The elusive cost function for tuning adaptive Helmholtz resonators

Sarabjeet Singh, Colin H. Hansen and Carl Q. Howard

School of Mechanical Engineering, The University of Adelaide, Adelaide, S.A. 5005, Australia

ABSTRACT

One of the problems associated with the use of Helmholtz resonators to control tonal noise propagation inside a pipe or duct is that any slight frequency changes in the tonal noise as a result of environmental changes or load changes on the device generating the noise will severely compromise the performance of the resonator. Thus, it is desirable to use an adaptive resonator whose volume or neck length can be adjusted to maintain optimal tuning as the excitation frequency or environmental conditions change. The ideal cost function would be a measure of the sound power propagating down the duct so the control system could minimise this quantity by driving motors that change the geometry of resonator. In practice, it is highly desirable to have available a self-contained adaptive resonator that does not need any external inputs or measures of quantities outside the resonator package. A cost function based on pressure measurements in the resonator, which corresponds to sound power in the duct has been found and verified experimentally and numerically. The effect of resonator damping on the cost function and a method to correct for the effect is also discussed.

INTRODUCTION

Background

The work presented in this paper is an extension to the work reported by the authors in a previous paper (Singh, Howard and Hansen 2006). The motivation for the ongoing investigations is the desire for the development of a self-contained adaptive Helmholtz resonator, which does not require any external sensors, to attenuate tonal noise propagation in ducts. The current paper is focused on the development of a cost function that uses only measurements from transducers located on or in the resonator to determine a quantity to be minimised.

Many authors have reported the successful implementation of adaptive Helmholtz resonators in small-scale laboratory setups. The optimal tuning of Helmholtz resonators has been achieved by using an electronic controller, which drives a motor in order to change the dimensions of either the cavity or neck of the resonator. The control algorithms reported in previous studies (Bedout *et al*. 1997) for changing the geometry of the resonators have used pressure measurements from microphones located in the duct downstream of the resonator. However, there are number of problems related to the in-duct mounting of the microphones and these have been discussed previously (Singh, Howard and Hansen 2006).

A cost function based on pressure measurements at the top of the closed end of the cavity and at the neck wall in close proximity to the neck-duct interface was presented previously (Singh, Howard and Hansen 2006). The cost function was empirically derived by relating the measured damping of the duct-HR system and the phase difference between the two pressure measuring locations in the resonator described above.

A new cost function is presented in this paper, which relates a different measure of the quality factor of the HR to the phase difference between pressure measurements at the top of the closed end of the cavity and at the neck wall in close proximity to the neck-duct interface. The quality factor, which is the reciprocal of twice the critical damping ratio, is typically estimated by using the bandwidth of frequencies bounding the resonance. Here the quality factor is estimated by using information at a single frequency only, at which the noise needs to be attenuated. This is a much more practical approach because in an actual system, the energy in the duct is dominated by the tonal noise to be controlled and it is often not convenient to introduce a loudspeaker to excite frequencies around the frequency to be controlled to enable the 3 dB bandwidth to be measured.

In the next section, the theoretical basis for estimating the resonance frequency and performance of a Helmholtz resonator is summarised. This is followed by a numerical analysis based on finite element analysis (FEA) using ANSYS software. Results obtained numerically are then compared with the theoretical results and the relationship between the pressures measured in the resonator and the in-duct sound power transmission is also discussed. Finally the cost function derived in our previous paper (Singh, Howard and Hansen 2006) is described and this is followed by the derivation and description of the new cost function.

THEORETICAL ANALYSIS

The following theoretical analysis is concerned with calculations of the resonance frequency of a HR as a stand-alone device and the acoustical performance of the HR mounted on a duct.

Helmholtz resonators as stand-alone devices

The resonance frequency, f_r, of a HR can be accurately calculated by using the well known classical formula given by:

$$f_r = \frac{c}{2\pi}\sqrt{\frac{\pi r^2}{l_{\text{eff}} V}} \tag{1}$$

where, c is the speed of sound, r is the radius of the neck, l_{eff} is the effective length of the neck, which includes two end-corrections, one at each end, and V is the volume of the cavity. However, Panton and Miller (Panton and Miller 1975) have shown that equation (1) is no longer valid when the dimensions of the HR exceed 1/16 of the wavelength of sound at the resonance frequency. Panton and Miller (Panton and Miller 1975) derived a new formula to calculate the resonance frequencies of cylindrical HRs, given by:

$$f_r = \frac{c}{2\pi}\sqrt{\frac{\pi r^2}{l_{\text{eff}}V + \frac{1}{3}L_c^2 A_n}} \qquad (2)$$

where, L_C is the length of the cavity, and A_n is the cross-sectional area of the neck. Unlike the assumptions for the classical formula, equation (2) is accurate for a length of the cavity comparable to or longer than a wavelength but the cavity diameter and neck dimensions must be kept smaller than a wavelength. Also, for the derivation of equation (2), it was considered that the length of the neck was very small compared to the length of the cavity.

As an extension to the work accomplished by Panton and Miller (Panton and Miller 1975), Li (Li 2003) proposed another model for calculating the resonance frequencies of cylindrical HRs. His derivation was also based on wave-tube theory but was more general than the model derived by Panton and Miller, and is given by:

$$f_r = \frac{c}{2\pi}\sqrt{-\frac{3L_{\text{eff}}A_c + L_c A_n}{2L_{\text{eff}}^3 A_c} + \sqrt{\left(\frac{3L_{\text{eff}}A_c + L_c A_n}{2L_{\text{eff}}^3 A_c}\right)^2 + \frac{3A_n}{L_{\text{eff}}^3 L_c A_c}}} \qquad (3)$$

where, A_c is the cross-sectional area of the cavity, and all the other variables have the same meaning as they do in equation (2).

As the dimensions of the HRs used in this study were all greater than 1/16 of the wavelength, the neck length was comparable or greater than the cavity length, and the other conditions related to the dimensions of the resonators conformed to those of Li's model, equation (3) was used to calculate the resonance frequencies.

Helmholtz resonator attached to a duct

When a Helmholtz resonator is mounted onto a duct, a coupled system is created whose resonance frequency is different to that of the stand-alone HR. HRs work by causing an impedance change in the acoustic system at its point of insertion. Thus, HRs act like passive bandstop filters barring the transmission of acoustic power past their location at frequencies in close proximity to their resonance frequencies.

The transfer matrix method, also referred to as transmission matrix or four-pole parameter representation (Munjal 1987), was used to calculate the net acoustic power transmission in the duct to which the HR was attached. The complete transfer matrix equation of the duct-HR system was built by discretising the duct-HR model into three elements: (1) section of the duct upstream of the HR, (2) section of the duct downstream of the HR, and (3) the HR. Figure 1 shows a schematic of the duct-HR system illustrating the three elements described above, along with the neck-cavity and neck-duct interfaces.

The key issue related to the theoretical analysis of the duct-HR system was the incorporation of the end-correction factors of the neck of the HR in addition to the actual dimensions of the elements. The end-correction factor at the neck-cavity interface, also referred to as interior end correction factor, δ_i, is well known and is given by (Ingard 1953):

$$\delta_i = \frac{8r}{3\pi}\left(1 - 1.25\frac{r}{R}\right) \qquad (4)$$

where, r is the radius of the neck, and R is the radius of the cavity. However, there exists some uncertainty concerning the estimation of the end-correction factor at the neck-duct interface.

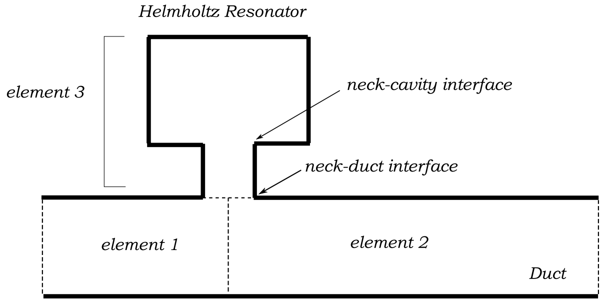

Figure 1. A schematic of the duct-HR system highlighting the neck-cavity and neck-duct interfaces.

Onorati (Onorati 1994) suggested that an end-correction factor of 0.3a, where a is the radius of the duct, should be added to the physical length of the neck in order to incorporate the mass loading of the fluid at the neck-duct interface. Ji (Ji 2005) presented an empirical value of the neck-duct interface end-correction factor that is related to the dimensions of the neck and the duct.

Figure 2 shows two plots of the in-duct net acoustic power transmission downstream of a HR calculated by using two measures of the neck-duct interface end-correction factor as per Onorati's (Onorati 1994) and Ji's (Ji 2005) model. The dimensions of the duct-HR system were: duct diameter = 0.1555 m, duct length = 3 m, cavity diameter = 0.131 m, cavity length = 0.070 m, neck diameter = 0.0525 m and physical neck length = 0.093 m.

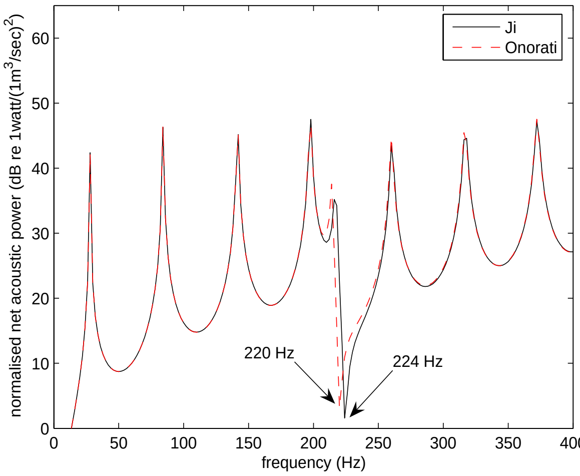

Figure 2. Theoretical results of the net acoustic power transmission in the duct downstream of the HR estimated by using estimates of the neck-duct interface end-correction factor as per Onorati's and Ji's models.

The frequency at which the maximum reduction of in-duct net acoustic power transmission was calculated as 220 Hz when Onorati's estimate of the neck-duct interface end-correction factor was used in the transfer matrix equation of the duct-HR system. This frequency changed to 224 Hz when Ji's model was used to estimate the neck-duct interface end-correction factor. It can be seen that using different values of the neck-duct interface end-correction factor results in different estimates of the in-duct net acoustic power transmission and in particular it leads to errors in the estimate of the frequency corresponding to the minimum sound power transmission. Therefore, the transfer matrix method was not considered a reliable option for analysing the duct-HR system.

To overcome the limitations of the transfer matrix method and the associated uncertainty in the neck-duct interface end-correction estimate, a numerical analysis of the duct-HR system was conducted and this is described in the next section.

NUMERICAL ANALYSIS

Numerical analysis of the duct-HR system was facilitated by using the ANSYS FEA software package. Unlike the theoretical analysis in which the end-correction factors have to be calculated, ANSYS automatically determines and incorporates the end-correction factors during its solution phase. It will be seen that the effective end-correction factor determined using ANSYS is different to both theoretical estimates described previously and used in the transfer matrix analysis.

One limitation with numerical analysis is associated with difficulty in obtaining an accurate estimate of system damping to include in the analysis. For the analysis undertaken here, the inclusion of damping is discussed following equation (5). The dimensions of the duct analysed were identical to those used for the transfer matrix analysis and are stated in the previous section.

In the ANSYS model, the source end of the duct was modelled as being driven by a piston by applying unit volume acceleration (denoted by the label FLOW in ANSYS). The other (right) end of the duct was modelled as open and radiating into free space. This was done by applying the frequency dependent complex radiation impedance boundary condition for the unflanged open end of a duct (Imaoka 2004). The theoretical expression used for the calculation of radiation impedance was that of the radiation impedance of an unflanged open duct with plane waves propagating inside it, and can be found in acoustic text books (Munjal 1987, Kinsler *et al*. 1982):

$$Z_r = \frac{\rho c}{S}\left[\frac{(ka)^2}{4} + j(0.6)ka\right] \tag{5}$$

Also, the frequency dependent viscous losses, which occur in the neck due to the oscillations of the fluid particles, were incorporated in the finite element model in order to model the damping in the HR. The estimate of the viscous losses was calculated by using the following expression for the resistance of the fluid in the neck (Bies and Hansen 2003):

$$R_A = \frac{\rho c}{A}\left[\frac{ktDw}{2A}\left(1+(\gamma-1)\sqrt{\frac{5}{3\gamma}}\right) + 0.288kt\log_{10}\left(\frac{4A}{\pi h^2}\right) + \varepsilon\frac{Ak^2}{2\pi} + M\right] \tag{6}$$

The physical meaning of each term and definitions of each variable can be found in Bies and Hansen (2003).

Figure 3 shows a schematic of a circular duct with an attached cylindrical HR, including the locations of the microphones 'A', 'B', '1' and '2' used for pressure estimates using ANSYS, which correspond to the locations used for the experimental work. The descriptions of the microphone locations are listed below.

- microphone 'A' - located at the top of the closed end of the cavity,
- microphone 'B' - located at the neck wall at a distance of 5 mm from the neck-duct interface,
- microphones '1' and '2' - flush mounted onto the duct wall downstream of the HR. Microphone '1' was located at a distance of 1.2 m from the mounting location of the HR and distance between microphones '1' and '2' was 0.3 m.

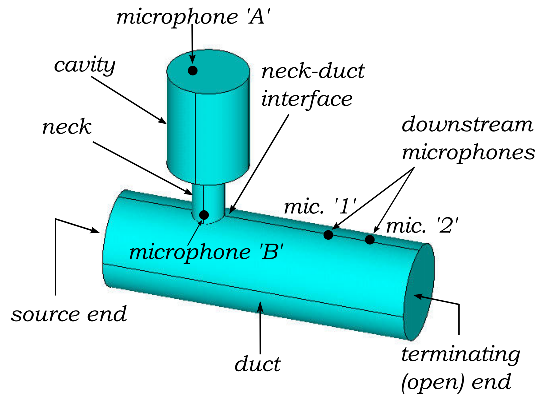

Figure 3. A schematic of the duct-HR system showing pressure measurement locations.

The sound pressures at these microphone locations were used to estimate power transmission in the duct (mic. '1' and mic. '2') and also the transfer function between locations 'A' and 'B' in the resonator, which is discussed in the next section. The duct and resonator dimensions used for the numerical and experimental analyses are identical to those used for the theoretical analysis in the previous section.

RESULTS

The experimental setup and procedure was described previously (Singh, Howard and Hansen 2006) and is not repeated here.

Figure 4 shows the experimental, numerical and theoretical results of the in-duct net acoustic power transmission downstream of the HR along with the experimental net acoustic power in the duct without the HR. The net acoustic power transmission in the duct was calculated using the in-duct modal decomposition of the sound field by measuring the acoustic pressure at the downstream microphones '1' and '2' (Chung and Blaser 1980, Åbom 1989).

The experimental results show a reduction of 18 dB at 226 Hz in the in-duct net acoustic power transmission as a result of mounting the HR onto the duct. A reduction of 22 dB at 226 Hz and 23.5 dB at 224 Hz was predicted by using ANSYS and the transfer matrix model, respectively.

The transfer matrix estimates of the in-duct net acoustic power shown in figure 4 were calculated by using the neck-duct interface end-correction factor as per Ji's model (Ji 2005). As stated earlier, because ANSYS automatically incorporates end-corrections based on first principles, the ANSYS results are considered more reliable than the transfer matrix results. However, the transfer matrix estimates can be made to exactly match the ANSYS results in the vicinity of the resonance frequency by adjusting the value of the neck-duct interface end-correction factor to 0.6*a*. It is likely that this relation may change for different duct and resonator dimensions.

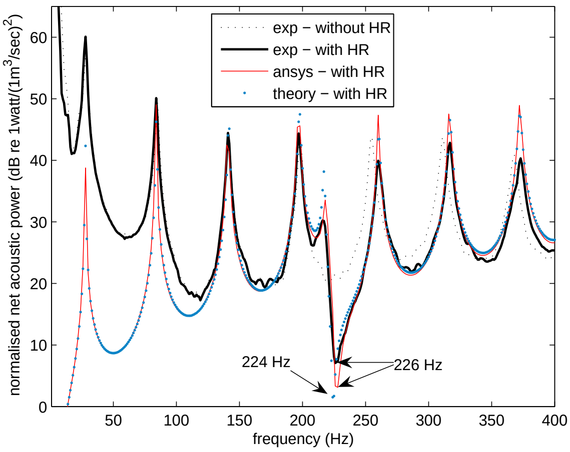

Figure 4. Experimental, numerical and theoretical results for the net acoustic power transmission in the duct with and without the HR, as a function of frequency.

The numerical and theoretical estimates of the net acoustic power transmission shown in figure 4 compare favourably with the experimental results except at the peaks, at their corresponding frequencies of minimum acoustic power (226 Hz and 224 Hz) and below 120 Hz. The reason for the differences of numerical and theoretical estimates from the experimental results at peaks is the inaccuracy in damping estimates for the theoretical and numerical models of the duct-HR system. At 226 Hz the difference is due to an inaccurate estimate of the neck-duct interface end correction used in the theoretical analysis. Below 120 Hz, the experimental results differ from the numerical and theoretical results due to the poor quality of the signal generated by the loudspeaker, leading to significant noise contamination of the signal and subsequent large errors in phase measurements between microphones '1' ans '2', which translates to large errors in the power transmission estimates.

For a stand-alone HR with the same dimensions as the one used in the duct-HR system, the resonance frequency measured in an anechoic chamber was found to be 222 Hz. This indicates that the presence of the duct has a significant effect on the HR resonance frequency, moving it from 222 Hz to 226 Hz.

Also for the same duct-HR system, the experimentally measured acoustic pressure at the top of the closed end of the cavity (microphone 'A') was a maximum at 218 Hz, and the amplitude of the pressure transfer function between microphone 'A' and microphone 'B' was a maximum at 246 Hz.

It was shown previously (Singh, Howard and Hansen 2006) that the dimensions of the HR (cavity length = 70 mm) which correspond to the maximum reduction of in-duct net acoustic power transmission differs from:

(1) the dimensions of the HR which correspond to the maximum acoustic pressure at the top of the closed end of the cavity (cavity length = 63 mm), and

(2) the dimensions of the HR which correspond to the maximum amplitude of the pressure transfer function between microphone 'A' and microphone 'B' (cavity length = 84 mm).

COST FUNCTION

Previous Cost Function

The cost function presented in our previous paper (Singh, Howard and Hansen 2006) was based on the measured damping of the duct-HR system and the measured value of the phase difference between microphone 'A' and microphone 'B' corresponding to the frequency at which the maximum reduction of the in-duct net acoustic power transmission was measured by microphones '1' and '2'. The damping of the duct-HR system was measured by using the half-power points method on the frequency response curve corresponding to the acoustic pressure at the top of the closed end of the cavity of the HR (microphone 'A').

Figure 5 shows a schematic of a typical frequency response curve in the vicinity of a system resonance (Tse, Morse and Hinkle 1978). The solid circular markers on either side of the resonance peak denote the frequencies that correspond to half-power points. These half-power points occur at $1/\sqrt{2}$ of the maximum response, $R_{\max}$.

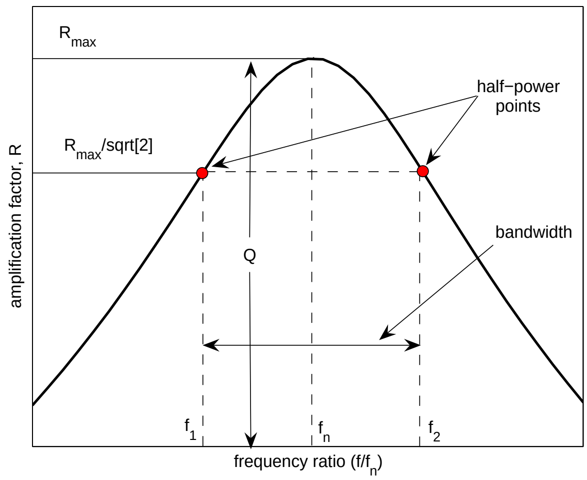

Figure 5. Frequency response of a system showing bandwidth and half-power points.

The damping ratio and the corresponding quality factor of a system are given by (Tse, Morse and Hinkle 1978):

$$Q = \frac{1}{2\zeta} = \frac{f_n}{f_2 - f_1} \tag{7}$$

where, Q is the quality factor of the system, ζ is the damping ratio of the system, f_n is the resonance frequency of the system corresponding to the maximum value in the frequency response, and f_2 and f_1 are the frequencies corresponding to the half-power points.

For the case of the duct-HR system, f_n is the frequency corresponding to the maximum acoustic pressure at the top of the closed end of the cavity (microphone 'A'). Frequencies f_2 and f_1 correspond to $1/\sqrt{2}$ of the maximum value of the acoustic pressure at microphone 'A'. Figure 6 shows a plot of the experimentally measured damping ratios of the duct-HR system as a function of the cavity length, using two different measurement methods. All the dimensions of the duct-HR system were identical to those mentioned earlier except the cavity length, which was varied from 60 mm to 170 mm in 5 mm increments.

For each cavity length, the corresponding resonance frequency and 3 dB down frequencies were determined. It is evident from figure 6, curve (a) that the measurement of the damping of the duct-HR system is strongly affected by the wave guide and the variation with frequency of the pressure at the entrance of the resonator neck. The maximum difference of 30% in the damping measures as the cavity length varies from 90 mm to 130 mm is an artefact of the measurement procedure rather than a real phenomenon. It is a result of the strong variation as a function of frequency of the acoustic pressure standing wave in the duct, resulting in a strong variation in the pressure at the entry to the resonator as a function of frequency. This results in a distortion of the frequency response curve. Thus, it is desirable to normalise the cavity pressure measurement with a measure of the acoustic pressure at the entrance to the resonator by taking a transfer function where the pressure at location A is divided by that measured at location B. This gives a much more consistent and accurate measure of the critical damping ratio as illustrated in Figure 6, curve (b).

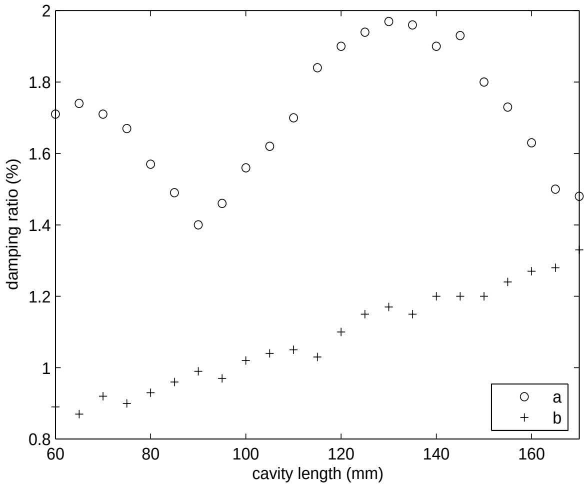

Figure 6. Experimental measures of the damping ratio of the duct-HR system estimated using the half-power point bandwidth method as a function of the cavity length, using:
(a) acoustic pressure measured at location A (Figure 3)
(b) pressure at location A divided by that at location B.

Measuring the damping (or the quality factor) of a system by using the half-power points method requires a system to be excited by a band limited noise signal so that information at the band of frequencies bounding the resonance or maximum response is available. Considering the practical case of a duct-HR system, where a centrifugal fan or a blower is installed at one end of the exhaust duct, noise is often generated at the fan blade passage frequency (BPF), and is tonal in nature. The signals at all the other frequencies can either be spurious or meaningless. Hence, estimating the damping (or quality factor) of such a system by using the half-power points method would involve some kind of provision to introduce a broadband signal in the duct, which seems unrealistic from the practical point of view. Thus, an alternative method for estimating the critical damping ratio or quality factor is needed

New Cost Function

Approximating Damping at a Single Frequency

Figure 5 shows that the height of the resonance peak is a function of damping in a system. This may be quantified by reference to a single degree-of-freedom (SDOF), spring-mass-damper, system. The equation which represents a SDOF system is given by:

$$M\frac{dx^2}{dt}+C\frac{dx}{dt}+Kx=F(t) \tag{8}$$

where *M, K* and *C* denote the mass, stiffness and damping of the system, *x* is the harmonic displacement of the mass resulting from the application of the forcing function, *F(t)*. For harmonic excitation of the system, the forcing function *F(t)* = *F* Sin *(ωt)*, and equation (8) becomes:

$$M\frac{dx^2}{dt}+C\frac{dx}{dt}+Kx=F\,\mathrm{Sin}\,(\omega t) \tag{9}$$

A solution to equation (9) is given by

$$x=X\,\mathrm{Sin}\,(\omega t-\phi) \tag{10}$$

where X and ϕ are the amplitude and phase of the frequency response, respectively, and ω is the driving (or excitation) frequency. The value of X can be obtained by substituting equation (10) in equation (9) and is given by (Tse, Morse and Hinkle 1978) as:

$$X=\frac{F}{\sqrt{(K-\omega^2M)^2+(\omega C)^2}} \tag{11}$$

Rewriting equation (11) with slight manipulation gives:

$$X=\frac{F/K}{\sqrt{\left(1-\frac{\omega^2M}{K}\right)^2+\left(\frac{\omega C}{K}\right)^2}} \tag{12}$$

Using the expressions $\frac{K}{M}=\omega_n^2$ and $\zeta=\frac{C}{2\sqrt{KM}}$, equation (12) can be arranged to give

$$\frac{X}{F/K}=\frac{1}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2+\left(2\zeta\frac{\omega}{\omega_n}\right)^2}} \tag{13}$$

where $\omega_n=2\pi f_n$ and f_n is the natural circular frequency of the system and ζ is called the damping ratio and $\frac{X}{F/K}$ is the amplification factor of the system, denoted by R in figure 5. The amplification factor is a maximum when the excitation frequency equals the natural frequency of the system so that:

$$\frac{X}{F/K}=R_{\max}=\frac{1}{2\zeta}=Q \tag{14}$$

If the pressure at the input to the resonator neck is considered to be the forcing function and the pressure in the cavity is the response, then the left side of Equation (14) may be represented as the ratio of the two or the transfer function. This demonstrates that the transfer function between the acoustic pressure in the resonator cavity and the pressure at the entrance to the neck can be used to estimate the quality factor of the resonator duct system. The next step is to find the maximum value of the frequency response function given that only a fixed excitation frequency is available. This is done by varying the cavity depth until the maximum transfer function value is obtained.

Figure 4 shows that for a particular geometry of the duct-HR system considered here, the maximum reduction of in-duct net acoustic power transmission downstream of the resonator occurred at 226 Hz. Considering tonal noise propagation at 226 Hz in a duct which needs to be controlled by using a self-contained HR, or by measuring the acoustic pressure at microphones 'A' and 'B', it would be convenient from the practical point of view to approximate the damping (or the quality factor) of such a duct-HR system at 226 Hz only.

Figure 7 shows experimental measurements of the amplitude of the pressure transfer function between microphone ‘A’ and microphone ‘B’ corresponding to 226 Hz as a function of the cavity length of the HR. The maximum amplitude of the pressure transfer function is 56 and it occurs at a cavity length of 84 mm.

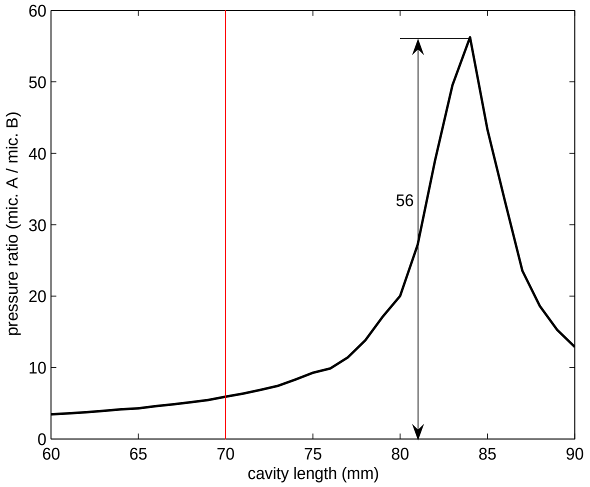

Figure 7. Experimental measurements of the ratio of pressures at microphone 'A' and microphone 'B' at a frequency of 226 Hz, as a function of the HR cavity length. The vertical line at 70 mm corresponds to the cavity length required for the maximum reduction of acoustic power transmission in the duct at 226 Hz.

The vertical line at a cavity length of 70 mm corresponds to the minimum acoustic power transmission in the duct and indicates that the maximum value of the transfer function is in itself not a suitable cost function for an active noise control system to minimise the transmitted in-duct acoustic power. As will be shown later, this conclusion applies whether or not there is damping in the system.

As a check on the validity of Equation (14) for estimating the quality factor, the experimental measure of the ratio of pressures at microphone ‘A’ and microphone ‘B’ corresponding to the cavity length of 84 mm is plotted as a function of frequency in figure 8. The maximum value of the pressure ratio occurring at 226 Hz equals 56 which approximately corresponds to the quality factor of the HR. For verification purposes, the quality factor for the same duct-HR system was also estimated by using the half-power point bandwidth scheme, and was found to be equal to 57.

For further verification, the quality factors of different HRs obtained by varying the neck diameter and cavity length were measured using the height of the frequency response and estimated by using the half-power point bandwidth method, with the results shown in figure 9. The excellent agreement indicates that estimating the quality factor of a HR by using the maximum value of pressure ratio (mic ‘A’/ mic ‘B’) for a fixed frequency as the cavity volume is varied is a valid method more convenient as compared to the half-power point bandwidth method.

Considering that pressure measurements at locations ‘A’ and ‘B’ are available at one frequency only at which the noise needs to be controlled, and there is provision to change the length of the cavity (necessary for the system to be adaptive), approximating the quality factor of the duct-HR system using the ratio of the two measured pressures as a function of cavity length proved to be a very practical approach.

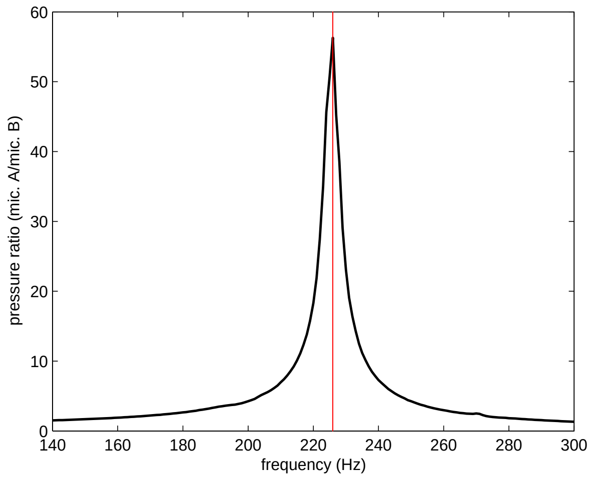

Figure 8. Experimental measure of the ratio of pressures at microphone 'A' and microphone 'B' corresponding to a cavity length of 84 mm, as a function of frequency. The vertical line at 226 Hz corresponds to the maximum pressure ratio.

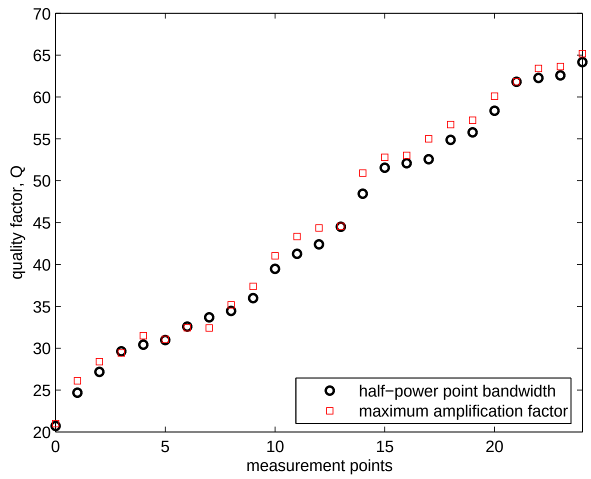

Figure 9. Quality factors of different HRs obtained by using 2 different methods: (1) by measuring the maximum pressure ratio of microphone ‘A’ to microphone ‘B’, and (2) by using the half-power point bandwidth method.

To develop a cost function for minimising tonal noise transmission in the duct by utilising the pressure measurements only in the resonator (microphone ‘A’ and ‘B’), experiments were conducted with varying dimensions of the HR. The cavity diameter and neck length throughout the experiments were kept constant and three different neck diameters (0.0405 m, 0.0525 m and 0.0675 m) were tested. For each different neck diameter, the cavity length was varied between 0.050 m and 0.170 m corresponding to a volume variation of $6.74\times10^{-4}\,m^3$ to $2.29\times10^{-3}\,m^3$.

Step-by-Step Method for Developing the Cost Function

The steps which were taken to develop the cost function are detailed below:

1. For each configuration of the HR, the in-duct net acoustic power transmission downstream of the resonator was measured by using microphones ‘1’ and ‘2’. Corresponding to each configuration of the HR, all the frequencies which achieved the maximum reduction of in-duct net acoustic power transmission were recorded. For the tested range of the dimensions of the HRs, these frequencies varied from 120 to 270 Hz.

2. For each configuration of the HR, the value of the phase difference between microphone ‘A’ and microphone ‘B’ corresponding to the frequency at which the maximum reduction of in-duct net acoustic power transmission, was noted.
3. For frequencies at approximately 5 Hz increments throughout the frequency range stated in step 1 (corresponding to the maximum in-duct net acoustic power reduction), the response curve of the ratio of the pressure measurements at microphone ‘A’ and microphone ‘B’ (microphone ‘A’/microphone ‘B’) was plotted as a function of the cavity length. The corresponding quality factor the HR was then approximated by using the height of the response curve, as shown in figure 7.
4. All the values of experimentally measured quality factors of the duct-HR system and the phase differences between microphone ‘A’ and microphone ‘B’ were plotted and shown in figure 10.

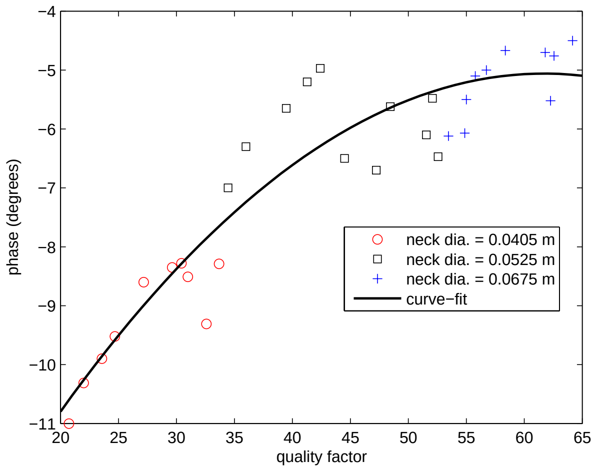

Figure 10. Quality factor vs phase difference corresponding to the frequency at which the maximum reduction of in-duct acoustic power transmission occurs, for three different HR neck sizes.

5. An empirical curve fitting relation between the phase difference and the quality factor was found, using a second order polynomial (with a correlation coefficient of 0.87), and is given by:

$$\text{phase difference} = -0.0033\,(\text{quality factor})^2 + 0.4072\,(\text{quality factor}) - 17.62 \qquad (15)$$

Equation (15) is a cost function that can be used by an electronic controller to optimally tune the HR for minimising the acoustic power transmission in the duct downstream of the resonator. The controller would adjust the resonator cavity length until the phase difference between signals from microphones ‘A’ and ‘B’ matched the phase difference in figure 10, which corresponds to the measured quality factor. As stated previously, the quality factor is determined by adjusting the cavity length until a peak in the A/B transfer function is obtained, and then it is equal to that peak value.

With a view to reduce the scatter in the data in figure 10, attempts were made to normalise the data by multiplying the quality factor by a combination of the duct-HR system parameters. The parameters included neck diameter, duct diameter, cavity length and the frequency of maximum reduction of the in-duct net acoustic power transmission. The parameters were arranged so as to yield a non-dimensional term on the abscissa. The values of the phase difference presented in figure 10 were plotted as a function of the modified quality factor, which was obtained by multiplying the actual quality factor by the non-dimensional term as described above. As the results did not show any appreciable reduction in scatter over the results presented in figure 10, they are not included here. It should be noted that the maximum scatter in figure 10 is about 1.3 degrees from the curve of best fit. This means that in many cases there will be a phase error associated with using the curve of best fit as the cost function. For a 1.3 degree error, the approximate compromise in the reduction in sound power transmission is 3 dB which is acceptable as the total reduction can range from 15 to 25 dB.

The cost function given by equation (15) is only valid for the dimensions of the duct-HRs that were covered in this study. Until now, all of the different HRs have been mounted on a single duct of diameter 0.1555 m. Although a finite element model of the duct-HR system was developed, obtaining a cost function for a wide range of dimensions of the duct-HR system was not possible. This is because accurate estimation of damping in the finite element model is a problem for systems for which no measurements are available.

Practical Implementation of the Cost Function

The steps involved in utilising the developed cost function to minimise the tonal in-duct net acoustic power transmission are detailed below:

1. The length of the cavity is varied until the ratio of pressures at microphone ‘A’ and microphone ‘B’ (microphone ‘A’/microphone ‘B’) reaches its maximum value at the frequency of tonal noise that is to be controlled.
2. The quality factor of the HR is then estimated by using the maximum value of the pressure ratio of microphone ‘A’ and microphone ‘B’.
3. The measured/estimated quality factor is substituted into equation (15) to calculate a value of the optimal phase difference between microphone ‘A’ and microphone ‘B’. This calculated phase difference will approximately correspond to the optimum length of the cavity required to minimise the in-duct net acoustic power transmission.
4. Finally, the length of the cavity is tuned until the calculated phase difference is reached.

Demonstration

Figure 11 shows the experimentally measured normalised net acoustic power transmission in the duct at 226 Hz as a function of cavity length, for several cost functions. The markers indicate the length of the cavity which corresponds to the:

* maximum value of the acoustic pressure at the top of the closed end of the cavity (microphone ‘A’) - (labelled 'maximum pressure in cavity'),

* maximum value of the amplitude of the pressure transfer function between microphone ‘A’ and microphone ‘B’ - (labelled 'maximum transfer function'),

* value of the phase difference, calculated by using equation (8), needed to be achieved by varying the cavity length for minimising the net acoustic power transmission in the duct downstream of the HR - (labelled 'new cost function'), and

* minimum acoustic power measured by mounting two microphones in the duct downstream of the HR - (labelled 'actual minimum power').

Figure 11 shows that in order to achieve the maximum reduction of the acoustic power transmission in the duct, the ‘new cost function’ gives the best possible cavity length when compared to the two alternative cost functions.

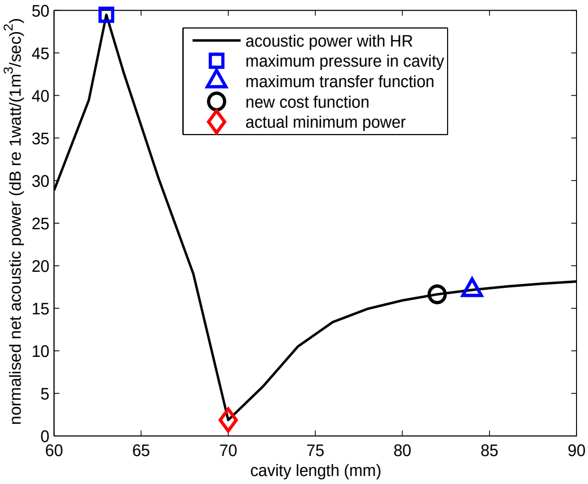

Figure 11. Net acoustic power transmission in the duct downstream of the HR at a frequency of 226 Hz, as a function of cavity length, with markers indicating the acoustic power level for various cost functions.

A similar figure to figure 11 was constructed using an ANSYS analysis for the same system with no damping, to demonstrate the effect of damping on the cost functions and the importance of estimating the damping or quality factor accurately.

The results are shown in figure 12 where it can be seen that both the new cost function and the maximum value of the pressure transfer function correspond to cavity lengths that are very much different to the one corresponding to the minimum in-duct acoustic power transmission.

CONCLUSIONS

A new cost function for minimising the in-duct net acoustic power transmission downstream of the HR has been developed. The new cost function relates the quality factor of the HR to the phase difference between the pressure measurements inside the resonator, which corresponds to the minimum in-duct sound power transmission. The duct-HR system quality factor was estimated using pressure measurements in the resonator at only a single frequency at which noise needs to be attenuated.

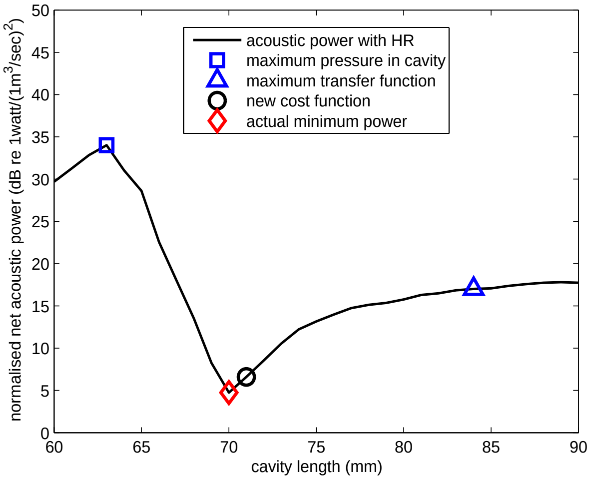

Figure 12. Net acoustic power in the duct downstream of the HR corresponding to 226 Hz estimated by using ANSYS modelling with no damping; markers indicating the acoustic power level for various cost functions.

REFERENCES

Åbom, M. 1989, Modal decomposition in ducts based on transfer function measurements between microphone pairs, Journal of Sound and Vibration, 135(1), 95-14

Bies, D.A. and Hansen, C.H. 2003, Engineering Noise Control, Spon Press

Chung, J.Y., and Blaser, D.A., 1980, *Transfer function method for measuring in-duct acoustic properties. I theory, II experiments*, Journal of the Acoustical Society of America, 68(3), 907-921

DeBedout, J.M., Franchek, M.A., Bernhard, R.J., Mongeau, L. 1997, *Adaptive-passive noise control with self-tuning Helmholtz resonators*, Journal of Sound and Vibration, 202(1), 109-123

Imaoka, S. 2004, Acoustic elements and boundary conditions

Ingard, U. 1953, *On the theory and design of acoustic resonators*, Journal of the Acoustical Society of America, 25(6), 1037-1061

Ji, Z.L. 2005, Acoustic length correction of a closed cylindrical side-branched tube, Journal of Sound and Vibration, 283, 1180-1186

Li, D. 2003, Vibroacoustic behaviour and noise control studies of advanced composite structures, PhD thesis, University of Pittsburgh

Munjal, M.L. 1987, Acoustics of Ducts and Mufflers, John Wiley and Sons

Onorati, A. 1994, Prediction of the acoustical performances of muffling pipe systems by method of characteristics, Journal of Sound and Vibration, 171(3), 369-395

Panton, R.L. and Miller, J.M. 1975, *Resonant frequencies of cylindrical Helmholtz resonators*, Journal of the Acoustical Society of America, 57(6), 1533-1535

Singh, S., Howard, C.Q. and Hansen, C.H. 2006, *Tuning a semi-active Helmholtz resonator*, Accepted for publication in Active 2006, The University of Adelaide, Adelaide, Australia

Tse, F.S., Morse, I.E. and Hinkle, R.T. 1978, Mechanical Vibrations, Allyn and Bacon, Inc.