Active random vibration control for stochastic piezoelectric truss structures

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ABSTRACT

This paper presents the optimization of the location and feedback gains of active bar in a closed loop control system for stochastic piezoelectric smart truss structures under stationary random excitation. The mathematical model with reliability constraints on the mean square value of the structural random dynamic displacement and stress response is developed based on maximization of dissipation energy due to the control action. The randomness of the structural physical materials and geometric dimensions are included in the analysis, and the applied forces are considered as stationary random excitation. The numerical characteristic of the stationary random responses of stochastic piezoelectric smart structures is developed by the random factor method. Numerical examples of piezoelectric truss structures are presented to demonstrate the rationality and validity of the active control model.

INTRODUCTION

The field of smart or intelligent structures has raised much interest over the past decade (Diwekar and Yedavalli 1996; Chopra 2000; Hurlebaus and Gaul 2006). Unlike the conventional engineering structures which are passive, smart structures has the ability to perform self-diagnosis and adapt to the environment change. A piezoelectric (PZT) smart truss structure is used in spacecraft deployable antenna, large antennas, and other important large-scale truss structures, in which the PZT active bar can be used as both an actuator for vibration excitation, and sensor for vibration measurement. Optimal placement of the PZT active bar is an important factor in the process of the structural design phase, and its shape and vibration control. The location of active bars in the smart truss structure directly affects the validity of active vibration control.

Recently, there has been much work published on the optimization of smart structures. Suk et al. (2001) introduced the Lyapunov control law for the slew maneuver of a flexible space structure by using a time-domain finite element analysis. To optimize the gain set of the control system, an energy-based performance index was adopted, and the gradients of the performance index with respect to each gain were derived. Quek et al. (2003) proposed a simple optimal placement strategy of piezoelectric sensor/actuator pairs for vibration control of laminated composite plate, where the active damping effect under a classical control framework is maximized using the finite element approach. Chen and Lin (2005) proposed a systematic method based on the impedance technique to determine the optimal locations and shapes of multiple induced strain actuators bonded on a host structure in respect of the smallest power consumption. Peng et al. (2005) proposed a performance criterion for the optimization of piezoelectric patch actuator locations on flexible plate structures based on maximizing the controllability grammian, and the Genetic Algorithm was used to implement the optimization. To date, the majority of modelling on optimization of active vibration control using piezoelectric smart structures has used deterministic models to model the dynamic response of smart structures, and optimal placement of the PZT actuators and sensors. In these cases, the structural parameters, applied loads and control forces are regarded as known parameters. However, deterministic models of the dynamic response associated with smart structures cannot reflect the influence of the randomness of the structural parameters. The dynamic response of an engineering structure can be sensitive to randomness in its parameters arising from variability in its geometric or material parameters, or randomness resulting from the assembly process and manufacturing tolerances. In addition, applied loads can be random process forces, such as wind, earthquakes and blast shock. The problem of stochastic smart structures subject to random applied excitation is of great significance in realistic engineering applications.

The dynamic response analysis of a closed loop control system for an intelligent structure is an important segment in the process of its design and vibration control. It is only in recent years that the dynamic response of stochastic structures under random excitation has received research attention. Zhao and Chen (2000) studied vibration of structures with random parameters to random excitation using Neumann stochastic finite element method (SFEM). Li and Liao (2002) investigated the use of the orthogonal expansion method with the pseudo excitation method for analysing the dynamic response of structures with uncertain parameters under external random excitation. Ma et al. (2004) solved the evolutionary earthquake response problem of an uncertain structure with bounded random parameters by a unified approach. Li and Chen (2005) proposed a new method based on probability density evolution method to construct the probability density evolution equation and obtain the numerical characteristics of random structural dynamic response.

In this paper, optimization of the location of the active bar and feedback gain in stochastic piezoelectric truss structures are investigated. The randomness of the structural materials and geometric dimensions are simultaneously considered. The applied force is taken as a stationary random excitation. Numerical expressions for the mean values and standard deviations of the natural frequencies and modeshapes, and displacement and stress responses of a piezoelectric truss structure are obtained. The performance function due to the control action is based on maximization of the dissipation energy. To formulate the optimal control problem, the algorithm for a linear quadratic regulator with output feedback has been employed in this paper. An optimal mathematical model with reliability constraints on the mean square value of structural dynamic displacement and stress response is developed. Numerical examples of stochastic piezoelectric truss structures are presented to demonstrate the rationality and validity of the active control model.

OPTIMAL MATHEMATICAL MODEL

Suppose that there are m elements and n degree of freedom in the piezoelectric smart structure under consideration, the equation of motion of the structure is given by

$$[M]\{\ddot{u}(t)\}+[C]\{\dot{u}(t)\}+[K]\{u(t)\}=\{F(t)\}+[B]\{F_C(t)\} \quad (1)$$

where $[M]$, $[C]$ and $[K]$ are the mass, damping and stiffness matrices respectively. $\{u(t)\}$, $\{\dot{u}(t)\}$ and $\{\ddot{u}(t)\}$ are displacement, velocity and acceleration vectors respectively. $\{F(t)\}$ is the load force vector generating the primary excitation. $\{F_C(t)\}$ is the control force vector. The matrix $[B]$ defines the location of the active bar on the smart structure under consideration. In the following analysis, the Wilson's damping hypothesis (Bathe 1995) is adopted. Using the modal expansion $\{u(t)\}=[\phi]\{z(t)\}$, the equation of motion takes the form

$$[I]\{\ddot{z}(t)\}+[D]\{\dot{z}(t)\}+[\Omega]\{z(t)\}=[\phi]^T\{F(t)\}+[\phi]^T[B]\{F_C(t)\} \quad (2)$$

where $[D]=diag\lfloor 2\zeta_j\omega_j\rfloor$, $[\Omega]=diag\lfloor\omega_j^2\rfloor$ for $j=1\ldots n$. $[\phi]=[\phi_1\cdots\phi_n]$ is the normal modal matrix of the structure, and ω_j, ζ_j are the j^{th} order natural frequency and damping ratio respectively.

For active control of the truss bar, a velocity feedback control law is considered. Since each active bar can be considered as a collocated actuator/sensor pair, the output matrix is the transpose of the input matrix. The output vector $Y(t)$ and control force vector $\{F_C(t)\}$ can be respectively expressed as

$$Y(t)=[B]^T[\phi]\{\dot{z}(t)\} \quad (3)$$

$$\{F_C(t)\}=-[G]Y(t)=-[G][B]^T[\phi]\{\dot{z}(t)\} \quad (4)$$

where $[G]=diag\{g_j\}$ is the gain matrix (Gao et al. 2003). Substituting equation (4) into equation (2) yields the equation of the closed-loop system

$$[I]\{\ddot{z}(t)\}+([D]+[\phi]^T[B][G][B]^T[\phi])\{\dot{z}(t)\}+[\Omega]\{z(t)\}=[\phi]^T\{F(t)\} \quad (5)$$

In the state-space representation, the equation of motion for the closed-loop system becomes

$$\{\dot{u}(t)\}=[A]\{u(t)\} \quad (6)$$

$$\{u(t)\}=\{z(t)\ \ \dot{z}(t)\}^T \quad (7)$$

$$[A]=\begin{bmatrix} 0 & [I] \\ -[\Omega] & -([D]+[\phi]^T[B][G][B]^T[\phi]) \end{bmatrix} \quad (8)$$

Both the optimal location of the active bar, and the optimal gain of the closed-loop control system are determined such that the total energy dissipated in the system is maximized. The total energy dissipated in the system is taken as the performance and it can be expressed as

$$J=\int_0^\infty\{\dot{z}(t)\}^T[\phi]^T([D]+[B][G][B]^T)[\phi]\{\dot{z}(t)\}dt \quad (9)$$

The equation (9) can also be expressed as (Abdullah 1998)

$$J=\{u(0)\}^T\int_0^\infty e^{[A]^T t}[Q]e^{[A]t}dt\{u(0)\} \quad (10)$$

where $[Q]=\begin{bmatrix}[\Omega] & 0\\ 0 & [I]\end{bmatrix}$. The performance function can be expressed as (Abdullah 1998)

$$J=tr[W] \quad (11)$$

where the matrix $[W]$ can be obtained by solving the Lyapunov equation

$$[A]^T[W]+[W][A]=[Q] \quad (12)$$

For the smart truss structure with random parameters, and where the load is a stationary random excitation, an optimization program is written with reliability constraints that implements the following steps. For a fixed gain ($g=g_j$), the optimal location of the active bar (that is, the optimal $[B]$ matrix) is obtained such that the total energy dissipated J is maximized. After the optimal placement of the active bar is determined, the feedback gain is then optimized. This is achieved by calculating the mean square displacement for each k^{th} degree of freedom and mean square dynamic stress for each e^{th} element. Reliability constraints are placed on the mean square displacement and stress respectively as follows

$$R^*_{\psi^2_{\sigma e}}-P_r\{\psi^{2\,*}_{\sigma e}-\psi^2_{\sigma e}\geq 0\}\leq 0\,,\quad e=1,2,...,m \quad (13)$$

$$R^*_{\psi^2_{uk}}-P_r\{\psi^{2\,*}_{uk}-\psi^2_{uk}\geq 0\}\leq 0\,,\quad k=1,2,...,n \quad (14)$$

$$[B]\subset[B^*],\quad [G]<[G^*] \quad (15)$$

$[B]$ and $[G]$ are the design variables. $R^*_{\psi^2_{\sigma e}}$ and $R^*_{\psi^2_{uk}}$ are given values of reliability of the mean square stress and displacement responses, respectively. $P_r\{\cdot\}$ is the reliability obtained from the actual calculation. $\psi^{2\,*}_{\sigma e}$ and $\psi^{2\,*}_{uk}$ are given limit values of the mean square stress and displacement responses, respectively. In this model, $[B]$, $[G]$, $R^*_{\psi^2_{\sigma e}}$, $R^*_{\psi^2_{uk}}$, $P_r\{\cdot\}$, $\psi^{2\,*}_{\sigma e}$ and $\psi^{2\,*}_{uk}$ can be random variables or deterministic values. $\psi^2_{\sigma e}$ and ψ^2_{uk} are the mean square dynamic stress of the e^{th} element, and displacement of the k^{th} degree of freedom, respectively. $[B^*]$ and $[G^*]$ are the upper bounds of $[B]$ and $[G]$ respectively.

In above model, the dynamic stress and response constraints are expressed by the probability form, which make the optimal problem difficult to solve. For this reason, the reliability constraints are transformed into normal constraints by means of the second order moment theory on the reliability (Chen and Duan 1994). Hence, the reliability constraints equation (13) and (14) can be respectively expressed as

$$\beta^*_{\psi^2_{\sigma e}}-\frac{\mu_{\psi^{2\,*}_{\sigma e}}-\mu_{\psi^2_{\sigma e}}}{(\sigma^2_{\psi^{2\,*}_{\sigma e}}+\sigma^2_{\psi^2_{\sigma e}})^{1/2}}\leq 0\,,\quad e=1,2,...,m \quad (16)$$

$$\beta^*_{\psi^2_{uk}} - \frac{\mu_{\psi^2_{uk^*}} - \mu_{\psi^2_{uk}}}{(\sigma^2_{\psi^2_{uk^*}} + \sigma^2_{\psi^2_{uk}})^{1/2}} \leq 0\,, \quad k = 1, 2, ..., n \tag{17}$$

where $\beta^*_{\psi^2_{\sigma e}} = \Phi^{-1}(R^*_{\psi^2_{\sigma e}})$ and $\beta^*_{\psi^2_{uk}} = \Phi^{-1}(R^*_{\psi^2_{uk}})$ are the given reliability of the mean square value of the stress and displacement of the k^{th} degree of freedom, respectively. $\Phi^{-1}(\cdot)$ denotes the inverse function of the normal distribution of random variables. $\mu_{\psi^{2*}_{\sigma e}}$ and $\sigma^2_{\psi^{2*}_{\sigma e}}$ are the limit values for the mean value and variance of the mean square stress of the e^{th} element $\psi^{2*}_{\sigma e}$, respectively. $\mu_{\psi^{2*}_{uk}}$ and $\sigma^2_{\psi^{2*}_{uk}}$ are the limit values for the mean value and variance of the mean square displacement of the k^{th} degree of freedom ψ^{2*}_{uk}, respectively. $\mu_{\psi^2_{\sigma e}}$ and $\sigma^2_{\psi^2_{\sigma e}}$ are the mean value and variance of the mean square dynamic stress of the e^{th} element, respectively. $\mu_{\psi^2_{uk}}$ and $\sigma^2_{\psi^2_{uk}}$ are the mean value and variance of the mean square displacement of the k^{th} degree of freedom, respectively.

STRUCTURAL STATIONARY RANDOM RESPONSE

In the structure, any element can be taken as either a passive or active bar, where a piezoelectric bar is used as the active bar. The stiffness matrix and the mass matrix of the smart truss structures in global coordinates can be expressed as

$$\begin{aligned}[K] &= \sum_{e=1}^{m}[K_e] \\ &= \sum_{e=1}^{m}\{[T_e]^T[\theta\frac{E_e A_e}{l_e} + (1-\theta)\frac{c_{33e} + (e_{33e})^2/\varepsilon_{33e}}{l_e^C}A_e^C][G][T_e]\}\end{aligned} \tag{18}$$

$$[M] = \sum_{e=1}^{m}[M_e] = \sum_{e=1}^{m}\{\frac{1}{2}(\theta\rho_e A_e l_e + (1-\theta)\rho_e^C A_e^C l_e^C)[I]\} \tag{19}$$

where θ is a Boolean algebra value defined by the following: when θ=0, the mixed element is a piezoelectric active element bar; when θ=1, the mixed element is a passive element bar. $[K_e]$ and $[M_e]$ are the stiffness matrix and mass matrix of the e^{th} element, respectively. ρ_e, A_e and l_e are the density, cross-sectional area and length respectively of the e^{th} passive bar element. ρ_e^C, A_e^C and l_e^C are the density, cross-sectional area and length respectively of the e^{th} active bar element. E_e is the Young's modulus of the e^{th} passive bar element. c_{33e}, e_{33e} and ε_{33e} are the Young's modulus, piezoelectric force/electrical constant and dielectric constant respectively of the e^{th} active bar element (Gao et al. 2004). $[I]$ is a 6^{th} order identity matrix, and $[G]$ is a 6×6 matrix. $[T_e]$ is the transformation matrix and $[T_e]^T$ is its transpose (Gao 2006). E_e^C is the generalized elastic modulus of the piezoelectric active bar which considers the mechanic-electronic coupling effect, and is given by

$$E_e^C = c_{33e} + (e_{33e})^2/\varepsilon_{33e} \tag{20}$$

Substituting the equation (20) into equation (18) yields

$$[K] = \sum_{e=1}^{m}[K_e] = \sum_{e=1}^{m}\{[T_e]^T[\theta\frac{E_e A_e}{l_e} + (1-\theta)\frac{E_e^C A_e^C}{l_e^C}][G][T_e]\} \tag{21}$$

In the closed loop control system, since the control force $\{F_C(t)\}$ is determined by the applied force $\{F(t)\}$, the control force is a random force vector, and these two variables have full positive correlation. Let

$$\{P(t)\} = \{F(t)\} + [B]\{F_C(t)\} \tag{22}$$

Equation (1) can be re-written as

$$[M]\{\ddot{u}(t)\} + [C]\{\dot{u}(t)\} + [K]\{u(t)\} = \{P(t)\} \tag{23}$$

Its formal solution can be obtained in terms of the decoupling transform and Duhamel integral (Gao et al. 2005), that is

$$\{u(t)\} = \int_{-\infty}^{t}[\phi][h(\tau)][\phi]^T\{P(t-\tau)\}d\tau \tag{24}$$

where $[h(t)]$ is the impulse response function matrix of the structure, and can be expressed as

$$[h(t)] = diag\{h_j(t)\} \tag{25}$$

$$h_j(t) = \begin{cases} \frac{1}{\omega_{jd}}\exp(-\zeta_j\omega_j t)\sin\omega_{jd}t & t \geq 0 \\ 0 & t < 0 \end{cases}, \quad j = 1, 2, ..., n \tag{26}$$

where $\omega_{jd} = \omega_j(1-\zeta_j^2)^{1/2}$. From equation (24), the correlation function matrix of the displacement response of the structure can be obtained

$$\begin{aligned}[R_u(\tau)] &= E(\{u(t)\}\{u(t+\tau)\}^T) \\ &= \int_{-\infty}^{t}\int_{-\infty}^{t}[\phi][h(\tau_1)][\phi]^T[R_P(\tau_1 - \tau_2 + \tau)][\phi][h(\tau_2)]^T[\phi]^T d\tau_1 d\tau_2\end{aligned} \tag{27}$$

where $[R_u(\tau)]$ is the correlation function matrix of the displacement response of the structure, $[R_P(\tau_1 - \tau_2 + \tau)]$ is the correlation function matrix of $\{P(t)\}$. By performing a Fourier transformation to $[R_u(\tau)]$, the power spectral density matrix of the displacement response $[S_u(\omega)]$ can be obtained as follows

$$[S_u(\omega)] = [\phi][H(\omega)][\phi]^T[S_P(\omega)][\phi][H^*(\omega)][\phi]^T \tag{28}$$

where $[S_P(\omega)]$ is the power spectral density matrix of $\{P(t)\}$. $[H^*(\omega)]$ is the conjugate matrix of $[H(\omega)]$, where $[H(\omega)]$ is the frequency response function matrix of the structure, and can be expressed as

$$[H(\omega)] = diag[H_j(\omega)] \tag{29}$$

$$H_j(\omega) = 1/(\omega_j^2 - \omega^2 + i2\zeta_j\omega_j\omega)\,, \quad j = 1, 2, ..., n \tag{30}$$

where $i = \sqrt{-1}$ is the complex number. Integrating $[S_u(\omega)]$ within the frequency domain, the mean square value matrix of structural displacement response $[\psi_u^2]$ can be obtained as

$$[\psi_u^2] = \int_{-\infty}^{\infty}[S_u(\omega)]d\omega = \int_{-\infty}^{\infty}[\phi][H(\omega)][\phi]^T[S_P(\omega)][\phi][H^*(\omega)][\phi]^T d\omega \tag{31}$$

The mean square displacement of the k^{th} degree of freedom becomes

$$\psi_{uk}^2 = \vec{\phi}_k \int_{-\infty}^{\infty} [H(\omega)][\phi]^T [S_P(\omega)][\phi][H^*(\omega)] d\omega \vec{\phi}_k^T \quad k = 1, 2, ..., n \tag{32}$$

where $\vec{\phi}_k$ is the k^{th} line vector of the modal matrix $[\phi]$. Using the relationship between node displacement and element stress, the stress response of the e^{th} element in the truss structure can be expressed as

$$\{\sigma_e(t)\} = E_e [B_1]\{u_e(t)\} \quad e = 1, 2, ..., m \tag{33}$$

where $\{u_e(t)\}$ is the displacements of the nodal points of the e^{th} element, $[B_1]$ is the element's strain matrix. From equation (33), the correlation function matrix of the e^{th} element stress response $[R_{\sigma e}(\tau)]$ can be obtained by

$$[R_{\sigma e}(\tau)] = E(\{\sigma_e(t)\}\{\sigma_e(t+\tau)\}^T) = E_e [B_1][R_{ue}(\tau)][B_1]^T E_e \tag{34}$$

Furthermore, the power spectral density matrix of the stress response of the e^{th} element $[S_{\sigma e}(\omega)]$ can be obtained

$$[S_{\sigma e}(\omega)] = E_e [B_1][S_{ue}(\omega)][B_1]^T E_e \tag{35}$$

Finally, the mean square value matrix of the e^{th} element stress response $[\psi_{\sigma e}^2]$ becomes

$$[\psi_{\sigma e}^2] = E_e [B_1][\psi_{ue}^2][B_1]^T E_e \tag{36}$$

NUMERICAL CHARACTERISTICS OF STOCHASTIC STRUCTURAL STATIONARY RANDOM RESPONSE

Structural dynamic characteristics analysis using random factor method (Gao et al. 2005)

The following parameters corresponding to ρ_e , A_e , l_e , E_e , ρ_e^C , A_e^C , l_e^C and c_{33e} are simultaneously considered as random variables. From equation (20), it can be easily observed that E_e^C is a random variable. The randomness of physical parameters and geometrical dimensions will result in randomness of the matrices $[K]$ and $[M]$, and consequently the natural frequencies ω_j and natural modal matrix $[\phi]$. In this paper, symbols μ_X , σ_X and ν_X denote the mean value, standard deviation and variation coefficient (the ratio of the standard deviation to the mean value) of the random variable X , respectively. In the following, the computing expression of the mean value and standard deviation of j^{th} order natural frequency can be respectively deduced by means of the algebra synthesis method

$$\mu_{\omega_j} = \overline{\omega}_j \{[1+\nu_Z^2+\nu_\rho^2+\nu_Z^2\nu_\rho^2 - c_{E\rho}\cdot\nu_E\cdot(\nu_Z^2+\nu_\rho^2+\nu_Z^2\nu_\rho^2)^{1/2}]^2 - \frac{1}{2}[\nu_E^2+\nu_Z^2+\nu_\rho^2+\nu_Z^2\nu_\rho^2 - 2c_{E\rho}\cdot\nu_E\cdot(\nu_Z^2+\nu_\rho^2+\nu_Z^2\nu_\rho^2)^{1/2}]\}^{1/4} \tag{37}$$

$$\sigma_{\omega_j} = \overline{\omega}_j \{[1+\nu_Z^2+\nu_\rho^2+\nu_Z^2\nu_\rho^2 - c_{E\rho}\cdot\nu_E\cdot(\nu_Z^2+\nu_\rho^2+\nu_Z^2\nu_\rho^2)^{1/2}] - \{[1+\nu_Z^2+\nu_\rho^2+\nu_Z^2\nu_\rho^2 - c_{E\rho}\cdot\nu_E\cdot(\nu_Z^2+\nu_\rho^2+\nu_Z^2\nu_\rho^2)^{1/2}]^2 - \frac{1}{2}[\nu_E^2+\nu_Z^2+\nu_\rho^2+\nu_Z^2\nu_\rho^2 - 2c_{E\rho}\cdot\nu_E\cdot(\nu_Z^2+\nu_\rho^2+\nu_Z^2\nu_\rho^2)^{1/2}]\}^{1/2}\}^{1/2}$$

$$\nu_z = \sqrt{4\nu_l^2 + 2\nu_l^4}\Big/(1+\nu_l^2) \tag{38}$$

where $\overline{\omega}_j$ can be obtained by the structural conventional dynamic characteristic analysis for deterministic structures.

Likewise, the randomness of each element (ϕ_{ij}) of modal matrix can be expressed as

$$\mu_{\phi_{ij}} = \overline{\phi}_{ij}[1+\frac{1}{2}(\nu_\rho^2+\nu_A^2+\nu_l^2+\nu_\rho^2\nu_A^2+\nu_\rho^2\nu_l^2+\nu_A^2\nu_l^2+\nu_\rho^2\nu_A^2\nu_l^2)]^{1/4} \tag{39}$$

$$\sigma_{\phi_{ij}} = \overline{\phi}_{ij}\{1+\nu_\rho^2+\nu_A^2+\nu_l^2+\nu_\rho^2\nu_A^2+\nu_\rho^2\nu_l^2+\nu_A^2\nu_l^2+\nu_\rho^2\nu_A^2\nu_l^2 - [1+\frac{1}{2}(\nu_\rho^2+\nu_A^2+\nu_l^2+\nu_\rho^2\nu_A^2+\nu_\rho^2\nu_l^2+\nu_A^2\nu_l^2+\nu_\rho^2\nu_A^2\nu_l^2)]^{1/2}\}^{1/2} \tag{40}$$

The deterministic values of natural modal shape and modal matrix can be obtained by means of the conventional dynamic analysis method.

Numerical characteristics of the structural stationary random response

The randomness of the structural natural frequencies, modeshapes and excitations will result in randomness in the structural dynamic responses of the closed loop control system, corresponding to the dynamic displacement and stress.

From equation (32), by means of the random variable's functional moment method (Gao et al. 2005), the mean value $\mu_{\psi_{uk}^2}$ and standard deviation $\sigma_{\psi_{uk}^2}$ of the mean square displacement for the k^{th} degree of freedom can be obtained as

$$\mu_{\psi_{uk}^2} = \int_{-\infty}^{\infty} \mu_{\vec{\phi}_k}\mu_{[H(\omega)]}\mu_{[\phi]^T}\mu_{[S_P(\omega)]}\mu_{[\phi]}\mu_{[H^*(\omega)]}\mu_{\vec{\phi}_k^T} d\omega \tag{41}$$

$$\begin{aligned}\sigma_{\psi_{uk}^2} = \{\int_{-\infty}^{\infty}\{&\{\sigma_{\vec{\phi}_k}\mu_{[H(\omega)]}\mu_{[\phi]^T}\mu_{[S_P(\omega)]}\mu_{[\phi]}\mu_{[H^*(\omega)]}\mu_{\vec{\phi}_k^T}\}^2 \\ &+\{\mu_{\vec{\phi}_k}\sigma_{[H(\omega)]}\mu_{[\phi]^T}\mu_{[S_P(\omega)]}\mu_{[\phi]}\mu_{[H^*(\omega)]}\mu_{\vec{\phi}_k^T}\}^2 \\ &+\{\mu_{\vec{\phi}_k}\mu_{[H(\omega)]}\sigma_{[\phi]^T}\mu_{[S_P(\omega)]}\mu_{[\phi]}\mu_{[H^*(\omega)]}\mu_{\vec{\phi}_k^T}\}^2 \\ &+\{\mu_{\vec{\phi}_k}\mu_{[H(\omega)]}\mu_{[\phi]^T}\mu_{[S_P(\omega)]}\sigma_{[\phi]}\mu_{[H^*(\omega)]}\mu_{\vec{\phi}_k^T}\}^2 \\ &+\{\mu_{\vec{\phi}_k}\mu_{[H(\omega)]}\mu_{[\phi]^T}\mu_{[S_P(\omega)]}\mu_{[\phi]}\sigma_{[H^*(\omega)]}\mu_{\vec{\phi}_k^T}\}^2 \\ &+\{\mu_{\vec{\phi}_k}\mu_{[H(\omega)]}\mu_{[\phi]^T}\mu_{[S_P(\omega)]}\mu_{[\phi]}\mu_{[H^*(\omega)]}\sigma_{\vec{\phi}_k^T}\}^2\}d\omega\}^{1/2} \\ &k = 1,2,\cdots,n\end{aligned} \tag{42}$$

$$\sigma_{[H(\omega)]} = diag\{\frac{(2\mu_{\omega_j}+i\cdot 2\zeta_j\omega)\cdot\sigma_{\omega_j}}{(\mu_{\omega_j}^2-\omega^2+i\cdot 2\zeta_j\mu_{\omega_j}\omega)^2}\} \quad j = 1,2,\cdots,n \tag{43}$$

From equation (36), and by means of the algebra synthesis method, expressions for the numerical characteristics of the mean square stress for the e^{th} element are obtained as

$$\mu_{[\psi^2_{\sigma e}]} = (\mu_E^2 + \sigma_E^2)[B_1]\mu_{[\psi^2_{ue}]}[B_1]^T \quad e = 1, 2, ..., m \tag{44}$$

$$\begin{aligned}\sigma_{[\psi^2_{\sigma e}]} = \{&(\mu_E^2 + \sigma_E^2)^2([B_1]\sigma_{[\psi^2_{ue}]}[B_1]^T)^2 \\ &+ (4\mu_E^2\sigma_E^2 + 2\sigma_E^4)([B_1]\mu_{[\psi^2_{ue}]}[B_1]^T)^2 \\ &+ (4\mu_E^2\sigma_E^2 + 2\sigma_E^4)([B_1]\sigma_{[\psi^2_{ue}]}[B_1]^T)^2\}^{1/2} \quad e = 1, 2, ..., m\end{aligned} \tag{45}$$

where $\mu_{[\psi^2_{\sigma e}]}$ and $\sigma_{[\psi^2_{\sigma e}]}$ are the mean value and standard deviation of the mean square stress for the e^{th} element respectively.

EXAMPLE

To illustrate the method, a 25-bar space piezoelectric smart truss structure shown in Figure 1 is used as an example. The material properties of the active and passive bars are given in Table 1. A ground level acceleration along with the positive direction of Y-axis acts on the structure, is a Gauss stationary random process and its mean value is zero. Its self-power spectral density can be expressed as (Gao et al. 2005)

$$S_{FF}(\omega) = \frac{1 + 4(\xi_g \omega / \omega_g)^2}{(1 - \omega^2/\omega_g^2)^2 + 4(\xi_g \omega/\omega_g)^2} S_0 \tag{46}$$

where $\omega_g = 16.5$, $\xi_g = 0.7$, $S_0 = 19.2$ cm^2/s^3.

In order to solve the optimal problem, two steps are adopted (Gao et al. 2003). In the first step, the reliability constraints of dynamic stress and displacement are neglected, and the feedback gains are kept constant. Then, each element bar is taken as an active bar in turn and the corresponding performance function value is calculated. Based on the computational results for the dissipated energy, the optimal location of the active bar can be determined. In the second step, after the optimal placement of the active bar is obtained, the reliability constraints are imposed, and the optimization of feedback gain, that is, minimization of feedback gain will be developed.

Optimal placement of the active bar

For the first step, and letting the closed loop control system feedback gains be $g = g_j = 100$, each element bar is taken as active bar in turn; the corresponding performance function value is given in Table 2.

From Table 2, it can be seen that if the sixteenth or twentieth element is used as the active bar, the active control performance of the smart truss structure is the best. The effect of active vibration control of the smart truss structure is the worst if the first element is used as the active bar.

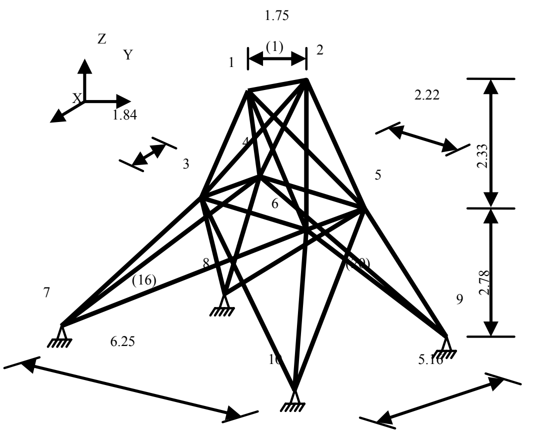

Figure 1. 25-bar space piezoelectric truss structure (units: m)

Table 1. Physical parameters of the smart truss structure

Mean value	Active bar (PZT-4)	Passive bar (steel)
mass density (kg/m^3)	7600	7800
Young's modulus (N/m^2)	8.807×10^{10}	2.1×10^{11}
e_{33} (C/m^2)	18.62	—
ε_{33} (C/Vm)	5.92×10^{-9}	—
cross section area (m^2)	3.0×10^{-4}	3.0×10^{-4}

Table 2. Computational results of the performance function

Element (node-node)	1(1-2)	2(1-3)	3(1-4)	4(1-5)	5(1-6)
Value of J	96.86	118.70	115.67	135.58	132.12
Element (node-node)	6(2-3)	7(2-4)	8(2-5)	9(2-6)	10 (3-4)
Value of J	137.33	129.88	126.55	122.31	141.26
Element (node-node)	11 (4-5)	12 (5-6)	13 (6-3)	14 (7-3)	15 (7-4)
Value of J	147.54	141.26	147.54	169.97	189.31
Element (node-node)	16 (7-6)	17 (8-3)	18 (8-4)	19 (8-5)	20 (9-4)
Value of J	196.28	176.49	163.22	182.65	196.28
Element (node-node)	21 (9-5)	22 (9-6)	23 (10-5)	24 (10-6)	25 (10-3)
Value of J	169.97	189.31	176.49	163.22	182.65

Optimization of the feedback gain

In order to assess the control performance with the reliability constraints imposed and optimization of the feedback gain, the control results using the 16th and 1st elements as the active bar respectively are compared. The structural parameters (material properties and geometric dimensions) and the limit values of the mean square stress and displacement, $\psi^{2\,*}_{\sigma e}$ and $\psi^{2\,*}_{uk}$, are all taken to be random variables, where $\mu_{\psi^{2*}_{\sigma e}} = 2800$ MPa^2, $\mu_{\psi^{2*}_{uk}} = 4.000$ mm^2 and $R^*_{\psi^2_{\sigma e}} = R^*_{\psi^2_{uk}}$ =0.95. Values from both deterministic and random models were obtained. In the deterministic model (DM), the mean values of the random variables are unity, and their standard deviations are zero. The optimal results for the feedback gains, and the mean displacement and stress responses are respectively given in Table 3 and Table 4. Results for two random models are presented, in which the variation coefficients of all random variables is equal to 0.05 in the first random model (1st RM), and 0.1 in the second random model (2nd RM). In addition, in order to verify our method, stationary random re-

Table 3. Computational results of the feedback gains (*dynamic analysis by the MCSM)

	16th element used as the active bar			
Design variables	Original value	DM	1st RM	2nd RM
G	50	89.26	117.55	136.91
*G			*117.56	*136.93
$\mu_{\psi^2_{\sigma e}}$	3744.9	2799.8	2144.4	1802.6
*$\mu_{\psi^2_{\sigma e}}$			*2145.2	*1803.7
$\mu_{\psi^2_{uk}}$	5.3694	3.9982	3.0506	2.4969
*$\mu_{\psi^2_{uk}}$			*3.0509	*2.4974
$R_{\psi^2_{\sigma e}}$		0.50	0.95	0.95
$R_{\psi^2_{uk}}$		0.52	0.97	0.97

sponses obtained using the Monte-Carlo simulation method (MCSM) are also presented in Table 3 and Table 4.

Table 4. Computational results of the feedback gains (*dynamic analysis by the MCSM)

1st element used as the active bar				
Design variables	Original value	DM	1^{st} RM	2^{nd} RM
G	50	97.11	135.79	162.98
*G			*135.83	*163.09
$\mu_{\psi^2_{\sigma e}}$	4007.2	2799.9	2144.5	1802.9
$^*\mu_{\psi^2_{\sigma e}}$			*2145.7	*1804.5
$\mu_{\psi^2_{uk}}$	5.7455	3.9984	3.0510	2.4971
$^*\mu_{\psi^2_{uk}}$			*3.0517	*2.4982
$R_{\psi^2_{\sigma e}}$		0.50	0.95	0.95
$R_{\psi^2_{uk}}$		0.52	0.97	0.97

From Table 3 and Table 4, it can be seen easily that the optimal results of the feedback gains obtained by the method proposed in this paper is in good agreement with that of the random structural stationary random responses analyzed by the Monte Carlo simulation method, by which the validity of our method is verified. The optimal results of the deterministic and random models are different, and the optimal value of feedback gain increases when the randomness of the structural parameters increases. The results show that the areas of the truss structure where the most energy is stored are the optimal location of an active bar in order to maximize its damping effect.

CONCLUSIONS

Energy dissipation in a piezoelectric smart truss structure has been maximized in order to determine the optimal location of a single piezoelectric active element. Results show that the effectiveness of using the active element is strongly dependent on its location in the truss structure. The effect of randomness of the structural parameters corresponding to the material properties and geometric dimensions on the feedback gain was also examined.

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