Consideration of the power flow re-direction for creating a zone of quiet (vibration) in structures

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ABSTRACT

The work is aimed to investigate the method to create a zone of quiet (vibration) on a structure by modifying its vibration power flow. Since the vibration behaviour can be related to how the vibration power flows, it is possible to control the vibration profile by forcing the power to flow to a certain direction over the structure. Therefore, it should also be possible to vary the location for the zone of quiet by re-directing the power flow. Numerical simulations on a panel structure are performed to investigate the effect of re-directing the power flow to change the structural vibration behaviour, where it is found that it is possible to achieve a zone of quiet by power flow modification.

INTRODUCTION

There has been numerous works that looks into the development of control strategies for structural vibration control such as direct velocity feedback control (Balas 1979), independent modal space control (Meirovitch and Baruh 1982), and the more recent optimal and robust control. These particular control methods attempt to control vibration by controlling the structural vibration modes, so they can be categorised into the modal control method (Tanaka 1996).

Alternatively, other methods consider the structural vibration control from the aspect of wave/power point of view such as the work in (Tanaka 1996, Tanaka et al. 1994, Tanaka and Sakano 2006). In particular, the work by Tanaka and Sakano (2006) considers the use of power flow control to create a zone of quiet on a structure. The control method works by suppressing the vibration intensity at a particular structural location so that a zone of quiet can be created. However, the method assumes a fixed location for the zone of quiet based on a particular fixed arrangement of sensors.

The question is whether there is a more flexible control method so that the location of zone of quiet can be changed if desired, without employing additional sensors. The work in this paper attempts to address this question by investigating an alternative control method via the modification (re-direction) of the vibration power flow. The strategy proposed in this paper is in contrast to the work by Tanaka and Sakano (2006) that attempts to completely suppress the vibration power flow by the vibration intensity suppression. Thus, the aim of the work is to look at the possibility of achieving vibration control at a particular structural region (a zone of quiet) by re-directing the power flow. The potential implication of the control method is that the location of zone of quiet can also be changed by modifying the power flow's direction to a desired direction.

DETERMINATION OF THE REAL VIBRATION POWER FLOW

The vibration power flow can be related to the rate of change of structural vibration energy, which can be determined from the product of the structural internal forces and the velocity parameters (Noiseux 1970). The following description will briefly discuss how the power flow can be calculated. Consider a flexible panel-type structure, the vibration intensity at a particular structural location *(x,y)* can be described by considering the internal forces acting at that point in the directions of x and y, i.e. $\boldsymbol{f}_x, \boldsymbol{f}_y$:

$$\boldsymbol{f}_x = \begin{bmatrix} Q_x & M_x & T_{xy} \end{bmatrix}^T \qquad \boldsymbol{f}_y = \begin{bmatrix} Q_y & M_y & T_{xy} \end{bmatrix}^T \tag{1}$$

where Q, M, T are the vertical shear forces, bending moments and torsional moments per unit length respectively, with respect to x or y directions. Note that x and y coordinates are taken as two orthogonal coordinates along the surface of a panel, which may be curved. These internal forces correspond to internal stresses acting within the structure.

The vibration intensity with respect to x and y directions, I_{vx}, I_{vy}, can be obtained from multiplying the internal forces with the corresponding vibration velocity terms, and taking the real component of the power term (Noiseux 1970, Tanaka 1996):

$$\begin{aligned} I_{vx}(r) &= -\frac{1}{2}\mathrm{Re}\left\{\boldsymbol{f}_x^{\,H}\boldsymbol{v}_x\right\} \\ I_{vy}(r) &= -\frac{1}{2}\mathrm{Re}\left\{\boldsymbol{f}_y^{\,H}\boldsymbol{v}_y\right\} \\ \boldsymbol{v}_x &= \begin{bmatrix} \dot{w}_x & \dot{\theta}_x & \dot{\theta}_y \end{bmatrix}^T \\ \boldsymbol{v}_y &= \begin{bmatrix} \dot{w}_y & \dot{\theta}_y & \dot{\theta}_x \end{bmatrix}^T \end{aligned} \tag{2}$$

where $\dot{w}, \dot{\theta}$ respectively represent the normal velocity and angular velocity with respect to x or y directions. The vibration intensity indicates the direction of the vibration power flow at any given point over the structure.

Since the power flow can be related to the way the vibration energy is distributed, it is thus interesting to investigate the potential of modifying this vibration intensity parameter for active vibration control purposes. If the vibration intensity parameter (including its direction) at a number of structural locations can be controlled by active means, the vibration power flow can be modified so to change the vibration behaviour of the structure. Consequently, it should be possible to modify the structural vibration to achieve vibration reduction at a particular structural region (a zone of quiet). What is also beneficial is that by changing the direction of the power flow, the zone of quiet can potentially be moved depending on requirements.

In this work, the investigation will concentrate on the power flow control of a panel structure. The flexural vibration of the panel w is of interest here, whose dynamics can be described by the following partial differential equation:

$$D\nabla^4 w(x,y,t) + \rho h\ddot{w}(x,y,t) = \sum_{i=1}^{K} \delta(x\text{-}x_i)\delta(y\text{-}y_i)f_i(t)$$
$$D = \frac{Eh^3}{12(1-v^2)} \tag{3}$$

where $\rho, h, v, E, K, \delta, f$ respectively denote the panel density, thickness, Poisson ratio, Young's modulus, the number of point forces used, the delta function and the applied external forces.

The internal forces of the panel can be obtained from the partial spatial derivatives of the normal/vertical vibration amplitude, w as in the followings:

$$\begin{aligned}
Q_x &= -D\frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \\
Q_y &= -D\frac{\partial}{\partial y}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \\
M_x &= -D\left(\frac{\partial^2 w}{\partial x^2} + v\frac{\partial^2 w}{\partial y^2}\right) \\
M_y &= -D\left(\frac{\partial^2 w}{\partial y^2} + v\frac{\partial^2 w}{\partial x^2}\right) \\
T_{xy} &= -D(1\text{-}v)\frac{\partial^2 w}{\partial x \partial y}.
\end{aligned} \tag{4}$$

The angular velocities of the panel can be simply obtained from:

$$\theta_x = -\frac{\partial w}{\partial x}$$
$$\theta_y = -\frac{\partial w}{\partial y}. \tag{5}$$

Using Eqs. (1) and (2), the vibration intensity at any *(x,y)* location over the panel can be obtained. Having described the procedure to compute the power flow, the following section will discuss on the optimisation strategy that will be used to modify the power flow.

OPTIMISATION PROBLEM FOR THE MODIFICATION OF STRUCTURAL POWER FLOW

The aim of this optimisation is to modify the strength and direction of the vibration intensity at a number of locations, with the view of modifying the overall power flow for changing the vibration behaviour of the structure. In here, we consider a feed forward control configuration that can be described as:

$$z(x,y) = \mathbf{G}_{zd}(x,y)\mathbf{d} + \mathbf{G}_{zu}(x,y)\mathbf{u} \tag{6}$$

where $z, \mathbf{d}, \mathbf{u}, \mathbf{G}_{zd}, \mathbf{G}_{zu}$ represent the relevant vibration parameters, disturbance inputs, control inputs, and the transfer matrices from the disturbance and control inputs to the vibration amplitude at location *(x,y)*. The vibration parameters such as the transverse and angular vibration parameters required for the power flow computation can be obtained using this approach.

The optimisation task is then defined to find the optimal control inputs so that the desired vibration intensity at several locations can be achieved as close as possible. Suppose the vibration intensity levels are observed at K number of locations:

$$\boldsymbol{I}_v = \begin{bmatrix} I_{v1} & I_{v2} & \cdots & I_{vK} \end{bmatrix}^T$$
$$I_{vk} = I_{vx} + j\, I_{vy} \tag{7}$$

where j is the imaginary number. The desired vibration intensity values at those K locations can be specified by considering a set of vibration intensity reference values at K number of locations:

$$\boldsymbol{I}_r = \begin{bmatrix} I_{r1} & I_{r2} & \cdots & I_{rK} \end{bmatrix}^T$$
$$I_{rk} = I_{rx} + j\, I_{ry}. \tag{8}$$

An optimisation problem can now be set up by finding an optimal control input to minimise the error between the desired/reference vibration intensity vector and the controlled vibration intensity vector:

$$\min_{\boldsymbol{u}} \boldsymbol{\Delta I}^H \boldsymbol{\Delta I}$$
$$\boldsymbol{\Delta I} = \begin{bmatrix} \Delta I_1 & \Delta I_2 & \cdots & \Delta I_K \end{bmatrix}^T$$
$$\Delta I_k = I_{vk} - I_{rk}. \tag{9}$$

Having set up the required optimisation problem, the following section will discuss the numerical investigation into the power flow modification for achieving a zone of quiet in a structure.

NUMERICAL SIMULATION FOR THE POWER FLOW RE-DIRECTION OF A PANEL

Consider a simply-supported steel rectangular panel structure (400mm x 350mm x 2.8mm), where all disturbance and control sources are in the form of point forces. Modal analysis (de Silva 2000) is used to model the simply-supported panel with 12 vibration modes. The damping ratio is assumed to be 0.005 and the eigen function for this panel is:

$$\varphi(r) = \frac{2}{\sqrt{L_x L_y}} \sin\left(\frac{m\pi\pi}{L_x}\right) \sin\left(\frac{n\pi\pi}{L_y}\right) \tag{10}$$

where L_x and L_y are the dimensions of the panel in x and y directions respectively. In addition, m and n are the mode indices in x and y directions respectively. The natural frequency of mode *(m,n)* is:

$$\omega_{mn} = \sqrt{\frac{D}{\rho}} \left\{ \left(\frac{m\pi}{L_x}\right)^2 + \left(\frac{n\pi}{L_y}\right)^2 \right\}. \tag{11}$$

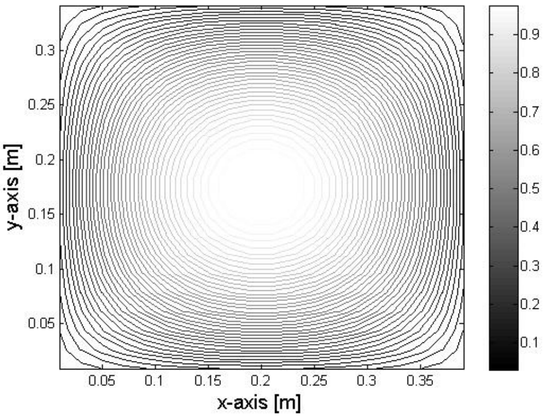

Figure 1. Vibration amplitude (contour) of the panel excited by the disturbance point force at the natural frequency of mode (1,1) at 99.3 Hz.

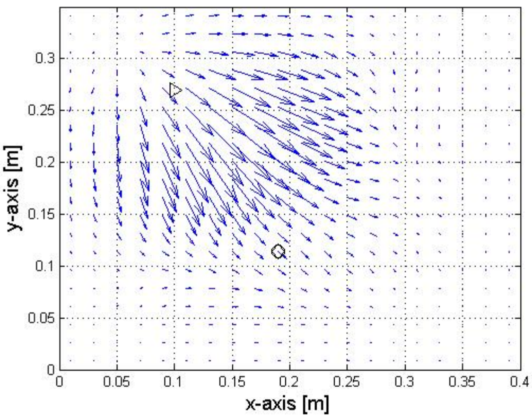

Figure 2. Vibration intensity plots of the panel excited by the disturbance point force at the natural frequency of mode (1,1) at 99.3 Hz. The disturbance and control source locations are denoted by the triangle and the circle respectively.

For this numerical simulation, we concentrate on the first vibration mode (1,1) (frequency of 99.3 Hz) in order to make it easier to investigate the power flow behaviour under the control action. Figure 1 shows a typical contour of vibration amplitude of mode (1,1) for the un-controlled panel. The magnitude of the contour is shown on the bar on the left of the figure, in which the maximum vibration level has been normalised to 1.0. As expected, the vibration peak occurs at the middle of the panel.

Figure 2 shows the vibration intensity plot of the uncontrolled panel, which represents how the vibration power flows over the panel. The disturbance source and control (actuator/sensor) source locations are denoted by a triangle and a circle respectively, where the actuator and sensor are co-located. It is interesting to note that the direction of vibration intensity indicates that the vibration power radiates from the location close to the disturbance point, and flows to the location of the highest vibration amplitude. This power flow behaviour can be expected since the disturbance point can be seen as a 'source' that injects energy into the system (Tanaka, 1996).

In this case, only a single excitation source is considered so relatively simple power flow behaviour is observed. However, when there are more than one excitation sources, the behaviour of the power flow will be more complex since the strength and phase of each excitation source will affect how the vibration power will flow. This is what happens when an additional control source is included into the system.

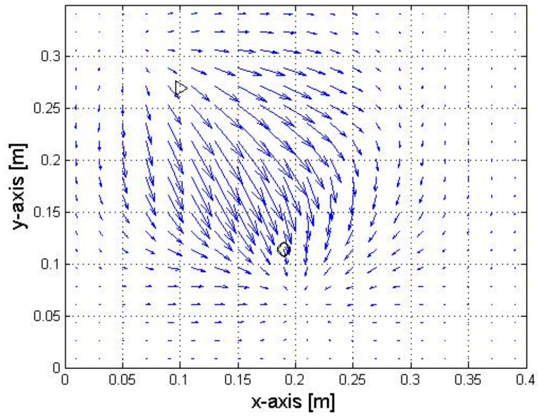

Figure 3. Vibration intensity plot for the controlled plate. Direction of vibration intensity at the actuator location (shown as circle) is -78.5 degrees.

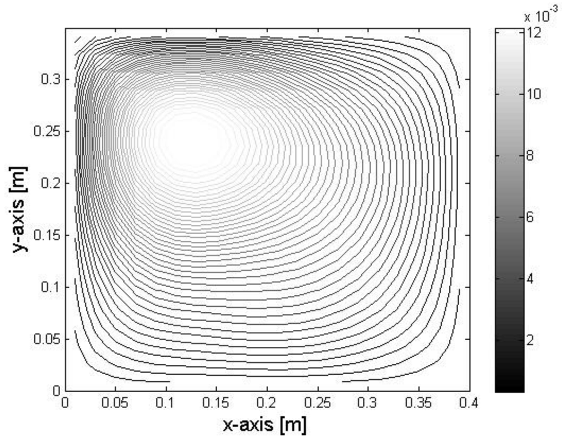

Figure 4. Vibration amplitude (contour) of the controlled plate. Vibration intensity direction is -78.5 degrees.

Figure 3 illustrates the power flow behaviour when an additional excitation source is added in the form of a control source whose control input is obtained from the optimisation. In this study, an optimisation process is performed to direct the vibration intensity at the actuator location to a downward direction. The achieved direction of the vibration intensity at the actuator location is -78.5 degrees, indicating that the power flows downward at this location. In this case, the vibration intensity level at the actuator location has been reduced by a factor of 100 with respect to that for the uncontrolled case. Note that different vector scaling is used for the uncontrolled and controlled cases.

As for the uncontrolled case, the vibration power flows from the location close to the disturbance point towards the centre of the panel where the vibration is at the highest, and then it flows towards the control point. At the control point, the strength of the power has been reduced significantly indicating that the control source has absorbed a significant part of the power flow at this location. This behaviour confirms the observation by Tanaka (1996) who considers that the control source may act as a 'sink'. The consequence of this power flow behaviour is that the region of high vibration moves towards the top left-hand corner of the panel as seen from Figure 4. It should be noted that the overall vibration still being minimised as can be seen by comparing the level of vibration amplitudes in Figures 1 and 4 (see the bars on the left of the figures). The level of overall vibration is also determined by the strength of the control force, relative to that of the disturbance force. However, the important thing that can be observed from this simulation result is that the location of the highest vibration has been moved due to the power flow modification. In this case, the region at the top left-hand-corner of the panel has received less vibration control than region at the bottom right-hand-corner, for example. The results imply that a zone of quiet at the regions away from the top left-hand corner can be achieved using this power flow configuration.

It should also be noted, however, that by changing the strength and phase of the disturbance and control sources/forces, it is possible that a reverse 'sink/source' condition to occur. In other words, the control point may act as a 'source' while the disturbance point may act as a 'sink'. This may happen when the power 'influence' of the control source is greater than that of the disturbance source. The power 'influence' of a source does not solely depend on the strength of the source, but also on the location of the source with respect to the structural vibration profile. For example, for the excitation case at mode (1,1), a source that is located at the middle of the panel will have more power 'influence' than a source with similar strength located away from the centre.

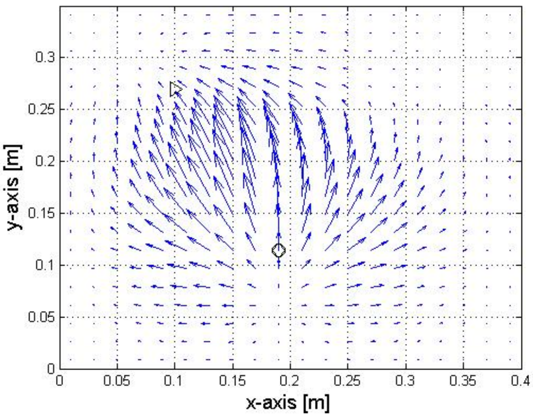

Figure 5. Vibration intensity plot for the controlled plate. The direction of vibration intensity at the actuator location (shown as circle) is 90 degrees.

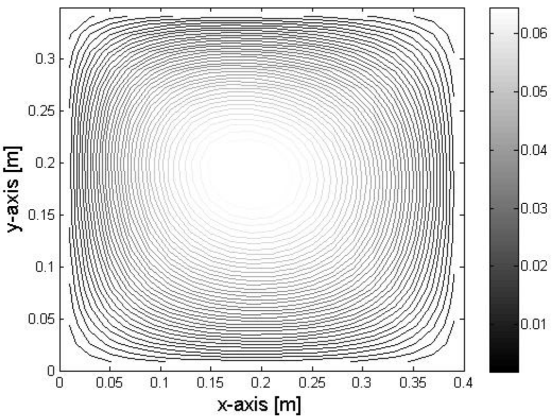

Figure 6. Vibration amplitude (contour) for the controlled plate. The vibration intensity direction is 90 degrees.

Figure 5 shows a different power flow configuration. In this case, the vibration intensity direction at the control point is 90 degrees, forcing the vibration power to flow upwards. As can be seen the power still flows towards the centre of the panel, but now the control point is acting a 'source' while the disturbance point acting as a 'sink'. In this case, the peak of vibration still occurs near the centre of the panel although the overall vibration for the controlled plate is less than that for the uncontrolled case, as shown in Figure 6. From the results, the regions away from the centre received more vibration control than that at the centre, indicating that the vibration profile is affected by the direction of the vibration intensity at the actuator location.

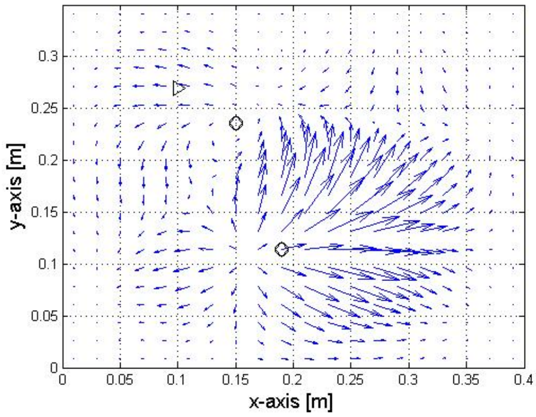

Figure 7. Vibration intensity plot for the controlled plate using two actuators. The locations of 2 actuators are shown as circles.

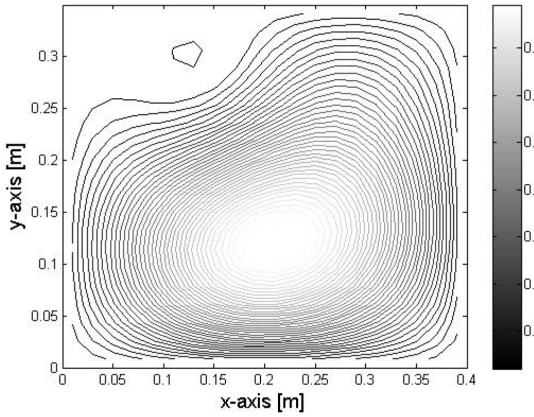

Figure 8.Vibration amplitude (contour) for the controlled plate using two actuators.

The next simulation results investigate the use of more than one control sources for power flow modification. In this case, 2 control sources are used and the vibration intensity level at the location of those control points is monitored and used for the optimisation. Figure 7 shows the vibration intensity plot in which a more complex power flow pattern is now observed. In this case, the strength of the vibration intensity at the top control point has been reduced, while the power flow at the location of the bottom control point is forced to move away from the top left-hand corner of the panel, by re-directing the power flow towards the right-hand side of the panel.

It can be seen that the top control point acts to absorb the power that may be directed towards the top left-hand corner of the panel. As a consequence, only a small amount of power flows toward the top left-hand region, reflecting small vibration amplitude in this region, as confirmed from the vibration amplitude shown in Figure 8. The peak of vibration has now been moved to a lower position as the region of the high vibration intensity level has been moved to this location (see Figure 7). In this case, the zone of quiet has been moved to the top left-hand corner of the panel. The results also indicate that using more actuators and sensors it is possible to create a more defined shape for the zone of quiet.

From the simulation results, it can be observed that the vibration behaviour of a structure can be changed by modifying/re-directing the vibration power flow. The zone of quiet can thus be moved according to how much the vibration power can be 'forced' to flow to a certain direction by the action of control forces. Although the shape of the obtained zone of quiet is still irregular, by re-directing the power flow, the location of the zone of quiet can also be varied by adjusting the strength and direction of the vibration intensity level at a number of locations. As indicated in the previous simulation results, more complex power flow can be achieved using more actuators and sensors, so it is possible to create a zone of quiet with a particular geometrical shape (a circle, for instance) by using a particular geometrical arrangements of structural sensors and actuators.

CONCLUSIONS

The investigation into the feasibility of using the power flow re-direction to create a zone of quiet in structures has been presented. The vibration power flow can be used to indicate the vibration behaviour of a structure, and the direction of the power flow can be modified by the action of control forces to achieve a zone of quiet. Numerical simulations on a panel structure demonstrate the potential of creating a zone of quiet by power flow modification. Although the zone of quiet obtained from the simulations does not have a defined shape, there is a potential for improvement by using more actuators and sensors. Future research will involve an extended investigation into the development of an efficient control method for power flow modification.

ACKNOWLEDGMENTS

Support from the Australian Academy of Science, Japan Society for the Promotion of Science, and Australian Research Council is gratefully acknowledged by the authors.

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