Mid-Frequency Modelling of the Vibroacoustic Responses of Structures with Uncertainties

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ABSTRACT

Modelling the vibro-acoustic responses of vehicles in the mid to high frequency range is particularly difficult. This is due in part to the structural and acoustic responses of sections of a vehicle being highly sensitive to uncertainties arising from the assembly process and manufacturing tolerances. For example, the dynamic response of body panels can be greatly altered by small variations in spot welds. This leads to significant variation in interior noise levels of successive vehicles from a production line. This paper investigates the use of techniques for predicting the responses of structures with inherent uncertainties in the medium frequency range. The dynamic characteristics and responses of an ensemble of coupled plate structures are examined, where uncertainty is generated by adding small masses at random locations. A measure of the uncertainty is obtained by observing the variation in the natural frequencies of an ensemble member from their mean value across the ensemble. An ergodic hypothesis is used to compare the frequency averaged response of a single member in the ensemble with the ensemble averaged response. Finally, structure-borne sound pressure levels obtained experimentally are compared with results obtained from an SEA model.

INTRODUCTION

For a deterministic system, it is a relatively straightforward task to determine its dynamic response from a known excitation. However, for most practical engineering systems, there are degrees of uncertainty about the properties of the system. This uncertainty can cause a deterministic approximation of the response of the system to be unrepresentative of the actual case, as two samples of the same product may have vastly different responses. Kompella and Bernhard (1996) observed this by measuring the frequency response functions (FRFs) across 98 identical vehicles. They found that across each measurement set, the FRFs differed by up to 20 dB. The variation in this case is in part attributed to local material property changes due to spot welding, and variation in the manufacturing tolerances and assembly process.

Deterministic modelling methods such as finite element analysis (FEA) are also limited in the mid to high frequency range by the computational expense required to model a structure at these frequencies. For complex models, a very large number of degrees of freedom are required to accurately capture the short wavelength deformation at high frequencies. The computational time and power required to solve such a model is beyond the range of a current desktop computer. Deterministic methods are also unable to easily account for variation in structural parameters and material properties, which makes them impractical for modelling in the mid to high frequency range. The stochastic finite element method using Monte-Carlo simulations can account for structural uncertainty (Kleiber and Hein, 1992; Papadrakakis and Kotsopulos, 1999; Shorter *et al*., 2003). However, it is still restricted by the computational expense and the amount of information required to model joints between subsystems (Shorter *et al*., 2003). To reduce the computational effort, a number of methods have been recently proposed (Venkiteswaran and Junk, 2005; Helton et al., 2005). Perturbation methods based on the finite element method are also often used (Kaminski, 2000). Including the second and higher order terms in the perturbation analysis is often necessary however this increases the time required to obtain solution. An improved finite element method which only uses the mean, variance and covariance of the properties of the uncertainty has been developed to reduce the computation time and increase the accuracy of the results (Stefanou and Papadrakakis, 2004). Fuzzy structure theory has been proposed by Soize (1993, 2000) to predict the FRF and responses of complex structural acoustic systems in the medium frequency range. In this technique, a complex structure is modelled in terms of a master structure to which minor subsystems are attached. The master structure represents the component of the system with properties that are exactly known and the minor subsystems represent the uncertainty inherent in the structure (fuzzy substructures).

To overcome the computational expense and amount of detailed required, the dynamic responses of structures in the mid to high frequency ranges are generally modelled using energy methods such as Statistical Energy Analysis (SEA) (Guyader *et al*., 1982; Lyon and DeJong, 1995; Langley, 2000; Shorter *et al*., 2003). Energy flow methods relate the total energy in a subsystem to the input power of a subsystem by a matrix of energy influence coefficients (Shorter *et al*., 2003), where these influence coefficients can be calculated experimentally or computationally.

This paper investigates the vibro-acoustic responses of structures with uncertainty in the medium frequency range. The dynamic characteristics and responses of an ensemble of coupled plate structures is both computationally and experimentally examined, where uncertainty is generated by adding small masses at random locations. A measure of the uncertainty across the ensemble is obtained using a non-dimensional parameter called the statistical overlap factor (Manohar and Keane, 1991). Statistical overlap occurs when there is sufficient random variation in an individual natural frequency of a system from its mean value across the ensemble. Using an ergodic hypothesis, the frequency averaged response of a single system in the ensemble is compared with the response at a single frequency averaged across the ensemble. The mean energy levels are also compared with those obtained from a Statistical Energy Analysis model.

BACKGROUND METHODOLOGIES

Statistical Overlap Factor

There are many causes of uncertainty which will affect the vibro-acoustic responses of an ensemble of nominally identical vehicles. The dynamic response is sensitive to any uncertainty in the system parameters, which may be due to variation in material parameters, geometry, manufacturing tolerances, assembly process, as well as any variation in operating conditions (Langley, 2004). In addition, the vibro-acoustic responses become increasingly sensitive to any uncertainty in the system parameters as the frequency increases. Attempting to predict and then model all the various causes of uncertainty in a structure would be a very onerous if not impossible task. However if the uncertainty becomes large enough, then the response of the system under consideration becomes independent of the details of the uncertainty (Langley, 2004). This would allow the responses of an ensemble of nominally identical structures to be predicted from only the gross properties, which would be the same for each member of the ensemble.

A measure of a structure's uncertainty can be obtained using a parameter known as the statistical overlap factor and is given by (Manohar and Keane, 1991):

$$S = \frac{2\sigma}{\mu} \tag{1}$$

where σ is the standard deviation of a particular natural frequency of a structure from its mean value measured across the ensemble, and μ is the mean spacing between the natural frequencies of the structure. Figure 1 shows the variation of natural frequencies between nominally identical vehicles, where ω_n represents the n^{th} natural frequency and $\overline{\omega}_n$ is the mean n^{th} natural frequency measured across the ensemble.

It has been suggested that when S is greater than 1, there is sufficient mixing between the modes and veering of the modes for the structure to be considered 'uncertain enough' (Langley, 2004). Statistical overlap will usually increase with frequency because the sensitivity of a system to uncertainty increases with frequency. For small values of S (much lower than 1) the natural frequencies are in well defined positions and are mainly unaffected by structural uncertainty. However if S exceeds unity then the uncertainty will significantly shift these modes. When a structure has a sufficient statistical overlap, the probability density function of the spacings between successive resonant frequencies will follow a Rayleigh distribution, as shown in figure 2. A Rayleigh distribution is given by (Mehta, 1991):

$$p(s) = \frac{s}{c^2} e^{-s^2/2c^2} \tag{2}$$

where $p(s)$ is the probability density function and s is the spacing between successive natural frequencies. c is described in terms of the mean frequency spacing by $c = \mu\sqrt{2/\pi}$. The Rayleigh distribution shows that there is a low probability of small frequency spacings, which is due to the tendency of modes to repel and thereby veer from each other. A Rayleigh distribution of the modal spacings of a system indicates that there is sufficient uncertainty in that system such that the structural response is independent of the properties of its uncertainty.

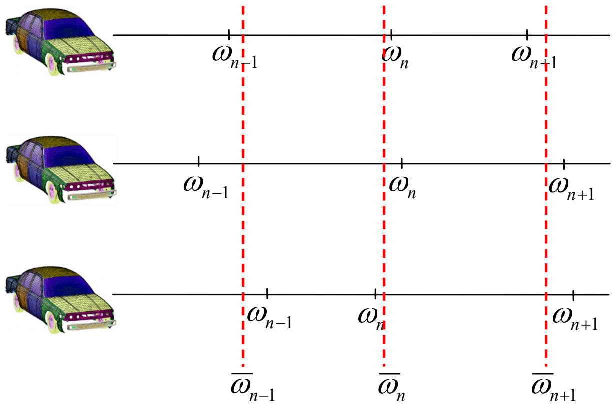

Figure 1. Variation of natural frequencies between nominally identical vehicles

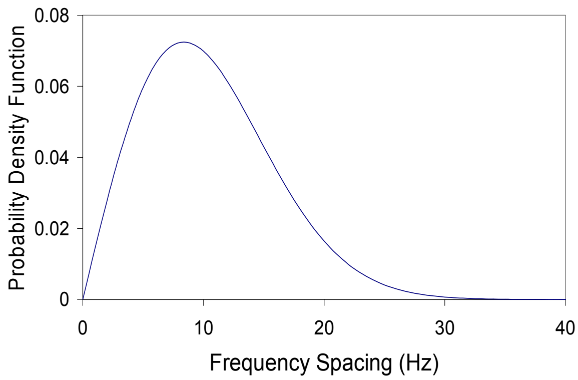

Figure 2. A Rayleigh distribution

Statistical Energy Analysis

Statistical Energy Analysis (SEA) is a high frequency modelling method which utilises the property that the response of a structure can be independent of the specific type of uncertainty inherent in it (Langley, 2004). SEA is a technique whereby a complete system is described in terms of energy flow between its various subsystems (Lyon and DeJong, 1995). Unlike FEA, a complex structure is modelled using only a small number of degrees of freedom and only the gross system properties are used. The main SEA equation is based on a power balance for a particular subsystem and is given by (Lyon and DeJong, 1995; Langley, 2000):

$$P_i = \omega\eta_i E_i + \omega \sum_{j\neq i} \eta_{ij} n_i \left(\frac{E_i}{n_i} - \frac{E_j}{n_j} \right) \tag{3}$$

where the terms P_i, E_i, η_i, η_{ij} and n_i respectively refer to the input power into subsystem i, energy in subsystem i, loss factor of subsystem i, coupling loss factor between subsystems i and j, and the modal density of subsystem i. The first term on the right hand side of equation (3) refers to the energy dissipated by subsystem i and the second term on the right hand side of the equation describes the energy flow between adjacent subsystems i and j.

For the SEA equation to be valid, the excitation of the structure needs to be high in frequency and random, to ensure equal partition of energy between modes. The coupling between subsystems must also be weak so that no global mode dominates, and the modal overlap must be high to allow for good mixing between modes (Langley, 2000). The use of SEA to model the mean energy levels of a system is considered reasonably accurate for statistical overlap factors of 1 or greater. The structural response determined using SEA is generally both frequency and ensemble averaged. Using an ergodic hypothesis, the frequency and spatially averaged mean energy levels of a structure obtained using SEA are compared with the energy levels averaged across an ensemble of nominally identical structures. This is described in more detail in the proceeding section.

Ergodic Hypothesis

The ergodic hypothesis states that if a structure has sufficient uncertainty, then the ensemble response of nominally identical structures is equal to the frequency averaged response of one member of this ensemble. This is mathematically represented by (Langley, 2000):

$$\left\langle e_i(x,\omega,p)\right\rangle = \left(\frac{1}{\Delta\omega}\right)\int_{\Delta\omega} e_i(x,\omega,p)\,d\omega \qquad (4)$$

where e_i is the kinetic energy density for subsystem i as a function of location x, frequency ω and any uncertainty across the ensemble p. The frequency band $\Delta\omega$ required for averaging should be sufficiently wide to encompass three or more modes (Langley, 2000).

The ergodic hypothesis is practically very important to the automotive industry as it could allow the mean response of an ensemble of similar vehicles at a single frequency to be calculated from the frequency averaged response of a single vehicle. It can also be used to compare with and verify the mean energy levels predicted using SEA.

COUPLED PLATE STRUCTURE WITH UNCERTAINTY

Computational Model

To investigate implementation of the various aforementioned methods to a structure with uncertainties, a coupled plate structure consisting of two plates connected at right angles in an L-shape with small added masses has been examined. An ensemble of L-shaped plates with uncertainty has been simulated by adding small masses at random locations on the plates. Due to the low damping properties of aluminium, a large number of modes can be obtained in the measured results.

The modes of an L-shaped plate were computationally determined by using a finite element model, as shown in figure 3. The two aluminium plates were modelled by quad 8, plate elements of 10mm in length. Simply supported boundary conditions were applied to the four long edges with the remaining two edges left free. Twenty 3 gram masses have been randomly located, 10 across each plate. 50 different configurations of these masses have been solved. The natural frequencies of the L-shaped plate were determined in the frequency range up to 4000 Hz, which includes 380 modes. The frequency spacings between each successive mode were then obtained. The probability density function of the spacings between successive natural frequencies has been compared with a Rayleigh distribution calculated using the mean frequency spacing of the L-shaped plate. It is worthwhile to note that the mean frequency spacing does not vary significantly between each ensemble member. The mean and standard deviation of the natural frequencies across the ensemble have also been determined, allowing the statistical overlap factor to be calculated.

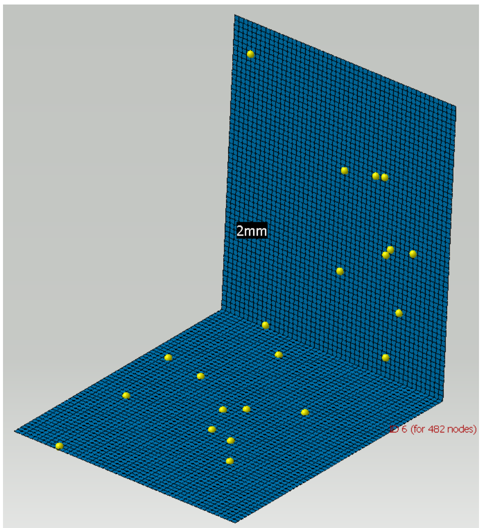

Figure 3. Finite element model of an L-shaped plate with masses attached at random locations

Experimental Set-up

Experiments were conducted on an L-shaped plate with small added masses to validate the various methodologies and computational results. The L-shaped plate was constructed from two 2mm thick aluminium plates welded together at right angles. The simply supported boundary conditions were constructed using 0.9mm thick aluminium Z-sections. The top flange of the Z-section was screwed at regular intervals along the four long edges of the L-plate, and the lower flange of the Z-sections was clamped between concrete blocks. These Z-sections have been shown to give a good approximation of simply supported boundary conditions as they are stiff for in plane vibration but allow rotation of the plate edges (Farag, 1979). The remaining two plate edges were left free, as shown in figure 4.

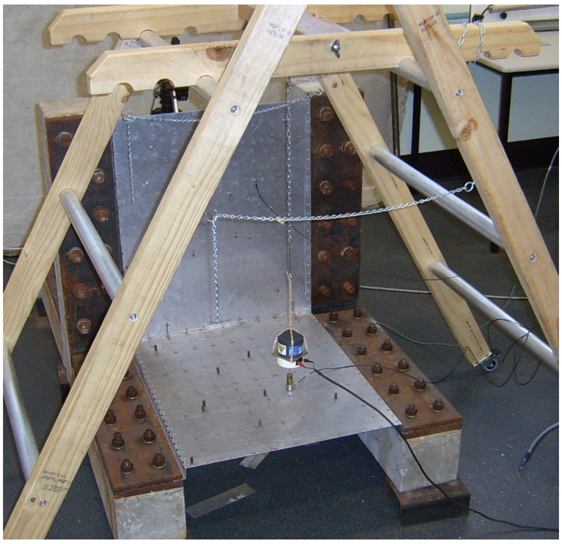

Figure 4. Photograph showing the experimental set-up of the L-plate with masses attached at random locations

Twenty 3 gram masses were attached to the two plates in random locations. A schematic diagram of the experimental set up is shown in figure 5. A shaker was used to excite the plate and was driven by random noise generated through a Pulse FFT analyser. The shaker was mounted vertically over the horizontal plane to simulate a point force excitation. A microphone was mounted behind the vertical plate, as shown in figure 5. The structural and acoustic responses were simultaneously measured. Frequency response functions were obtained between the force transducer at the shaker input and an accelerometer located on plate 2. Vibro-acoustic transfer path functions have been determined between the force transducer and the microphone (*Transfer Path Analysis*, 1995). The signals from both the accelerometer and force transducer passed through a charge conditioning amplifier before being sampled by the FFT analyser. 50 different configurations have been measured by randomising the locations of the small masses.

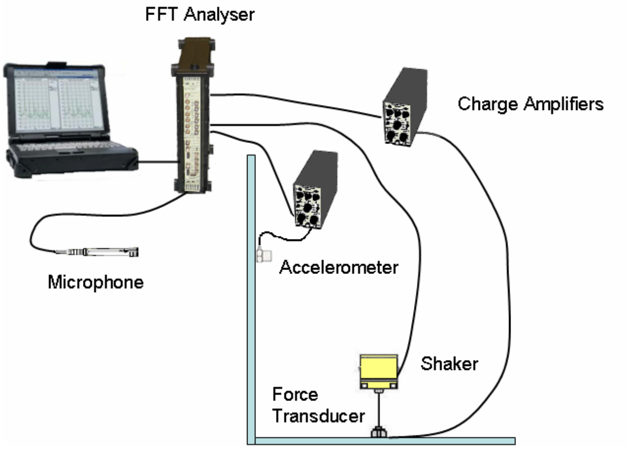

Figure 5. A schematic experimental set-up for the L-shaped plate measurement

RESULTS

Modal Spacing Distribution and Statistical Overlap Factor

In the computational results obtained from the FE model, it was found that for a modal range of 200 to 380 modes, the distribution of the spacings between successive natural frequencies followed a Rayleigh distribution, as shown in figure 6. The lower modes were discarded as they are in well defined positions and are therefore much less susceptible to the uncertainty in the plates. The results showing a Rayleigh distribution of modal spacings for a single plate loaded by masses has been previously reported (Langley and Brown, 2004).

Figure 7 presents the statistical overlap factor as a function of mode number, obtained computationally. A trend line approximated by a second order polynomial which approaches unity is also shown. The statistical overlap factor shows a steady increase with mode number. The levelling off of the statistical overlap factor is attributed to the inertia of the masses increasing as frequency increases, to the degree that each mass is acting as a clamped point such that no further increase in statistical overlap factor will occur with increasing frequency. It is interesting to note that the Rayleigh distribution in figure 6 is clearly observed for a modal range occurring before the statistical overlap factor approached unity, implying that very little uncertainty in a system is required to result in a Rayleigh distribution of successive modal spacings.

Figure 8 presents the probability density function of the spacings between successive natural frequencies obtained experimentally, for modes 40 to 155. A Rayleigh distribution of the mean frequency spacing for this modal range is also given. The statistical overlap factor for the mass loaded L-shaped plate ensemble is given in figure 9. A linear trend line was shown to better approximate the statistical overlap factor.

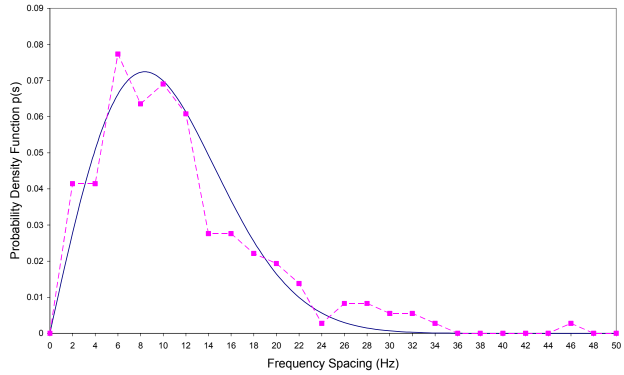

Figure 6. Probability density function of successive frequency spacings for modes 200 to 380 (dashed line) and a Rayleigh distribution of the mean frequency spacing (solid line) – computational results

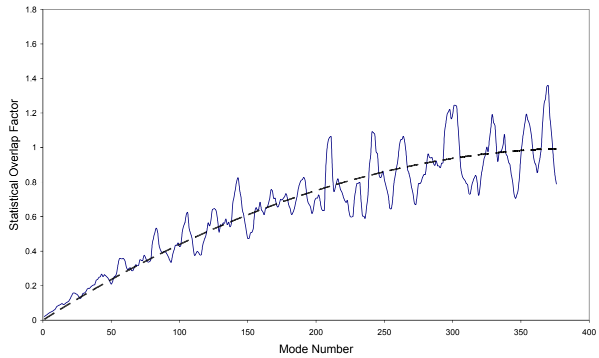

Figure 7. Statistical overlap factor (solid line), trend line (dashed line) – computational results

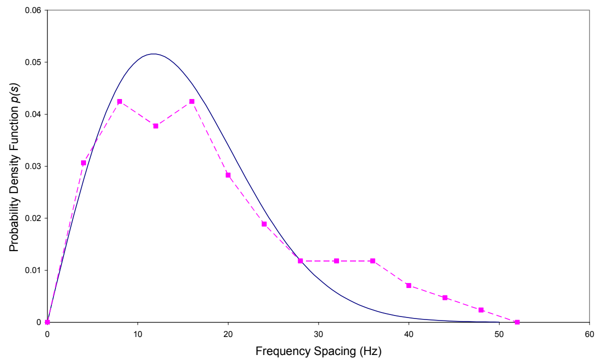

Figure 8. Probability density function of successive frequency spacings for modes 40-155 (dashed line) and a Rayleigh distribution of the mean frequency spacing (solid line) – experimental results

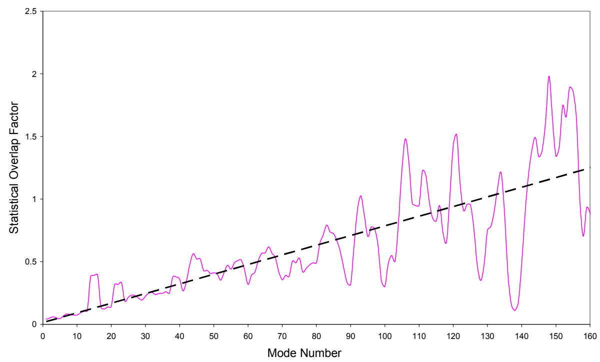

Figure 9. Statistical overlap factor (solid line), trend line (dashed line) – experimental results

The results obtained through experimental measurement have restrictions which are described in what follows. Firstly, as the frequency range increases, the damping in the plates, though very low, does cause each resonant peak to spread into its neighbouring one. In this case it was found that above 2500 Hz, it was impossible to distinguish the individual resonances as the modal density of the plates was so high that many resonances were merging together. It was however possible to determine the statistical overlap factor up to this frequency and obtain a meaningful spread of information. Another restriction to the experimental measurements is that not each of the resonant modes was observed for each randomised mass configuration. This has reduced the population size used for determining both the frequency spacing distribution and statistical overlap factor, increasing the error in both cases.

Despite the two limitations mentioned above, the experimental results match well with the simulated results. The frequency spacing distribution shows a clear Rayleigh distribution below the expected statistical overlap factor of unity. Results for the statistical overlap factor suggest that the level of uncertainty in the practical situation increases more quickly than the simulated case, attributed to uncertainty inherent in the measurements for each ensemble member.

Comparison of Frequency Averaging and Ensemble Averaging

In an attempt to validate the ergodic hypothesis, response measurements taken from the L-shaped plate are used to compare the mean levels found by frequency averaging (Plunt, 1999) and ensemble averaging. Both structural and acoustic measurements were simultaneously taken. The frequency response function (FRF) was measured between two points on the structure. The excitation was random and the frequency range was measured up to 3.2 kHz. The frequency response function has been converted into a velocity by integrating with respect to time and assuming a unit force input. The second measurement was of the vibro-acoustic transfer path between the excitation point and a microphone located behind the plate, as shown in figure 5. The transfer path measurement has been converted into an energy density response. A unit input force has been assumed. The frequency averaged response of a single ensemble member was obtained by averaging the response using a proportional frequency bandwidth of 4% of the frequency range. The ensemble averaging has been achieved by averaging the response measurement for each ensemble member at each discrete frequency.

From the experimentally measured FRFs, it was found that the frequency averaged result of the single ensemble member gives a close match to the ensemble averaged result, as shown in figure 10. Ensemble averaged and frequency averaged results have also been calculated using FEA for the structural response. Very similar results were obtained computationally in comparison to those determined experimentally as shown in figure 11, again a 4% proportional averaging bandwidth was used.

Vibro-acoustic measurements obtained experimentally were also used to compare the ensemble averaged acoustic energy density at each frequency and the frequency averaged acoustic energy density (figure 12). In this case a wider averaging band of 10% bandwidth was used.

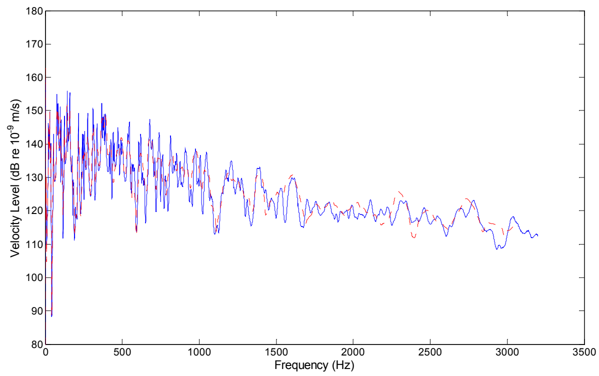

Figure 10: Ensemble averaged velocity at each frequency (solid line), frequency averaged velocity with a 4% proportional averaging bandwidth (dashed line) – experimental results

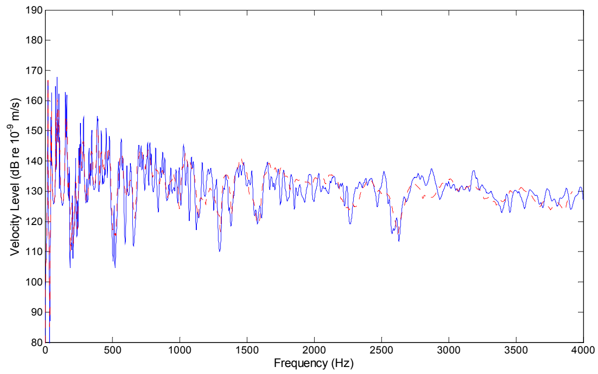

Figure 11: Ensemble averaged velocity at each frequency (solid line), frequency averaged velocity with a 4% proportional averaging bandwidth (dashed line) – computational results

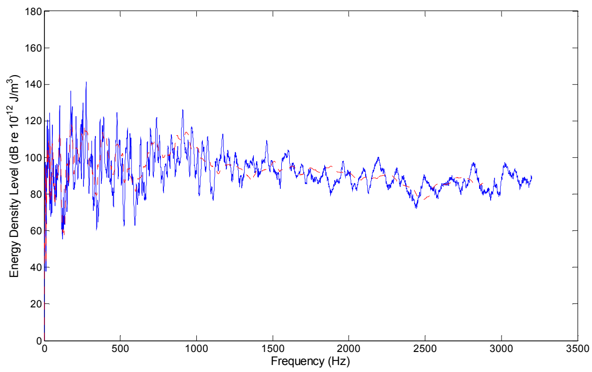

Figure 12: Ensemble averaged acoustic energy density at each frequency (solid line), frequency averaged acoustic energy density with a 10% proportional averaging bandwidth (dashed line) – experimental results

Frequency Averaged Experimental Results compared with SEA Results

The frequency averaged vibro-acoustic response obtained experimentally and shown in figure 12 is compared to the mean structurally radiated acoustic response of an L-shaped plate calculated using SEA. A unit input force was applied to the SEA model. The SEA response was calculated using the PAM-VA One software (Langley, 2004; Langley *et al*., 2005; Shorter *et al*., 2005), by means of the predefined properties of an aluminium plate. Figure 13 shows that the frequency averaged acoustic response converges towards the SEA prediction as frequency increases. SEA tends to slightly over predict the mean vibro-acoustic responses of the structure, al-though this is not the case in figure 13. This discrepancy is attributed to the fact that the experimental results were obtained out in a semi reverberant field whilst the SEA calculations assume an anechoic chamber.

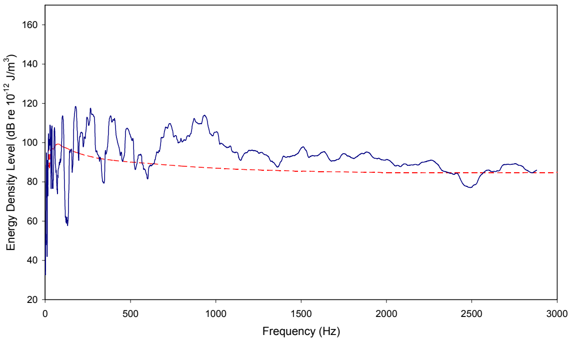

Figure 13. Frequency averaged sound energy level calculated experimentally (solid line), SEA prediction (dashed line)

CONCLUSIONS

Several statistical methodologies corresponding to the statistical overlap factor, probability density function of modal spacings and statistical energy analysis have been investigated both computationally and experimentally for an L-shaped plate. It was found that in the presence of structural uncertainty, the spacings between successive natural frequencies of the structure followed a Rayleigh distribution, indicating that the response of the structure is independent of the properties of the uncertainty. An ergodic hypothesis was employed which compared the mean response of an ensemble of structures at each frequency with the frequency averaged response of a single ensemble member. The frequency averaged sound energy levels obtained experimentally from measuring the vibro-acoustic transfer function of the structure were compared with the results obtained from a commercially available SEA software. This paper summarises its preliminary computational and experimental findings of an ongoing body of work into modelling structures with uncertainty in the mid frequency range.

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