An active control strategy for achieving general cluster control in structural-acoustic systems

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ABSTRACT

The work is aimed to develop a strategy for controlling general multi-objective functions using a cluster control method. A set of multiple quadratic objective functions is described as a set of clusters, in which each cluster represents a particular objective function. The proposed clustering method will thus allow a simple control procedure since only the associated cluster needs to be controlled. This general cluster control method can be used for a variety of vibration and structural-acoustic control, such as for vibration or acoustic control at varying locations yielding a multi-objective control problem. An optimisation procedure is developed to simultaneously determine the locations of sensors used and the clustering parameter. A case study on a structural-acoustic system is provided to demonstrate the effectiveness of the proposed cluster-control method to target the sound power contributed by a particular set of structural modes.

INTRODUCTION

When a structure vibrates in an acoustic medium, it radiates sound, which at times need to be controlled either by passive or active means. Studies on the structural sound radiation and on how to control the sound radiation have been performed intensively, such as the work in (Dimitriadis and Fuller 1992; Clark and Fuller 1991; Elliot and Johnson 1993, Burgan et al. 2002, Fuller et al 1992, Wallace 1972), just to name a few.

Of a particular interest in this work is the implementation of cluster control for structural acoustic control proposed by Tanaka and Snyder (2002). The cluster control can be obtained by clustering/grouping structural modes with the same properties. The cluster control proposed is considered to be a 'middle authority control' that has the stability and control law simplicity of 'low authority control' (such as direct velocity feedback), and with the high control performance similar to 'high authority control' (such as the optimal and robust control) (Tanaka and Snyder 2002). However, this work and other closely related work such as (Elliot and Johnson 1993, Snyder and Tanaka 1993) have generally been in the area of structural-sound radiation control, and not in more general vibration or structural-acoustic areas.

Thus, the work in this paper would look into generalising the cluster control method to deal with a generic quadratic objective function commonly used for vibration and structural-acoustic control purposes. In this case, the locations of collocated sensors/actuators as well as the clustering parameter will be optimised so multiple quadratic objective functions can be represented by multiple clusters. Since each objective function can be represented by a cluster, it would be relatively straightforward to control this particular objective function by controlling the associated cluster.

CLUSTERING METHOD FOR MULTIPLE OBJECTIVE FUNCTIONS

In active control of structural-acoustic systems, one generally needs to define an objective function which describes the performance criteria to be minimised by active control. This objective function may be in the form of the sound power radiated by a vibrating panel structure or the vibration energy of the panel, usually in a quadratic form.

In this work, however, a combination of objective functions, instead of a single objective function commonly used, will be investigated by clustering vibration/acoustic measurements obtained from structural/acoustic sensors used. Each objective function can be clustered separately so each cluster control can target each particular objective function by utilising the same set of sensors. This multi-objective approach will be useful when one deals with minimising a number of objective functions during the control operation, such as to dynamically change the locations of zones of quiet.

The task is to represent each objective function by a number of clusters, whose number depends on how accurate the objective function will be represented by the clusters. For practical purposes, it may be sufficient to use less number of clusters to represent the objective function so less complex controller can be used. Let L to be the number of objective functions of interest, this work consider the l-th objective function that can be estimated by:

$$\mathrm{J}_l = \tilde{\boldsymbol{v}}_l^{\mathrm{H}} \boldsymbol{\alpha}_l(\omega) \tilde{\boldsymbol{v}}_l \tag{1}$$

where $\boldsymbol{\alpha}_l(\omega), \tilde{\boldsymbol{v}}_l$ respectively represent a diagonal positive definite matrix that may be frequency dependent, and a vector consisting a number of cluster amplitudes. Suppose one utilises a set of K sensors whose outputs are contained in vector $\bar{\boldsymbol{v}}$, the cluster vector can be obtained from:

$$\tilde{\boldsymbol{v}}_l = \boldsymbol{T}_l \bar{\boldsymbol{v}} \tag{2}$$

Note that for each objective function, the same set of sensors is used together with the appropriate clustering matrix $\boldsymbol{T}_l$. Also:

$$\tilde{\boldsymbol{v}}_l = \boldsymbol{T}_l \boldsymbol{G}(r) \boldsymbol{v} \tag{3}$$

where r, v, G are the locations of K sensors in the structural-acoustic system, the modal amplitude vector, and the appropriate transfer matrix relating the sensor outputs to the modal amplitudes respectively.

Then, the cluster vector can be expressed by a linear combination of each sensor output $\bar{v}_k$:

$$\tilde{\boldsymbol{v}}_l = \sum_{k=1}^{K} \boldsymbol{t}_{lk} \bar{v}_k$$
$$\boldsymbol{T}_l = \begin{bmatrix} \boldsymbol{t}_{l1} & \boldsymbol{t}_{l2} & \cdots & \boldsymbol{t}_{lK} \end{bmatrix} \quad (4)$$
$$\bar{v}_k = \sum_{j=1}^{N} g_{kj} v_j$$

where g_{kj} is the k-th row and j-th column of matrix $\boldsymbol{G}$. Note that $\boldsymbol{t}_{lk}$ can be a vector depending on how many cluster elements are needed to represent the j-th objective function.

The following expression for the cluster's modal sensing strength can be shown by changing the order of the summations used in Eq. (4):

$$\tilde{\boldsymbol{v}}_l = \sum_{j=1}^{N} \boldsymbol{\beta}_{lkj} v_j$$
$$\boldsymbol{\beta}_{lkj} = \sum_{k=1}^{K} \boldsymbol{t}_{lk} g_{kj}. \quad (5)$$

The expression shows that the cluster vector can be shown to be a linear combination of modal amplitudes. Here, $\boldsymbol{\beta}_{lkj}$ can be seen as the j-th modal sensing strength of the l-th cluster vector due to the k-th sensor output. In the following, it will be shown that a general quadratic objective functions commonly used in structural-acoustic systems can also be estimated by the cluster modal sensing representation.

OBJECTIVE FUNCTIONS FOR STRUCTURAL-ACOUSTIC SYSTEMS

For simplicity of the derivations, consider a set of structural sensors utilised to measure the structural vibration. Note that a combination of structural and acoustic sensors can also be used. Although structural-acoustic systems are spatially distributed systems whose dynamics is governed by partial differential equations (PDEs), in this work, structural modal amplitudes are used to represent the dynamics of such systems. In this case, a modal solution from the relevant PDEs can be obtained using a standard modal analysis approach.

Structural vibration objective functions

Typical quadratic objective function can be represented by:

$$J_s = \boldsymbol{v}^H \boldsymbol{P} \boldsymbol{v} \quad (6)$$

where $J_s, \boldsymbol{P}, \boldsymbol{v}$ are the structural objective function, a (N x N) matrix $\boldsymbol{P}$ and a (N x 1) vector $\boldsymbol{v}$ containing the structural modal displacement/velocity amplitudes. Here, N is the number of structural modes of interest. This type of objective function can be used to represent various structural performance criteria such as strain, velocity, or displacement vibration energy of a structure. In addition, to target a structural zone of quiet can also be achieved using this objective function by considering the vibration at the structural zone of interest. Matrix $\boldsymbol{P}$ can be made to be frequency independent if compatible sensors are used, such as velocity sensors used to represent the velocity energy of a structure.

An eigenvalue decomposition of $\boldsymbol{P}$ can be done so the objective function can be expressed as:

$$J_s = \boldsymbol{v}^H \boldsymbol{R}^T \boldsymbol{\Lambda} \boldsymbol{R} \mathrm{v} = \boldsymbol{y}^H \boldsymbol{y}$$
$$\boldsymbol{y} = \boldsymbol{\Lambda}^{1/2} \boldsymbol{R} \boldsymbol{v} = \boldsymbol{Q} \boldsymbol{v} \quad (7)$$

where $\boldsymbol{R}$ and $\boldsymbol{\Lambda}$ are respectively the matrices containing the eigenvectors and eigenvalues of matrix $\boldsymbol{P}$. Here, $\boldsymbol{y}$ is the vector obtained from the product of matrix $\boldsymbol{Q}$ and modal amplitude vector $\boldsymbol{v}$.

Structural-acoustic objective functions

A quadratic objective function can again be used, for example to represent the sound radiation power from a vibrating panel structure. In this case, the objective function J_a can be related to the structural modal velocities, which are denoted by v:

$$J_a = \boldsymbol{v}^H \boldsymbol{M}(\omega) \boldsymbol{v} \quad (8)$$

Here, $\boldsymbol{M}$ is a (N x N) matrix that varies with frequency ω since the radiated sound pressure would not necessarily in-phase with the structural velocity measurements.

Eigenvalue decomposition on matrix $\boldsymbol{M}$ can be done to achieve the following:

$$J_a = \boldsymbol{v}^H \boldsymbol{Q}^T \boldsymbol{\Lambda} \boldsymbol{Q} \boldsymbol{v} = \boldsymbol{y}^H \boldsymbol{\Lambda} \boldsymbol{y} \quad (9)$$

where $\boldsymbol{Q}$ and $\boldsymbol{\Lambda}$ are respectively the matrices containing the eigenvectors and eigenvalues of matrix $\boldsymbol{M}$.

When the above cost function relates to sound radiation power from a vibrating structure, Elliot and Johnson (1993) proposed that radiation modes that have independent contribution to the sound radiation power. Since the radiation mode shapes, contained in matrix $\boldsymbol{Q}$, do not change significantly at low frequencies, a common approach (Snyder et al 1993, Elliot and Johnson 1993) is to consider a frequency-independent eigenvector matrix $\boldsymbol{Q}$ with frequency-dependent eigenvalue matrix $\boldsymbol{\Lambda}$.

It should be noted that a more general structural-acoustic objective function can also be represented into a quadratic form described in Eqs. (2) and (3), such as the sound radiation at a particular far-field region, or a region within an acoustic enclosure.

Cluster representation for multi-objective functions

One can utilise an estimate of the objective function by taking into account only the contributions from the few largest eigenvalues in $\boldsymbol{\Lambda}$ so the number of cluster elements required would be less than the number of modes considered. Suppose that an estimate of the quadratic objective function uses the first m eigenvalues/eigenvectors, where $m<N$. Let $\boldsymbol{q}$ to be the eigenvector of matrix $\boldsymbol{Q}$ in Eqs. (2) or (4), then a new

vector $\hat{\boldsymbol{y}}$ can be expressed as the contribution of each modal amplitude $\boldsymbol{v}_{\mathrm{j}}$:

$$\hat{\boldsymbol{y}} = \sum_{j=1}^{N} \boldsymbol{q}_{\mathrm{j}} \boldsymbol{v}_{\mathrm{j}} \tag{10}$$

where j denotes the j-th element of vectors $\boldsymbol{y}, \boldsymbol{q}_{\mathrm{i}}, \boldsymbol{v}$. Comparing Eq. (10) to the first equation in Eq. (5), it can be shown that the cluster vector can be used as an estimate of the general quadratic objective functions. Note that an estimate of an objective function can be achieved by taking into account only the contributions from the few largest eigenvalues in $\boldsymbol{\Lambda}$ so the number of cluster elements required would be less. If all the contributions of eigenvalues in $\boldsymbol{\Lambda}$ are considered (m=N), it can be simply shown that $\tilde{\boldsymbol{v}}_l = \hat{\boldsymbol{y}}$.

Having shown that the l-th cluster vector can be used as an estimate of the l-th objective function, the challenging task is how one can find a satisfactory cluster matrix $\boldsymbol{T}$ that can be used to represent not just one objective function, but all L objective functions having relied on the same set of sensor output. This task will be discussed in the following section.

DETERMINATION OF THE CLUSTER MATRIX AND SENSOR LOCATIONS FOR MULTI-OBJECTIVE FUNCTIONS

It is important to note that determining the cluster matrix $\boldsymbol{T}$ for obtaining a good representation of multi-objective functions is only a partial solution of the control strategy. It is equally important to place sensors at strategic locations so an efficient clustering can be achieved. For this purpose, an optimisation needs to be set up to simultaneously determine the locations of sensors and the value of the cluster matrix.

The modal sensing strength for each cluster is represented in Eq. (5). The desired modal sensing strength in fact can be linked to $\boldsymbol{q}_{\mathrm{j}}$ in Eq. (10). Let the vector of the desired j-th modal sensing strength to be $\boldsymbol{\gamma}_{\mathrm{j}}$ that consists of the desired modal sensing strength $\boldsymbol{q}_{\mathrm{j}}$ for each cluster vector. Then an optimisation problem can be set up as follows:

$$\begin{aligned} &\min_{r_{\mathrm{k}}} \boldsymbol{E}^{\mathrm{T}} \boldsymbol{E} \\ &\boldsymbol{E} = \begin{bmatrix} \boldsymbol{E}_1^{\mathrm{T}} & \boldsymbol{E}_2^{\mathrm{T}} & \cdots & \boldsymbol{E}_{\mathrm{N}}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \\ &\boldsymbol{E}_{\mathrm{j}} = \sum_{k=1}^{K} \boldsymbol{t}_{\mathrm{k}} g_{\mathrm{kj}}(r_{\mathrm{k}}) - \boldsymbol{\gamma}_{\mathrm{j}} \end{aligned} \tag{11}$$

where $\boldsymbol{t}_{\mathrm{k}}$ is a vector containing $\boldsymbol{t}_{l\mathrm{k}}$ for each l-th cluster vector. Therefore, the optimisation problem is performed to find the optimal locations of sensors contained in $g_{\mathrm{kj}}(r_k)$ and the cluster matrix $\boldsymbol{T}$ to achieve the desired modal sensing strength for each cluster. Note that since this is a generic optimisation method, the use of different types of structural sensors can also be accommodated.

Reduction of the spillover effects via clustering optimisation

Due to the bandwidth limitation of practical sensors/actuators used, as well as the digital implementation of the control system, it is important that the high frequency spillover effect can be minimised as much as possible. The advantage of the proposed clustering formulation is that the spillover effect can be reduced by forcing the modal sensing strength of higher frequencies to be close to zero. In this case, the impact of higher frequency modes will be negligible, so the effect of spillover can be minimised. In other words, if one wants to reduce the spillover effect due to the modal sensing strength of the last (N-n) modes (where N>n), then:

$$\boldsymbol{\gamma}_{\mathrm{j}} = 0, \; j = n+1, \ldots, N. \tag{12}$$

CLUSTER CONTROL

After the cluster matrix $\boldsymbol{T}$ is obtained with the optimised locations of sensors, the cluster control process can now be completed. The cluster control forces can be done by collocating the actuators and sensors, and the cluster control forces can be simply expressed as (Tanaka and Snyder 2002):

$$\tilde{\boldsymbol{f}} = -\boldsymbol{K}_{\mathrm{c}} \tilde{\boldsymbol{v}} \tag{13}$$

where $\boldsymbol{K}_{\mathrm{c}}$ can be chosen to be a diagonal positive definite matrix, whose diagonal matrix contains the control gain for controlling each cluster vector.

The actual control forces can be obtained by pre-multiplying the cluster control forces with the transpose of the cluster matrix.

$$\boldsymbol{f} = \boldsymbol{T}^{\mathrm{T}} \tilde{\boldsymbol{f}} = -\boldsymbol{T}^{\mathrm{T}} \boldsymbol{K}_{\mathrm{c}} \tilde{\boldsymbol{v}} = -\boldsymbol{T}^{\mathrm{T}} \boldsymbol{K}_{\mathrm{c}} \boldsymbol{T} \boldsymbol{v}. \tag{14}$$

Since $\boldsymbol{T}^{\mathrm{T}} \boldsymbol{K}_{\mathrm{c}} \boldsymbol{T} > 0$, it can be shown that the controlled system is unconditionally stable.

NUMERICAL STUDIES OF CLUSTER CONTROL WITH MULTIPLE OBJECTIVES

In this section, consider a simply-supported steel rectangular panel structure (400mm x 350mm x 2.8mm). Table 1 shows the first 8 natural frequencies of the panel. Point force actuators and velocity sensors are used in this numerical studies.

Table 1. The first 8 natural frequencies of the panel structure used for cluster control

Mode	Frequency[Hz]
(1,1)	99.3
(2,1)	228.4
(1,2)	268.0
(2,2)	397.1
(3,1)	443.7
(1,3)	549.1
(3,2)	612.4
(2,3)	678.3

The first case study of cluster control

Here, the first 8 structural modes are considered and let consider a particular desired modal sensing strength to be:

$$\boldsymbol{\gamma} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}. \quad (15)$$

That is, the first cluster is desired to observed the first 3 modes with relatively equal strength, while the second cluster is desired to observe modes 4-6. To reduce the spillover effect, the sensing strengths of the last 2 modes are forced to be close to zero. It can be shown that to be able to effectively observe all vibration modes of interest, the number of sensors needs to be at least equal to the number of modes. However, in this work, less sensors will be considered although the more sensors used, a better optimisation result can be achieved. Three velocity sensors are considered in this optimisation based on Eq. (11). Due to possible local minima occurring in the optimisation, a number of different initial conditions for the sensor placements are investigated. The optimised locations of the 3 sensors are shown in Figure 1.

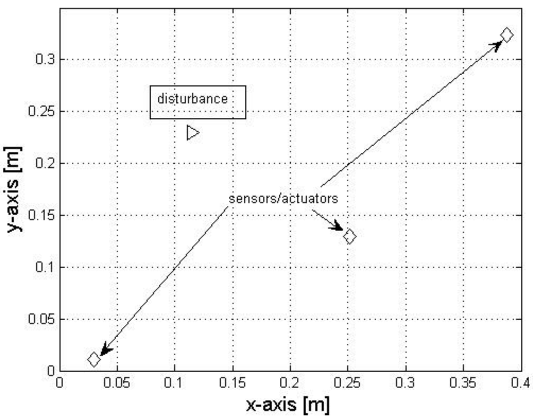

Figure 1. A rectangular plate with locations of sensors/actuators and disturbance.

The optimised results in Figure 2 demonstrate how close is the determination of the modal sensing strength for the first 8 modes. The cluster matrix obtained from the optimisation process is:

$$\boldsymbol{T} = \begin{bmatrix} -2.6449 & -1.1932 & -2.6053 \\ 5.7252 & -0.1064 & 5.6174 \end{bmatrix}. \quad (16)$$

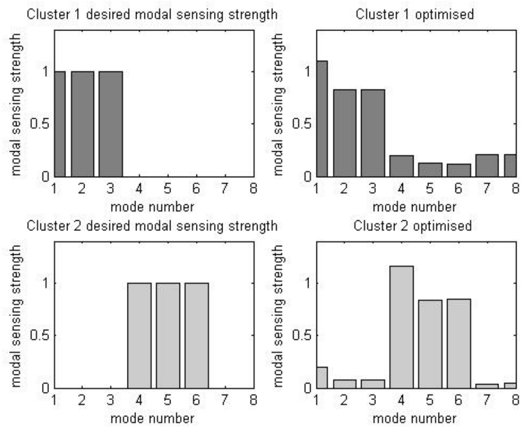

Figure 2. Modal sensing strength for the first 8 modes for cluster I and cluster II.

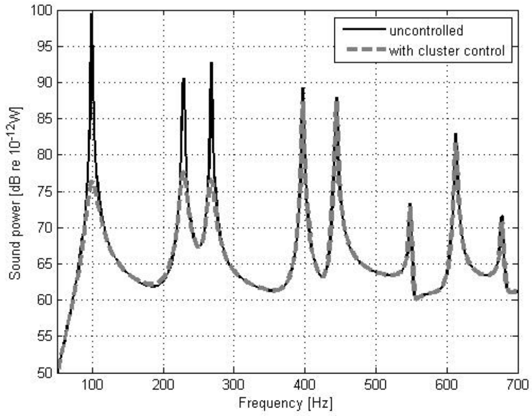

Figure 3. Sound radiation power when cluster 1 is controlled.

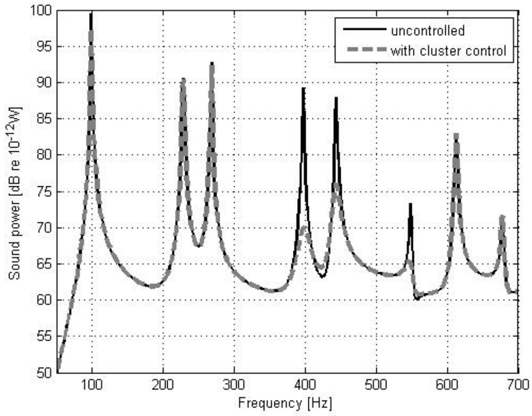

Figure 4. Sound radiation power when cluster 2 is controlled.

Next, the sound radiation power from the panel is considered to investigate the influence of the clustering for active control. The first analysis is done by setting the control gain of the second cluster to zero, thus only controlling the first cluster. The sound power of up to 700 Hz is shown in Figure 3, showing the effect of controlling the first cluster. It can be seen that only the sound power contribution of the first 3 vibration modes have been reduced, while the sound power due to the rest of the modes are left unchanged. The sound power due to the first 3 modes has been reduced by approximately 23, 13 and 16 dB respectively.

When the second cluster is controlled, a completely different sound power result is obtained as shown in Figure 4. The sound power contribution of modes 4, 5, and 6 have been reduced by about 19, 12, 8 dB respectively, while the contribution of the rest of the modes to the sound power stays almost unchanged. These results show the impact of the clustering in selecting the contribution of each mode to a particular objective function.

The first case study of cluster control

Next, let consider the desired modal sensing strength as follows:

$$\boldsymbol{\gamma} = \begin{bmatrix} 1 & 0 & 1 & 1.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0.5 & 1 & 0 & 0 \end{bmatrix}. \quad (17)$$

The obtained optimal cluster matrix is:

$$\boldsymbol{T} = \begin{bmatrix} -7.6157 & 3.0062 & 5.8701 \\ -5.9486 & 1.8497 & 4.2377 \end{bmatrix}. \quad (18)$$

where the locations of 3 sensors/actuators are shown in Figure 5.

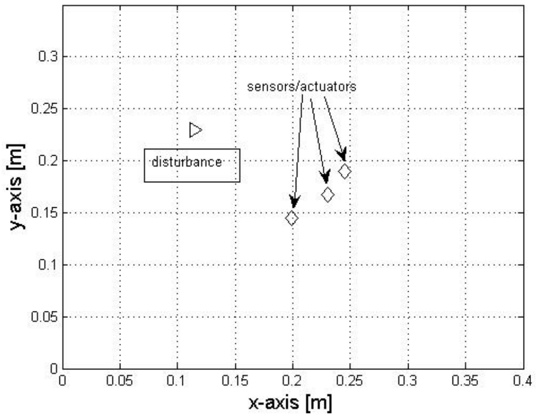

Figure 5. The locations of sensors/actuators, disturbance for the second study.

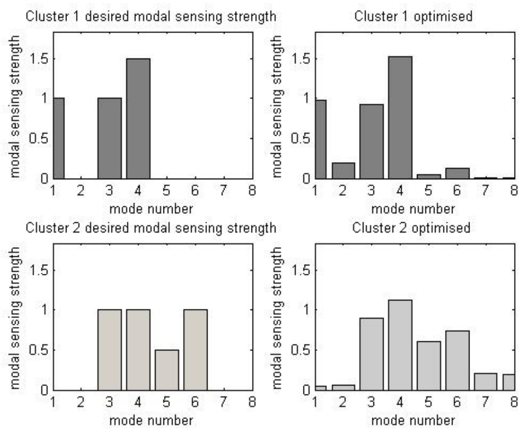

Figure 6. Modal sensing strength for the first 8 modes for cluster I and cluster II.

The obtained modal sensing strength for cluster 1 and cluster 2 is shown in Figure 6. Again, the results show that the optimised cluster matrix ***T*** and the locations of the sensors produce the modal sensing strength that is close to the desired one. In this case study, the strength of each mode is not set as simply 1 or 0, thus reflecting a more general representation of the modal sensing strength that may be needed to express a general quadratic objective function for vibration or structural-acoustic control.

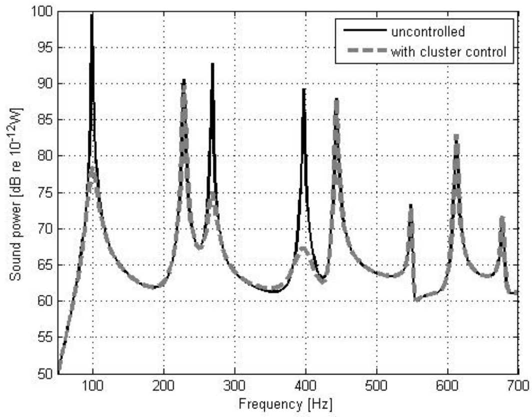

Figure 7. Sound radiation power when cluster 1 is controlled.

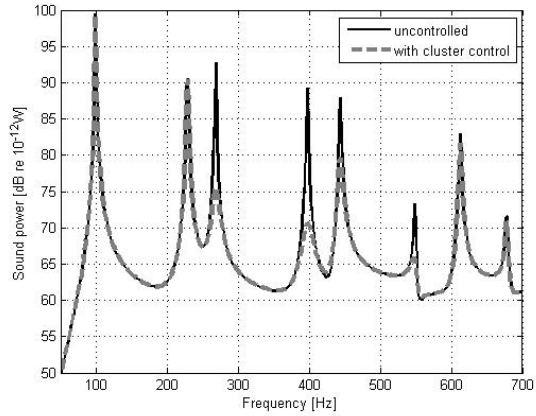

Figure 8. Sound radiation power when cluster 2 is controlled.

Figures 7 and 8 show the sound power results when each single cluster is controlled. When the first cluster is controlled, the sound power contribution of modes 1, 3, 4 has been reduced by approximately 21, 18 and 22 dB respectively. When the second cluster is controlled, the sound power reductions due to modes 3-6 are 18, 18, 8 and 7 dB respectively.

CONCLUSIONS

A method to cluster information from sensors for vibration or structural-acoustic control purposes has been discussed in this paper. The locations of the collocated sensors/actuators and the cluster matrix are optimised so that each cluster can represent a particular objective function, so that multiple objective functions can be easily controlled by considering each appropriate cluster. Case studies on sound radiation control of a panel demonstrated that the cluster control method can be useful for minimising the sound radiation power contributed by a particular set of vibration modes.

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