency of mode 0 [-] is 0.0 Hz. The eigenfrequency of mode 1 is 0.67 Hz for DM and 0.0 Hz for FBG. According to Leissa (1993:40): (i) Mode 1 corresponds to rigid body translation; the position of the shell axis oscillates but the instantaneous wall radial displacement relative to the moving axis is zero over all azimuths; (ii) the correct eigenfrequency for "the lowest radial-circumferential vibration mode in the case *n* = *1*" is zero. By "lowest" he presumably meant what is referred to here as family [-].

On comparing the [-] and [+] eigenfrequencies for a given mode, it can be seen that it would be inappropriate to call any [+] mode an overtone of its [-] counterpart. With this procedure of treating the two families separately, each mode has only one eigenfrequency and there are no overtones.

3.2 Submerged empty shell

When a shell is submerged, the equations of motion contain an extra term to account for radiation loading due to the impedance of the external medium. The term for each mode contains a ratio of a Hankel function to its derivative, and setting the resulting determinant to zero generally does not yield a tractable equation.

3.2.1 LF modes

The only expression for the eigenfrequency of a submerged shell in JF is their Eq. (9.24), which includes axial wavenumbers and is thus appropriate to a shell of finite length. If the axial wavenumbers are all set to zero then the expression will be applicable to an infinite shell (JF:370). When that is done, the expression's numerator is proportional to βn^2. For mode 0 the expression's numerator is zero and its denominator is infinite, whence $\Omega_0 = 0$. de Jong (1994:18) had used a LF assumption in a hybrid analytical-numerical model and noted that there is no solution for $n = 0$. The expression for the structural impedance of mode *n* (JF:222) contained the family [-] eigenfrequency for a shell in a vacuum, Ω_n [-]. Each such occurrence has been replaced by the corresponding eigenfrequency yielded by the LF model.

3.2.2 HF modes

For HF modes, a semi-analytical model is obtained by using the assumption that $ka >> n$. This allows each Hankel function to be replaced by its principal asymptotic form (Abramowitz et al, 1972: 364); whereupon the ratio of a Hankel function to its derivative is constant to a good approximation. The eigenvalue equation becomes a quartic equation and will have four coefficients (two complex) that in general cannot be simplified. Mode 0 is a special case because two coefficients are zero and the number of non-zero eigenfrequencies reduces to two. For a thin steel shell in water the eigenfrequencies for $n = 0$ will be imaginary if the shell material is lossless, and have a small real part if it is lossy. The quartic equation has been solved with the standard numerical routine "Poly-zeros" (Bini, 1996). The eigenfrequencies obtained with both DM and FBG are the same to at least 5 significant figures. As expected, the real part of the eigenfrequency for Mode 0 is zero. A surprising result is that the ring frequency now seems to belong to Mode 1 rather than mode 0, although it has a higher value. The eigenfrequencies for a water exterior deceed those for a vacuum exterior, but they converge as mode number increases. From mode 1 to mode 18 for example, the differences between the eigenfrequencies decrease from 98 Hz to 0.2 Hz. The expression for the structural impedance of mode *n* (JF:222) contained the family [+] eigenfrequency for a shell in a vacuum, Ω_n [+]. Each such occurrence has been replaced by the corresponding eigenfrequency yielded by the HF model.

3.3 Submerged fluid-filled shell

3.3.1 JF Chapter 2

In a section on fluid-filled cylindrical waveguides, this chapter contains expressions for radial wavenumber eigenvalues. Unfortunately, these expressions cannot contribute significantly to the present analysis since the derivation of those expressions assumed: (i) the exterior is a vacuum, (ii) $\beta^2 a^2 \ll 1$, where β denotes radial wavenumber (unrelated to the wall thickness parameter β^2 of Chapter 7), and (iii) the radial pressure gradient in the internal fluid is proportional to cylindrical range (r) from the shell axis over the interval $0 \le r \le a_i$, in which a_i denotes the internal radius ($a - h/2$). The pressure in a cylinder must be proportional to a Bessel function of this range, $J_n(kr)$, and the only $J_n'(kr)$ that is linear as $r \to 0$ is that for which $n = 0$ (the mode that is independent of azimuth). An ensonified cylindrical shell will however vibrate significantly in many modes. In addition, a realistic boundary condition for an air-filled steel shell is that radial velocity and hence radial pressure gradient be approximately zero at the inner wall. For every mode except $n = 1, J_n'(0) = 0$. It is not possible for the radial velocity to vary linearly with range and also to be zero at both the axis and the wall.

3.3.2 JF Chapter 9

Like all expressions in this chapter on Sound Radiation by Shells, Eq. (9.24) does not cater for an internal medium.

3.3.3 JF Chapter 11

This chapter contains a section on “Sound propagation in fluid-filled elastic waveguides” but does not present an expression for any eigenfrequency of a shell. It appears that an analytical model for eigenfrequencies that takes account of both internal and external media is unavailable. Since such a model would need to include ratios of Hankel to Bessel functions, obtaining an analytical expression may be achievable for $ka << 1$ (de Jong, 1994) but likely to be impossible otherwise.

4 ANALYTICAL MODEL OF INTERNAL ACOUSTIC MODES

In a fluid within a cylindrical shell the pressure at a single frequency may be expressed as an infinite sum of standing wave modes (JF:379). For an infinite shell, the expression simplifies to:

$$p(r,\phi,\omega) = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} P_{n,m}(\omega)\, J_n(\alpha_{nm}\, r/a_i)\ \cos n\varphi \tag{7}$$

where each α_{nm} is the *(m+1)*’th zero of the *(n+1)*’th root of a characteristic equation obtained by expressing the ratio of pressure to radial velocity ($p\ /\dot{w}$) in terms of properties of the fluid and shell. The wavenumber eigenvalue of each standing wave is $k_{n,m} = \alpha_{nm}\ /a_i$. The summations are over two indices because for each mode *n* there are many eigenfrequencies (or wavenumber eigenvalues) at which the boundary condition is satisfied. According to de Jong (1994:17) “The presence of the Bessel functions in the matrix causes the dispersion equation to have an infinite number of solutions …”. A useful way of describing vibration modes on a circle is to call *n* the number of nodal diameters and *m* the number of nodal circles; a nodal line being a line that does not move while the rest of the structure is vibrating (Russell, 2018). From a general expression for $P_{n,m}(\omega)$ (Atassi, 2014), it can be shown that each coefficient is proportional to the *n*’th component of the resultant pressure on the cylindrical surface, for which an expression is available (JF:323).

An approximate method is used here to produce an analytical expression for internal fluid eigenfrequencies. The shell responds to external excitation but is subdued (slightly) by loading from the internal fluid. It is assumed that the shell displacement is a negligible fraction of the shell radius. It is also assumed that the (radial) velocity of the shell is negligible. Thus, each α_{nm} is approximately the *(m+1)*’th zero of J_n'. Eigenfrequencies $f_{n,m}$ are related to wavenumber eigenvalues $k_{n,m}$ by:

$$f_{n,m} = c_i\, k_{n,m}/2\pi\ . \tag{8}$$

As is the case for structural modes, integer *n* defines the dependence of each mode on azimuth around the cylinder axis.

By convention, multiple eigenfrequencies for any mode are called the “fundamental” (identified with $m = 0$) and its “overtones” (identified with $m = 1,2,\ldots$). The fundamental will henceforth be referred to as “tone 0” and the overtones will be referred to as "tone 1", "tone 2", and so on. For a given *n*, the summand at each *m* in Eq. (7) is thus called tone *m* of mode *n*. An ordered pair of both indices "(*n*,*m*)" is used to identify a mode and tone.

Since the pressure and radial acceleration in a fluid are related by:

$$\partial p/\partial r \ = -\rho \ddot{w} \tag{9}$$

the radial particle velocity may be expressed as:

$$\dot{w}\ (r,\phi,\omega) = \ddot{w}/(-i\omega) = \partial p/\partial r\ \ /i\omega\rho = (1/i\omega\rho)\ \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} P_{n,m}(\omega)\ \ k_{n,m}\ \ J_n'\big(k_{n,m}\, r\big)\ \cos n\varphi \tag{10}$$

in which J_n' denotes differentiation with respect to the whole argument. From the boundary condition at the internal radius a_i, it follows that

$$k_{n,m} a_i = \ j'(n,m) \tag{11}$$

where $j'(n,m)$ is the (*m*+*1*)th zero of J_n' ($m = 0$ corresponds to the first zero). For $m = 0$, $j'(1,0) = 1.8412$; $j'(n,0) = 0$ if $n \neq 1$. The next five zeros ($m\ =\ 1$ to 5) of the derivatives J_0' to J_5' are listed by Wolfram Mathworld (2019), although for J_1' they overstate the value of *m* by 1. This came about because they ignored any zero *j'* that equals 0 but overlooked the fact that $j'(1,0)$ is not zero. For *n* > 5 and *m* > 5, "McMahon's expansion" for large zeros is used (Abramowitz et al, 1972: 371). As an example, the tone-0 eigenfrequency ($m = 0$) of the ($n = 1$) mode of the fluid in a rigid shell is the frequency at which

$$k_{1,0}\ a_i = \ j'(1,0) = \ 1.8412 \tag{12}$$

The tone-0 eigenfrequency of the ($n = 1$) fluid mode is therefore approximately:

$$f_{1,0} \approx 1.8412\ \ c_i/2\pi\, a_i \tag{13}$$

which yields a value of 31 Hz for the shell described in Table 2.

If the interior is a vacuum ($c_i\ =\ 0$) then none of these resonances will be present.

5 SPECTRA

The total (measurable) pressure is the coherent sum of the pressures of the incident plane wave and the scattered wave. Whether the scattered or the total spectra is being discussed will vary with the context.

5.1 Submerged empty shell

Spectra of total sound pressure forward and backward from the empty steel shell in water have been computed. For each azimuth, spectra have been computed over the band from 0 to 5000 Hz for both lossless and lossy steel, as defined in Table 2.

5.1.1 Forward scatter

The forward spectra (both scattered and total) exhibit resonances in good agreement with the structural eigenfrequences. For example, there is a resonance at 552 Hz, and this agrees more closely with the HF model's eigenfrequency of 554 Hz for mode 2 than it does with the in-vacuo eigenfrequency of 596 Hz. For the lossless steel the peaks remain sharp over the whole band, whereas for lossy steel the peaks become negligible as frequency increases past around 3000 Hz. This occurs because as frequency increases, diffraction becomes less important, the wave transmission takes on the property of rays, and the total pressure at a point within the acoustic shadow beyond a large obstacle will approach zero.

5.1.2 Backscatter

The backscatter spectra exhibit a strong interference pattern. There are 22 peaks in this interference pattern; of which the first occurs at 115 Hz and the final at 4950 Hz; the average spacing is thus 230 Hz. At a range of twice the radius the scattered pressure will be around 71% (3 dB down) of that at the shell surface. The total pressure should therefore oscillate between around 29% and 171% (the actual variation is between around 40% and 160%). These large peaks are not related to the shell eigenfrequencies. Small peaks superimposed on the large peaks are related to the eigenfrequencies. As expected for this direction, there is no indication of a HF acoustic shadow.

5.2 Submerged fluid-filled shell

A spectrum of scattered pressure forward of the air-filled steel shell has been computed using JF's Eq. (11.61), in which the internal specific acoustic impedance (not mentioned by JF) has been subtracted from the external specific acoustic impedance as discussed in the Introduction. The result for frequency varying from 1 mHz to 120 Hz in steps of 1 mHz is shown in Figure 1. The steel is lossy. The spectrum is observed to contain nine peaks with very narrow bandwidths ("spikes") superimposed on the structural background scattering. This band does not contain any [+] structural resonance since the in-vacuo ring frequency is 266 Hz. In principle it does contain [-] modes 0 – 13. For modes [-] 2, 3 and 4 the LF model based on the FBG equations yields eigenfrequencies from 1.1 Hz to 7.3 Hz, for which ka increases from 0 to 0.1. The LF assumption should be valid if $ka < 0.1$. However, no peak is noticeable at or near these frequencies.

Each of the spikes in Figure 1 has been assigned a mode number and tone number in the format "*(n,m)*" by computing eigenfrequencies in the same manner as Eq. (13). In theory, there should be a spike at 0 Hz, but its amplitude is zero (linear acoustic variables do not have a DC component). Except for mode 1, the eigenfrequencies of tone 0 are all zero. It is clear that the 31-Hz spike is the (1,0) vibration as demonstrated in Eq. (13). There is no *(n,m)* that yields an eigenfrequency such that $0.0 < f_{n,m} <$ 31 Hz. Near 89 Hz there are two spikes, a small one followed by a larger one; their eigenfrequencies $f_{4,1}$= 89.08 Hz and $f_{1,1}$= 89.32 Hz are separated by 0.24 Hz and the spikes are separated by 0.20 Hz. There are three further spikes at 107, 112 and 118 Hz.

In the band 0 - 120 Hz, the axisymmetric mode $(n = 0)$ has three tones $(m = 0, 1 \text{ and } 2)$: $f_{0,0}$= 0 Hz (zero amplitude), $f_{0,1}$= 64 Hz (very small amplitude), and $f_{0,2}$ = 118 Hz. The "rigid" mode $(n = 1)$ has two tones: $f_{1,0}$= 31 Hz and $f_{1,1}$= 89.3 Hz. Mode 2 has three tones: $f_{2,0}$ = 0 Hz, $f_{2,1}$ = 51 Hz, and $f_{2,2}$ = 112 Hz. The three noticeable spikes omitted so far are $f_{3,1}$ = 70 Hz, $f_{4,1}$ = 89.1 Hz, and $f_{5,1}$ = 108 Hz (very small). The positions of all nine spikes agree closely with the eigenfrequencies computed from the zeros of the Bessel function derivative, although it is possible that the spikes at 89 Hz have not been identified correctly. This analysis can be extended to any bandwidth. Once the mode number of any spike in a spectrum is known, the azimuthal variation of that peak is also known.

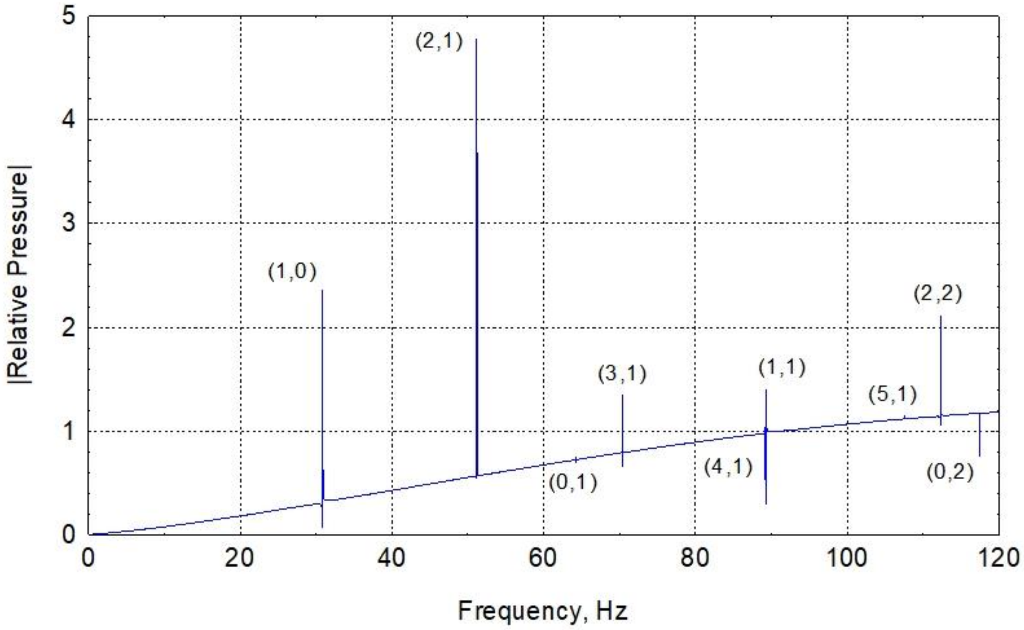

Figure 1. Spectrum of forward scattered pressure over the band 0 – 120 Hz. Nine spikes due to standing waves in the internal fluid are evident. Each spike has been assigned a mode number and tone number.

6 CONCLUSIONS

The spectrum model for scattering by a shell (which of course must have a fluid exterior) can cater for either a vacuum or fluid-filled interior. When applied to an empty steel shell it yields resonance peaks at approximate multiples of the shell ring frequency. When applied to an air-filled steel shell it yields those same resonance peaks (with additional spikes due to the internal fluid), indicating that changing the interior from vacuum to air causes negligible difference to the structural modes. If the steel is lossy however then the resonances become negligible as frequency exceeds around 3000 Hz.

Approximate LF and HF analytical models for shell eigenfrequency can cater for a fluid exterior but not for a fluid interior. When applied to scattering by a steel shell submerged in water, they are sufficiently accurate that the mode numbers of structural resonances in the spectra may be determined. Being submerged in water lowers structural eigenfrequencies, but the differences from the vacuum-exterior case approach zero as mode number increases. Taken together, the models for LF, and the new model for HF, structural eigenfrequencies provide a useful coverage for a submerged shell over a wide frequency band.

Although the analytical eigenfrequency models do not cater for a fluid-filled shell interior, excellent agreement between the spectral resonances (air interior) and the analytical structural eigenfrequencies (vacuum interior) reinforce the finding that changing the shell interior from vacuum to air causes negligible difference to the structural modes. Eigenfrequencies of internal fluid vibration are treated in a separate analysis that depends only on the shell's internal radius and the internal fluid sound-speed. Incident pressures are unlikely to be sufficiently high so as to significantly affect the radius of the shell. Spectrum resonances can generally be identified with eigenfrequencies of particular mode and tone numbers without ambiguity (although eigenfrequencies of different modes and tones are very close in some cases). Future work will focus on a model of spike amplitudes, so as to resolve such ambiguities when they arise.

ACKNOWLEDGMENT

The author would like to acknowledge Nicole Kessissoglou for her helpful discussions on cylindrical shells with application to noise control.

REFERENCES

Abramowitz, Milton, and Irene A. Stegun. 1972. *Handbook of Mathmatical Functions*. New York: Dover Publications.

Atassi, H. M. 2014. *Acoustic Waves in a Circular Duct*, https://www3.nd.edu/~atassi/Teaching/ame60612/Notes/Acoustic_circular_duct1.pdf

Biezino, C. B. and R. Grammell. 1962. *Engineering Dynamics, Vol. II*. London: Blackie and Sons Ltd.

Bini, Dario A. 1996. 'Numerical computation of polynomial zeros by means of Aberth's method'. Numerical Algorithms 13: 179-200.

de Jong, Christiaan A. F. 1994. *Analysis of pulsations and vibrations in fluid-filled pipe systems*. Thesis. Eindhoven, Netherlands: Eindhoven University of Technology.

Eggler, Daniel, Hyuck Chung, Fabien Montiel, Jie Pan and Nicole Kessissoglou. 2019. 'Active noise cloaking of 2D cylindrical shells'. *Wave Motion* 87: 106-122

Hall, Marshall V. 2015. 'An analytical model for the underwater sound pressure waveforms radiated when an offshore pile is driven'. *The Journal of the Acoustical Society of America* 138: 795 - 806.

Junger, Miguel C, and David Feit. 1993. *Sound, Structures and Their Interaction.* New York: Acoustical Society of America.

Leissa, Arthur. 1993. *Vibration of shells.* New York: Acoustical Society of America.

Russell, Daniel A. 2018. 'Vibrational Modes of a Circular Membrane'. https://www.acs.psu.edu/drussell/Demos/MembraneCircle/Circle.html

Wolfram Mathworld. 2019. 'Bessel Function Zeros'. http://mathworld.wolfram.com/BesselFunctionZeros.html.

Zhang, Cong, and Nicole Kessissoglou. 2012. 'Effect of excitation loads on the low frequency structural responses of a submerged hull'. In *Proceedings of Acoustics 2012 – Fremantle*, 163 – 167. Australian Acoustical Society.