# The directivity of sound radiated from a panel or opening excited by sound incident on the other side

# John Laurence Davy (1,2) and Vladimir Pavasovic (3)

School of Applied Sciences, RMIT University, GPO Box 2476V Melbourne Victoria 3001, Australia
 CSIRO Manufacturing & Infrastructure Technology, PO Box 56 Highett Victoria 3190, Australia
 Watson Moss Growcott Acoustics Pty Ltd, 7/696 High Street East Kew Victoria 3102, Australia

# ABSTRACT

This paper describes a theoretical method for predicting the directivity of the sound radiated from a panel or opening excited by sound incident on the other side. The method uses a two dimensional strip model and the low frequency result for a square piston. A cosine squared weighting function with a weighting angle parameter is used to account for the angular distribution of the incident sound. The method is compared with published results. The values of the weighting angle parameter which give the best agreement with each set of published results are determined. The directivity depends strongly on the length of the radiating object in the direction of the observer and only slightly on the width of the object at right angles to the direction of the observer. Above its critical frequency a panel radiates strongly at the angle at which coincidence occurs.

# INTRODUCTION

This paper describes a theoretical method for predicting the directivity of the sound radiated from a panel or opening excited by sound incident on the other side. This directivity needs to be known when predicting the sound level at a particular position due to sound radiation from a factory roof, wall, ventilating duct or chimney flue. There is surprisingly little information on how to predict this directivity in the scientific literature. Most of this information is based on limited experimental data or its basis cannot be determined.

# THEORY

## The equations used for calculation

The effective impedance  $Z_e(\phi)$  of a finite panel in an infinite baffle to a plane sound wave incident at an angle of  $\phi$  to the normal to the panel is (Rindel 1975)

$$Z_e(\phi) = Z_{wfi}(\phi) + Z_{wfi}(\phi) + Z_{wp}(\phi)$$
<sup>(1)</sup>

where

 $Z_{w\!\not\!n}(\phi)$  is the wave impedance of the fluid as experienced by the finite panel in an infinite baffle, whose vibration is due a plane sound wave incident at an angle of  $\phi$  to the normal to the panel, on the side from which the plane sound wave is incident (this is the fluid loading on the incident side),

 $Z_{w\!/\!t}(\phi)$  is the wave impedance of the fluid as experienced by the finite panel in an infinite baffle, whose vibration is due a plane sound wave incident at an angle of  $\phi$  to the normal to the panel, on the side opposite to which the sound is incident (this is the fluid loading on the non-incident or transmitted side) and

 $Z_{wp}(\phi)$  is the wave impedance of the finite panel in an infi-

nite baffle to a plane sound wave incident at an angle of  $\phi$  to the normal to the panel, ignoring fluid loading.

It will be assumed that the fluid wave impedances on both sides are the same and the imaginary part of the fluid wave impedance will be ignored (Rindel 1975). That is

$$Z_{wfi}(\phi) = Z_{wfi}(\phi) = \rho c \sigma(\phi) \tag{2}$$

where  $\rho$  is the density of the fluid, *c* is the speed of sound in the fluid and  $\sigma(\phi)$  is the radiation efficiency into the fluid of one side of the finite panel in an infinite baffle, whose vibration is due a plane sound wave incident at an angle of  $\phi$ to the normal to the panel.

Reflections at the panel edges are ignored (Rindel 1975). The rms normal velocity  $v_{rms}(\phi)$  of the panel due to a plane sound wave incident at an angle of  $\phi$  to the normal to the panel which exerts an rms pressure  $p_{irms}(\phi)$  is



Figure 1. Sound incident at an angle of  $\phi$  to the normal to a panel or opening and radiated at an angle of  $\theta$  to the normal.

The transmitted rms sound pressure  $p_{trms}(\theta, \phi)$  which is radiated by the panel on the non-incident side to a receiving point which is at an angle of  $\theta$  to the normal to the centre of the panel and a large distance from the panel is (Davy 2004)

20-22 November 2006, Christchurch, New Zealand

$$p_{trms}(\theta,\phi) \propto v_{rms}(\phi) \frac{\sin[ka(\sin\theta - \sin\phi)]}{ka(\sin\theta - \sin\phi)}$$
(4)

where k is the wave number of the sound and 2a is the length of the panel in the direction of the source. Thus

$$p_{trms}(\theta,\phi) \propto \frac{p_{trms}(\phi)}{2\rho c\sigma(\phi) + Z_{wp}(\phi)} \frac{\sin\left[ka(\sin\theta - \sin\phi)\right]}{ka(\sin\theta - \sin\phi)}.$$
 (5)

The case where the incident sound is generated by a sound source in a room or duct is now considered. We assume that the sound pressure waves are incident at different angles  $\phi$  with random phases and mean squared sound pressures which are proportional to a weighting function  $w(\phi)$ .

$$\left|p_{irms}(\phi)\right|^2 \propto w(\phi) \,. \tag{6}$$

The weighting function is to account for the fact that sound waves at grazing angles of incidence will have had to suffer more wall collisions and therefore be more attenuated before reaching the panel. The total mean square sound pressure  $|p_{Trms}(\theta)|^2$  at the receiving point is

$$\left| p_{Trms}(\theta) \right|^{2} \propto \int_{-\pi/2}^{\pi/2} \frac{w(\phi)}{\left| 2\rho c\sigma(\phi) + Z_{wp}(\phi) \right|^{2}} \left\{ \frac{\sin\left[ ka(\sin\theta - \sin\phi) \right]}{ka(\sin\theta - \sin\phi)} \right\}^{2} d\phi$$
(7)

The case when sound is incident from a source in a free field at an angle  $\theta$  to the normal to the panel and the panel radiates at all angles  $\phi$  into a room or duct is also of interest. In this case the weighting function  $w(\phi)$  is to account for the fact that sound waves radiated at grazing angles will have had more wall collisions and therefore be more attenuated before reaching the receiving position which is assumed to be a reasonable distance from the panel or opening which is radiating the sound. In this second case, we have to integrate over all angles of radiation  $\phi$  because of the reverberant nature of the sound. For this case, the impedance terms in the integral are functions of  $\theta$  rather than  $\phi$  and can be taken outside the integral. However in this study both cases are calculated using the formula for the first case which is shown above. This is because both cases should be the same by the principle of reciprocity and it is not clear which form of the formula is more correct.

For large values of ka, the two cases of the formula will be similar. If ka is much greater than 1, the function

$$\left\{\frac{\sin\left[ka(\sin\theta - \sin\phi)\right]}{ka(\sin\theta - \sin\phi)}\right\}^{2}$$
(8)

has a sharp maximum at  $\phi = \theta$  and is symmetrical in both  $\theta$ and  $\phi$  about the point  $\phi = \theta$ . We can exploit these facts by evaluating the impedance terms for the first case at  $\phi = \theta$ and taking them out side the integral. This gives the formula for the second case.

The relative sound pressure level  $L(\theta)$  in the direction  $\theta$  is

$$L(\theta) = 20\log_{10}\left(\left|p_{Trms}(\theta)\right|\right) - 20\log_{10}\left(\left|p_{Trms}(0)\right|\right)$$
(9)

A weighting function was needed, because the assumption of diffuse field incidence did not agree with the experimental results in the literature. In this study, the following weighting function  $w(\phi)$  was developed.

$$w(\phi) = \begin{cases} \frac{1}{2} \left( 1 + \cos\left(\frac{\pi\phi}{\phi_w}\right) \right) & \text{if } |\phi| < |\phi_w| \\ 0 & \text{if } |\phi| \ge |\phi_w| \end{cases}$$
(10)

This weighting function was chosen because it goes smoothly to zero at the weighting angle  $\phi_w$ . Weighting functions based on physical models are being investigated by a current postgraduate student at RMIT University. Note that this weighting angle  $\phi_w$  is different from the limiting angle  $\phi_l$  which will be introduced in equations (11) and (12) (Davy 2004).  $\phi_i$  is a function of ka which is used in the calculation the radiation efficiency  $\sigma(\phi) \cdot \phi_w$  is a parameter which is used to match the weighting function to the actual distribution of sound as determined by the best match between the theory and the directivity results from the literature.  $\phi_w$  may need to be a function of frequency, angle of radiation (or incidence), size of panel, size of opening, room dimensions or duct length. However this study will try to use a constant value of  $\phi_w$  when comparing with a particular set of experimental or theoretical data.

It is important to note that although the weighting angle  $\phi_w$  can be greater than  $\pi/2$ ,  $\phi$  is only integrated over the range from  $-\pi/2$  to  $\pi/2$ . Also, the weighting function does not have to be zero when  $\phi$  is equal to  $-\pi/2$  and  $\pi/2$ . Uniform weighting is obtained when  $\phi_w$  is infinite.

In this study we use the radiation efficiency of a strip of width 2a, which we approximate with the following equation (Davy 2004).

$$\sigma(\phi) = \begin{cases} \frac{1}{\frac{\pi}{2k^2a^2} + \cos\phi} & \text{if } |\phi| \le \phi_l \\ \frac{1}{\frac{\pi}{2k^2a^2} + \frac{3\cos\phi - \cos\phi}{2}} & \text{if } \phi_l < |\phi| \le \frac{\pi}{2} \end{cases}$$
(11)

where

$$\phi_l = \arccos\left(\sqrt{\frac{\pi}{2ka}}\right) \tag{12}$$

and k is the wave number of the sound and 2a is the length of the panel in the direction of the source.

For an opening with no panel in an infinite baffle we put  $Z_{wp}(\phi) = 0$ . For a finite panel in an infinite baffle we use the infinite panel result for  $Z_{wp}(\phi)$ . This result is expected to be the correct result when averaged over frequency, because this approach gives the correct result for point impedances when averaged over frequency and position on a finite panel (Cremer and Heckl 1973).

$$Z_{wp}(\phi) = m\omega \left\{ j \left[ 1 - \left(\frac{\omega}{\omega_c}\right)^2 \sin^4(\phi) \right] + \eta \left(\frac{\omega}{\omega_c}\right)^2 \sin^4(\phi) \right\} \quad (13)$$

where *m* is the surface density (mass per unit area) of the panel,  $\eta$  is the damping loss factor of the panel,  $\omega_c$  is the critical frequency of the panel and  $\omega$  is the angular frequency of the sound.

In this study the integral in equation (7) was performed by evaluating the integrand at  $1^{\circ}$  intervals from -90° to 90° and summing the values.

#### Why are openings and finite panels different?

The case when the sound is radiated from a room or duct is considered in this section. It is assumed that the sound is incident as described by the weighting function  $w(\phi)$ . The following analysis applies to finite size openings and finite size panels. Assumptions will be made which allow the expressions to be simplified in order to demonstrate the differences between openings and panels. In particular, assumptions will be made which allow the approximation  $\phi = \theta$ . To show why openings and finite panels are different a further approximation is made. In equation (7) for the total mean square sound pressure  $|p_{Trms}(\theta)|^2$  at the receiving point, note that if ka is much greater than 1, the function

$$\left\{\frac{\sin\left[ka(\sin\theta - \sin\phi)\right]}{ka(\sin\theta - \sin\phi)}\right\}^{2}$$
(14)

has a sharp maximum at  $\phi = \theta$ . This fact is exploited by evaluating the rest of the integral at  $\phi = \theta$  and taking it out side the integral. Note that the integral that is left is proportional to the radiation efficiency  $\sigma(\theta)$  of the finite panel (Davy 2004). Thus

$$\left|p_{Trms}(\theta)\right|^{2} \propto \frac{w(\theta)\sigma(\theta)}{\left|2\rho c\sigma(\theta) + Z_{wp}(\theta)\right|^{2}}.$$
(15)

For an opening  $Z_{wp}(\theta) = 0$  and thus

$$\left|p_{Trms}(\theta)\right|^2 \propto \frac{w(\theta)}{\sigma(\theta)}.$$
 (16)

For a panel  $|Z_{wp}(\theta)|^2 >> 2\rho c\sigma(\theta)$  and thus

$$\left|p_{Trms}(\theta)\right|^{2} \propto \frac{w(\theta)\sigma(\theta)}{\left|Z_{wp}(\theta)\right|^{2}}.$$
(17)

At coincidence, the trace wavelength of the incident sound measured between pressure maxima on the panel is equal to the wave length of free bending waves in the panel. Apart from near coincidence,  $|Z_{wp}(\theta)|^2$  is independent of  $\theta$ . Thus the result for a panel reduces to

$$\left| p_{Trms}(\theta) \right|^2 \propto w(\theta) \sigma(\theta) \,. \tag{18}$$

If 
$$ka >> 1$$
 and  $|\theta| < \phi$ 

$$\sigma(\theta) = \frac{1}{\cos(\theta)},\tag{19}$$

(Davy 2004).

In this case, if  $w(\theta)$  is constant, for an opening

$$\left| p_{Trms}(\theta) \right|^2 \propto \cos(\theta),$$
 (20)

and for a finite panel

$$\left|p_{Trms}(\theta)\right|^2 \propto \frac{1}{\cos(\theta)}$$
 (21)

Another possible simple weighing function is

$$w(\theta) \approx \cos(\theta)$$
. (22)

With this weighting, the approximate results become

$$p_{Trms}(\theta)|^2 \propto \cos^2(\theta),$$
 (23)

for an opening and

$$\left| p_{Trms}(\theta) \right|^2$$
 is independent of  $\theta$  (24)

for a finite panel except near coincidence. In both cases, the ratio of the result for an opening to the result for a finite panel is proportional to  $\cos^2(\theta)$ . Note that this result assumes ka >> 1 and  $|\theta| < \phi_l$  where  $\phi_l$  is given by equation (12).

## COMPARISON WITH PUBLISHED RESULTS

In this section, the prediction method described in the previous section is compared with experimental results and prediction methods for finite size panels and finite size openings from the literature. The weighting function given in equation (10) is used and the weighting angle  $\phi_w$  is varied to obtain the best agreement. The weighting function and the weighting angle  $\phi_w$  give an indication of the distribution of incident sound which is needed to produce the experimental results or the results of the predictive method. This will be a valuable guidance for future work which seeks to develop a physical model of the incident sound distribution in different situations. Results for all methods are presented on a logarithmic scale of Strouhal number. The Strouhal number is defined as the ratio of the distance across the finite flat panel or finite opening in the direction of the receiver to the wavelength of the sound in the air.

## 1 mm glass panel in room wall



**Figure 2**. The sound level at 45° relative to that at 0° as a function of Strouhal number for 1 mm thick glass installed in the wall of a box.



**Figure 3**. The sound level at 75° relative to that at 0° as a function of Strouhal number for 1 mm thick glass installed in the wall of a box.



**Figure 4**. The sound level at 90° relative to that at 0° as a function of Strouhal number for 1 mm thick glass installed in the wall of a box.



**Figure 5**. The sound level at 45° relative to that at 0° as a function of Strouhal number for 3 mm thick glass installed in the wall of a box.

Rindel (Rindel 1975) made 1:4 scale model measurements on the sound insulation of windows installed in one wall of a box in an anechoic room. The sound was incident at an angle to the window normal from outside the box. This is the opposite direction to the calculation method used in this paper, but as explained above is expected to give similar results because of the principle of reciprocity. The windows were mounted in an opening measuring 420 mm wide by 300 mm high by 75 mm deep. The wall of the box containing the window was part of a baffle measuring 3800 mm wide by 3100 mm high. The internal dimensions of the box were 1210 mm wide by 960 mm high by 740 mm deep. The loudspeaker was 4000 mm from the middle of the front of the opening. For the measurements plotted in this sub-section Rindel used 1 mm thick glass which would scale to 4 mm thickness in full scale. The weighting angle which gave the best agreement between theory and experiment in Figures 2 to 4 was 120°. The relative sound level in Figures 2 to 4 is basically constant apart from the coincidence peak.

#### 3 mm glass panel in room wall

Figures 5 to 7 show Rindel's measurements with 3 mm thick glass which would scale to 12 mm thickness in full scale. As in Figures 2 to 4, the weighting angle which gave the best agreement between theory and experiment in Figures 5 to 7 was 120°. Again the relative sound level in Figures 5 to 7 is basically constant apart from the coincidence peak.



Figure 6. The sound level at  $75^{\circ}$  relative to that at  $0^{\circ}$  as a function of Strouhal number for 3 mm thick glass installed in the wall of a box.



**Figure 7**. The sound level at 90° relative to that at 0° as a function of Strouhal number for 3 mm thick glass installed in the wall of a box.

#### 6 mm glass panel in room wall

Stead (Stead 2001) measured the sound insulation of a window installed in one wall of a room. The sound was incident at an angle to the window normal from outside the room. This is the opposite direction to the calculation method used in this paper, but as in the case of Rindel's measurements is expected to give similar results because of the principle of reciprocity. The window was 1450 mm wide by 2120 mm high. The wall of the room containing the window was part of the external wall of a larger building which served as a baffle. The internal dimensions of the room were 2880 mm wide by 3000 mm high by 5120 mm deep. The loudspeaker was 20 m from the middle of the widow. The glass was 6 mm thick.

The relative sound levels in Figures 8 to 13 are basically flat as a function of Strouhal number apart from the coincidence peak. However they do decrease with increasing angle. A weighting angle of 83° gave the best agreement between theory and Stead's experimental results. This is less than the 120° used with Rindel's results and is believed to be due to the fact that the absorption coefficients of the walls of Stead's room were greater than the absorption coefficients of the walls of Rindel's box. Sound incident at large angles to the normal from a source in the centre of the room will have to undergo a large number of reflections. The larger the wall absorption coefficients, the more attenuated this sound will be. A similar argument applies if source and receiver are interchanged as in the case of Stead's and Rindel's results.



Figure 8. The sound level at  $15^{\circ}$  relative to that at  $0^{\circ}$  as a function of Strouhal number for 6 mm thick glass installed in the wall of a room.



**Figure 9**. The sound level at 30° relative to that at 0° as a function of Strouhal number for 6 mm thick glass installed in the wall of a room.



Figure 10. The sound level at  $45^{\circ}$  relative to that at  $0^{\circ}$  as a function of Strouhal number for 6 mm thick glass installed in the wall of a room.



**Figure 11**. The sound level at 60° relative to that at 0° as a function of Strouhal number for 6 mm thick glass installed in the wall of a room.





More recent research is developing a physical model for the weighting function which gives better agreement with Stead's data. This research also shows that diffraction effects will need to be included for the 90°.



Figure 13. The sound level at  $90^{\circ}$  relative to that at  $0^{\circ}$  as a function of Strouhal number for 6 mm thick glass installed in the wall of a room.

# Opening in room wall (Rindel)



**Figure 14**. The sound level at  $45^{\circ}$  relative to that at  $0^{\circ}$  as a function of Strouhal number for opening in the wall of a box.



**Figure 15**. The sound level at  $75^{\circ}$  relative to that at  $0^{\circ}$  as a function of Strouhal number for opening in the wall of a box.

Rindel also made measurements on his opening with no window installed in it. In contrast to Rindel's window results, his opening results in Figures 14 to 16 decreases with increasing Strouhal number and increasing angle. As explained above this difference in behaviour between panels and openings is predicted by the theory. Surprisingly a weighting angle of 200° gave the best agreement between theory and experiment for Rindel's opening. This contrasted to the weighting angle of 120° which was needed with Rindel's window panel results. No explanation for this difference is immediately obvious.



**Figure 16**. The sound level at 90° relative to that at 0° as a function of Strouhal number for an opening in the wall of a box.

## **Opening in room wall (Roberts)**



**Figure 17**. The sound level at 90° relative to that at 0° as a function of Strouhal number for an opening in the wall of a room.

Roberts (Roberts 1983) made measurements on the opening of a sliding window in the wall of a room. The wall of the room baffled the opening. The opening width was set at 75, 150, 300 and 600 mm and the opening height was 970 mm. The direction of sound propagation was from inside the room to outside. Measurements were made at  $0^{\circ}$  and  $90^{\circ}$  to the normal to the opening at a distance of 1000 mm from the centre of the edge of the opening. The measurements were made in both the horizontal and vertical directions. A constant of -1.5 dB was added to the theoretical results to make them agree with the experimental results at small Strouhal numbers. In contrast to Rindel's opening, a weighting angle of 100° needed to be used for Roberts' results shown in Figure 17. This value of weighting angle is in between the 83° needed for Stead's window panel and the 120° needed for Rindel's window panels.

# Unbaffled duct end opening (Bies and Hansen)



**Figure 18**. The sound level at 30° relative to that at 0° as a function of Strouhal number for an unbaffled duct end opening.



**Figure 19**. The sound level at 45° relative to that at 0° as a function of Strouhal number for an unbaffled duct end opening.



**Figure 20**. The sound level at 60° relative to that at 0° as a function of Strouhal number for an unbaffled duct end opening.

Bies and Hansen (Bies and Hansen 1996) gave directivity curves for an unbaffled duct end opening. These curves are based on measurements made in an anechoic room. As with Roberts results, a constant of -1.5 dB was added to the theoretical results to make them agree with the experimental results at small Strouhal numbers. A weighting angle of  $55^{\circ}$ gave the best agreement between theory and experiment in Figures 18 to 21 overall. However better agreement is obtained for sound radiating at 90° to the normal to the duct end opening if a weighting angle of 19° is used as shown in Figure 22. This is thought to be due to the diffraction of sound into the shadow zone because the duct end opening is unbaffled. The smaller value of weighting angle is thought to be due to the different angular distribution of sound in a duct compared to sound in a room. It is perhaps surprising that the weighting angle is not much smaller for Strouhal numbers less than 0.5 where only plane propagation is supposed to occur inside the duct.



**Figure 21**. The sound level at 90° relative to that at 0° as a function of Strouhal number for an unbaffled duct end opening with a weighting angle of 55°.



**Figure 22**. The sound level at 90° relative to that at 0° as a function of Strouhal number for an unbaffled duct end opening with a weighting angle of 19°.

## Unbaffled duct end opening (Levine and Schwinger)

Levine and Schwinger (Levine and Schwinger 1948) presented the results of analytic calculations of the directivity of the radiation of a plane duct wave from a circular unbaffled duct end. The theoretical results in Figures 23 to 25 were calculated with a weighting angle of  $1^{\circ}$ . Because the integration was performed in  $1^{\circ}$  steps this limited the incident sound to an incidence angle of  $0^{\circ}$ .

## ASSUMPTIONS

The theory presented in this paper is based on the previous work of Rindel (1975) and Davy (2004). Davy's results are for a two dimension model based on an infinite strip in an infinite baffle. Both Rindel and Davy ignored reflections at the panel, opening or strip edges. Also they both ignored the imaginary part of the fluid wave impedance. None of these assumptions are expected to have a large effect on the predicted relative sound levels. This paper assumes an ad hoc weighting function for the distribution of incident sound energy whose weighting angle parameter must be chosen by comparison with experiment. Current research is developing



**Figure 23**. The sound level at 30° relative to that at 0° as a function of Strouhal number for an unbaffled duct end opening.



Strouhal Number

**Figure 24**. The sound level at 60° relative to that at 0° as a function of Strouhal number for an unbaffled duct end opening.

a physical model for the weighting function. This paper also approximates the wave impedance of a finite panel with that of an infinite panel. It also models the transmission from a room or duct to the outside while some of the experimental results are in the opposite direction.

# CONCLUSION

The theoretical model presented in this paper can be used to successfully predict the sound level radiated at a particular angle to the normal of a panel or opening, relative to the sound level radiated in the direction of the normal, if a suitable value of weighting angle parameter can be determined. The weighting angle parameters used in the comparisons between theory and experiment presented in this paper can be used as a guide in the selection of a suitable weighting angle parameter for a particular situation. These weighting angle parameters will also act as a valuable guide in the development of physical models for the angular distribution of sound incidence in particular situations. The theory depends on the length of the radiating object in the direction of the observer divided by the wavelength of the sound in air, and is independent of the width of the object at right angles to the direction of the observer. The relative sound level radiated from a panel is relatively independent of the Strouhal number and the angle of radiation apart from a strong peak at coincidence. The relative sound level radiated from an opening decreases as the both the Strouhal number and the angle of radiation increase.





## REFERENCES

- Bies, D.A. and Hansen, C.H., 1996, Engineering Noise Control: Theory and Practice, Second Edition, E. & F.N. Spon, London.
- Cremer, L., and Heckl, M., 1973, Structure-Borne Sound, Springer-Verlag Company, New York.
- Davy, J.L., 2004, The radiation efficiency of finite size flat panels, Acoustics 2004, Transportation Noise and Vibration – The New Millennium, Proceedings of the Annual Conference of the Australian Acoustical Society, Gold Coast, Australia, 3-5 November 2004, editors Mee, M.J., Hooker, R.J. and Hillock, I.D.M., pages 555-560, Book ISBN 909882-21-5, CDROM ISBN 0-909882-22-3 ISSN 1446-0998, publisher Australian Acoustical Society, Castlemaine, Victoria, Australia.
- Levine, H. and Schwinger, J., 1948, On the radiation of sound from an unflanged circular pipe, Physical Review 73(4): 383-406.
- Rindel, J.H., 1975, Transmission of traffic noise through windows – Influence of incident angle on sound insulation in theory and experiment, Report No. 9, The Acoustics Laboratory, Technical University of Denmark, Lyngby, Denmark.
- Roberts, J., 1983, The prediction of directional sound fields, Transactions of the Institution of Engineers, Australia, Mechanical Engineering, Paper M1153, ME8(1):16-22, ISSN 0727-7369.
- Stead, M., 2001, Sound Reduction for Reverberant, Direct and Diffracted Sound through Single Isotropic Glass Panels of Finite Size, Master of Engineering Science Thesis, Department of Mechanical Engineering, Monash University, Melbourne, Australia.