Control of the spatially-weighted vibration of an arbitrary structure using an adaptive control strategy

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ABSTRACT

The aim of the work is to develop a practical active control strategy in which vibration of an arbitrary structure can be spatially-weighted for achieving a vibration reduction at the structural region of interest. Multiple sensors are used to sense the vibration of the entire arbitrary structure and a filtered-reference Least Mean Square (FX-LMS)-based adaptive control strategy is used to minimise the instantaneous error energy representing the spatially weighted vibration energy of the structure. A numerical study for spatial vibration control of a plate structure is discussed to demonstrate the control effectiveness of the adaptive spatial controller for tonal and broadband cases.

INTRODUCTION

For structural vibration control applications, it may not be efficient to rely on active control methods that heavily depend on an accurate a-priori structural model, since obtaining such a model of a complex actual structure may not be practical. Furthermore, during the control operation, the dynamics of the structure may also change which means that a model-based control may not perform effectively, and in the worst case may cause the system to be unstable. Therefore, it is desirable to develop an active vibration control method that does not rely on the accurate a-priori model, which is usually being implemented with the use of some forms of adaptive algorithms (Haykin 2002; Elliot 2001)

The work in this paper thus considers the use of an adaptive control method for regulating structural vibration when vibration at only certain structural regions needs to be minimised. The control method considers a spatially weighted vibration objective function that utilises multiple structural sensors for estimating the instantaneous spatial vibration profile. Other vibration control research that utilises multiple structural sensors has also been common, such as in (Meirovitch and Baruh 1982; Meirovitch 1987; Pajunen et al. 1994), but the research generally has the objective for controlling the overall structural vibration.

Recent work in (Halim and Cazzolato 2005; 2006) considers the case of employing a continuous spatial weighting function for emphasising the structural regions that need to be controlled. The control method utilises the estimation of the spatial vibration profile via spatial interpolations which does not require a-priori dynamic model of a structure. The work in this paper considers the use of this method by incorporating an adaptive control strategy which can be useful for practical vibration control purposes.

ADAPTIVE SPATIAL CONTROL OF AN ARBITRARY STRUCTURE

Here, the approach used to estimate the vibration profile is briefly described. Consider a panel structure of an arbitrary shape in Figure 1, where there are multiple structural sensors distributed over the panel. Let the vibration signal measured at each sensor at location (x_i,y_i) to be v_i. An element/region (whose local coordinates are $(x^{(m)}, y^{(m)})$ for the m-th element shown in Figure 1) can be constructed from several adjacent sensors or nodes at structural boundaries as illustrated in Figure 1. The vibration profile within each element/region can be obtained via spatial interpolation functions, whose implementation is similar to the one used in numerical finite element analysis (Bathe and Wilson. 1976; Cheung and Leung 1991). The vibration profile of the structure can be estimated by considering the contributions of all elements/regions over the structure (Halim and Cazzolato 2006):

$$v(x, y, t) \approx M(x, y)v(t) \tag{1}$$

where v(t) is the vector containing all the measurements at the structural sensors, and *M(x,y)* contains a spatial interpolation matrix that relates the sensor measurements to the vibration output at any point over the structure, as well as the linear matrix for transforming the local elemental coordinates $(x^{(m)}, y^{(m)})$ to the structural coordinates *(x,y)*.

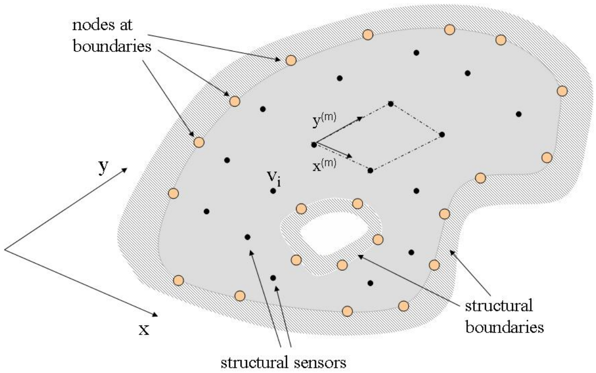

Figure 1. A structure with multiple structural sensors used for vibration profile estimation.

Consider a spatial vibration signal as a function of structural locations at the n-th sample time:

$$v(x, y, n) = v_d(x, y, n) + v_u(x, y, n) \tag{2}$$

where v, v_d, v_u are respectively the total structural vibration, vibration due to disturbance and vibration due to control at various locations *(x,y)* over the structure. The spatial vibration signal over the structure can then be estimated as follows:

$$\begin{aligned} v(x,y,n) &\approx M(x,y)v(n) \\ &= M(x,y)v_d(n) + M(x,y)v_u(n). \end{aligned} \tag{3}$$

The instantaneous spatial vibration is of interest here with the aim of reducing the spatially-weighted vibration energy across the entire structural region. In this case, a spatial weighting matrix, continuous in *(x,y),* can be introduced so that certain structural regions can be emphasised for control. The regions which are more important for control are given relatively high weightings. Introducing a continuous spatial weighting matrix $Q(x,y)>0$, the instantaneous spatial function that needs to be minimised is integrated over the region of the structure S:

$$\begin{aligned} J(n) &= \int_S v(x,y,n)^T Q(x,y) v(x,y,n) dS \\ &\approx (v_d(n)+v_u(n))^T \left[\int_S M(x,y)^T Q(x,y) M dS \right] \\ &\times (v_d(n)+v_u(n)) \\ &= (\Omega(v_d(n)+v_u(n)))^T (\Omega(v_d(n)+v_u(n))) \\ &= e(n)^T e(n) \end{aligned} \tag{4}$$

where

$$\begin{aligned} R &= \int_S M(x,y)^T Q(x,y) M(x,y) dS + \alpha I \\ &= UVU^T \\ \Omega &= V^{1/2} U^T \end{aligned} \tag{5}$$

and α, I are a small positive scalar and an identity matrix respectively with the purpose of ensuring a positive definite matrix R, since the numerical computation on R may produce small negative eigenvalues. Note that in practice, matrix $Q(x,y)$ can be chosen to be a diagonal matrix whose elements consist of continuous spatial weighting functions, which can be obtained by using polynomial functions in x and y.

The instantaneous spatial error signal $e(n)$ now can be expressed as:

$$e(n) = \Omega P(z) d(n) + \Omega S(z) u(n) \tag{7}$$

where $P(z)$ and $S(z)$ are the discrete primary and secondary transfer matrices respectively.

Employing the filtered-reference Least Mean Square adaptive (FX-LMS) algorithm, the filtered-reference signal $r(n)$ is expressed by:

$$r(n) = \Omega S(z) x(n) \tag{8}$$

where $x(n)$ is the reference signal and $S(z)$ is represented as a J-th order FIR (Finite Impulse Response) filter. For simplicity in the formulation, scalar reference and control input signals, $x(n)$ and $u(n)$ are considered in this paper.

The FIR controller, whose coefficients are contained in vector $w(n)$, generates a control input signal $u(n)$:

$$u(n) = w(n)^T x(n). \tag{9}$$

The controller's coefficients are obtained from an LMS adaptive optimisation of the instantaneous spatial cost function $J(n)$:

$$\begin{aligned} e(n) &= \Omega P(z) d(n) + r(n) w(n) \\ \frac{\partial J(n)}{\partial w(n)} &= \frac{\partial \left(\int_S v(x,y,n)^T Q(x,y) v(x,y,n) dS \right)}{\partial w(n)} \\ &= \frac{\partial e(n)^T e(n)}{\partial w(n)} = 2r(n)^T e(n). \end{aligned} \tag{10}$$

Thus, by employing the gradient previously obtained, the LMS adaptation becomes:

$$\begin{aligned} w(n+1) &= \gamma w(n) - \mu r(n)^T e(n) \\ \gamma &= 1 - \mu\beta \end{aligned} \tag{11}$$

where a leaky adaptation algorithm has been used to increase the stability of the adaptation (Elliot 2001). Here, μ, β are the convergence coefficient and the leakage factor respectively.

The implementation diagram of FX-LMS adaptive feed forwad spatial control is illustrated in Figure 2. For a feedback control option, the reference signal can be modified so that the disturbance signal can be estimated (Elliot 2001).

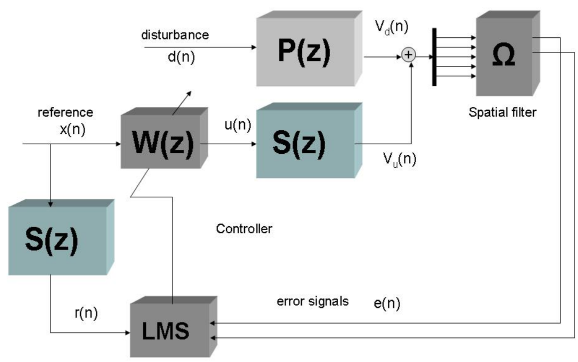

Figure 2. FX-LMS adaptive spatial control diagram.

NUMERICAL STUDIES: ADAPTIVE SPATIAL CONTROL OF A PANEL STRUCTURE

The developed FX-LMS adaptive spatial control is now implemented on a simply-supported rectangular steel panel with 4 x 4 structural velocity sensors distributed over the panel as shown in Figure 4. The dimensions of the panel are 400mm x 350mm x 2.8mm and point sources are used as disturbance and control sources as illustrated in Figure 3. Matlab and Simulink are used for this numerical simulation using a 15-taps FIR filter for the controller. The partial differential equation that governs the dynamics of the panel is:

$$\begin{aligned} D\nabla^4 v(x,y,t) + \rho h \ddot{v}(x,y,t) &= f(t) \\ D &= \frac{Eh^3}{12(1-\nu^2)} \end{aligned} \tag{12}$$

where ρ, h, ν, E, f denote the panel density, thickness, Poisson ratio, Young's modulus, and the applied external forces respectively.

The model of the panel is obtained using the modal analysis (de Silva, 2000) by including the first 12 vibration modes for frequencies up to 940 Hz. The natural frequencies of the first 7 vibration modes are shown in Table 1. The interpolation function used is a linear function as described by Halim and Cazzolato (2006). For these numerical studies, only the two largest eigenvalues of matrix R in Eq. (5) are used, which means that the error signals are reduced from potentially 16 to just 2 signals. The less number of error signals would obviously simplify the adaptive control process which is important for practical active control applications.

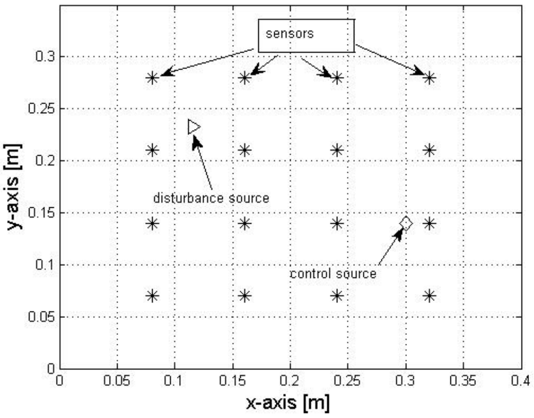

Figure 3. A rectangular plate with locations of sensors, disturbance and control sources.

Table 1. The first 7 natural frequencies of the panel

Mode	Frequency [Hz]
(1,1)	99.3
(2,1)	228.4
(1,2)	268.0
(2,2)	397.1
(3,1)	443.7
(1,3)	549.1
(3,2)	612.4

Adaptive spatial control results for spatial weighting I

The first numerical study considers a particular spatial weighting function *Q(x,y)* shown in Figure 4. The height of the weighting reflects the importance of the region for vibration minimisation objective. In this case, the region of interest for vibration control is located close to the top right-hand corner of the panel. Figure 5 shows the typical eigenvalues for matrix R, in which only a few eigenvalues dominate. In this study, as mentioned previously, the first 2 largest eigenvalues are used and the rest of eigenvalues are ignored.

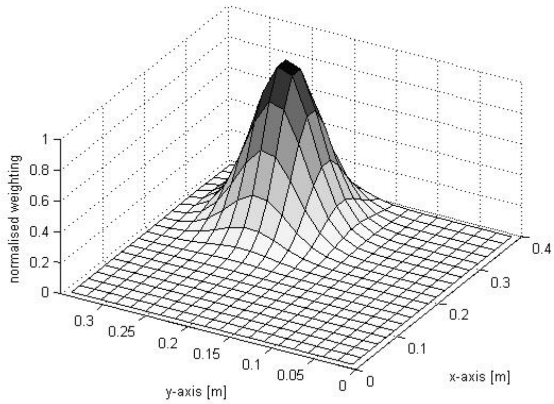

Figure 4. Spatial weighting function I.

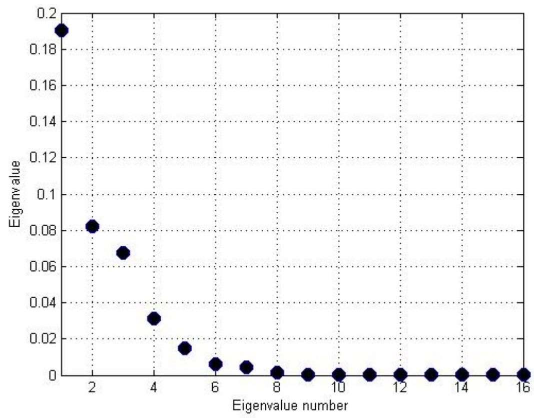

Figure 5. Eigenvalue plots of R.

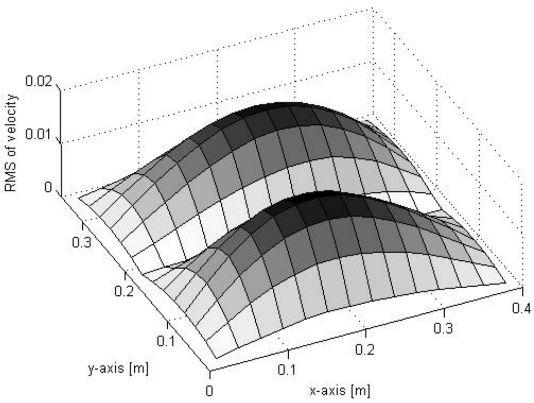

Figure 6. RMS vibration level over the entire panel without control: tonal case for mode (1,2).

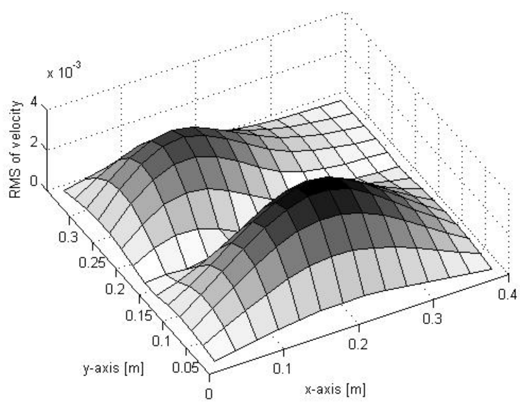

Figure 7. RMS vibration level over the panel with adaptive control for the first spatial weighting : tonal case for mode (1,2).

The results for tonal control for mode (1,2) at 268.0 Hz are shown in Figures 6 and 7. The results show the Root Mean Squared (RMS) vibration velocities across the panel when a tonal excitation disturbance is injected into the system. It can be seen that by comparing the results for un-controlled and controlled cases, the vibration at the structural region with the highest weighting at the top right-hand corner of the panel has been reduced more than that at other regions although the overall vibration has also been minimised.

The overall vibration can also be minimised because the adaptive controller attempts to control vibration modes that have a significant contribution to the region of interest, which means that vibration at other regions may also be reduced. However, it is also possible that the vibration level at other regions is increased, particularly if large control gain is allowed since the controller will force the vibration at the region of interest to be lower, even at the expense of vibration at other regions.

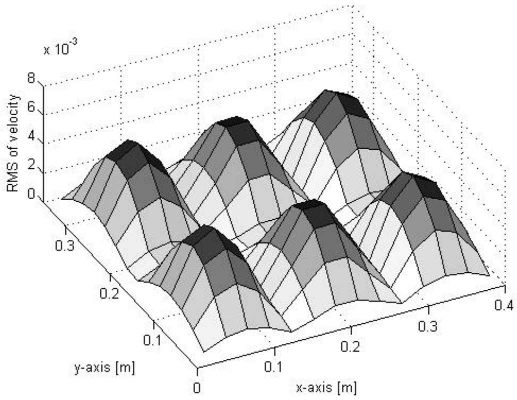

Figure 8. RMS vibration level over the entire panel without control: tonal case for mode (3,2).

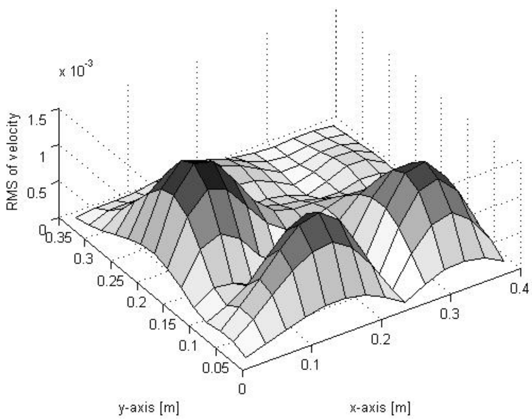

Figure 9. RMS vibration level over the panel with adaptive control for the first spatial weighting : tonal case for mode (3,2).

Figures 8 and 9 depict the results for the tonal control of mode (3,2) at 612.4 Hz. The region at the top right-hand corner of the panel has again been minimised as expected.

The results for broadband excitation are considered in the followings. A white noise disturbance input, low-pass filtered with the cut-off frequency of 800 Hz, is injected into the system, where the first 8 vibration modes are inside this frequency bandwidth. The RMS of the vibration velocities over the panel is shown in Figures 10 and 11 for the un-controlled case and controlled case respectively. The overall RMS across the panel has been reduced by the action of the adaptive spatial control, with a particular reduction occurred in the region around the centre of the plot (i.e. observe the concave shape of the RMS plot). The result can be expected based on the spatial weighting used for this study. Note that this broadband result implies that the adaptive controller attempts to control the first 8 vibration modes simultaneously to achieve minimum vibration energy at the region of interest.

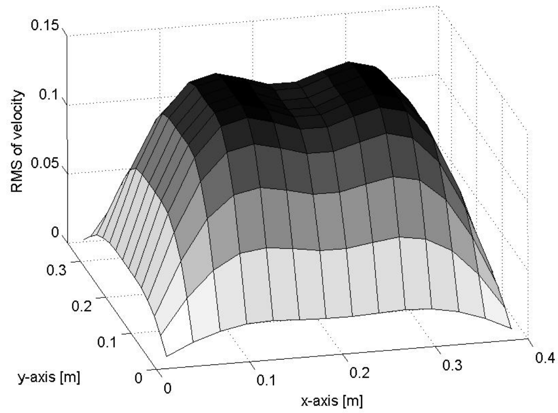

Figure 10. RMS vibration level over the entire panel without control: broadband case.

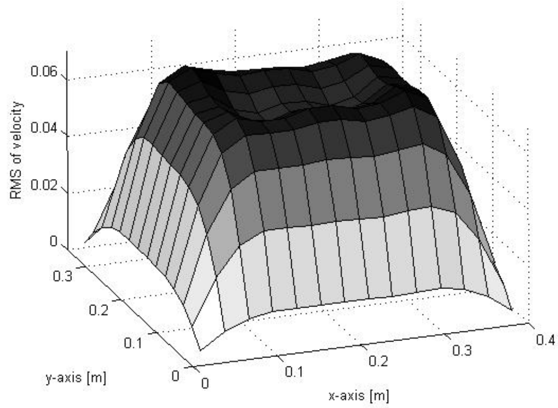

Figure 11. RMS vibration level over the panel with adaptive control for the first spatial weighting: broadband case.

Adaptive spatial control results for spatial weighting II

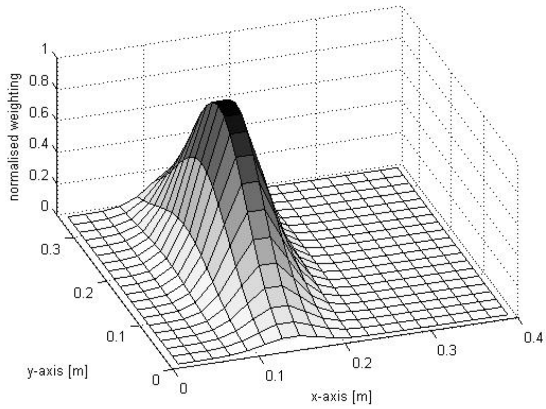

Figure 12. Spatial weighting function II.

The second spatial weighting function is shown in Figure 12 where the region of interest has been moved to the left-hand side of the panel, with a different geometrical shape. The region of the highest weighting is shifted closer to the top end of the panel. The tonal control results for mode (2,2) at 397.1 Hz are shown in Figures 13 and 14 where the region of minimal vibration occurs at the left hand side of the panel that corresponds to the region with the high weighting. The region at the top end of the panel also experiences a significant vibration minimisation.

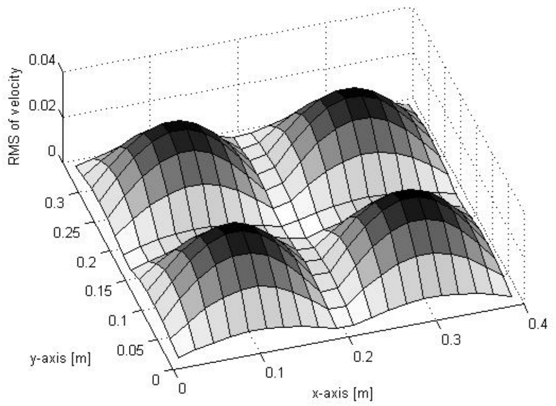

Figure 13. RMS vibration level over the panel without control : tonal case for mode (2,2).

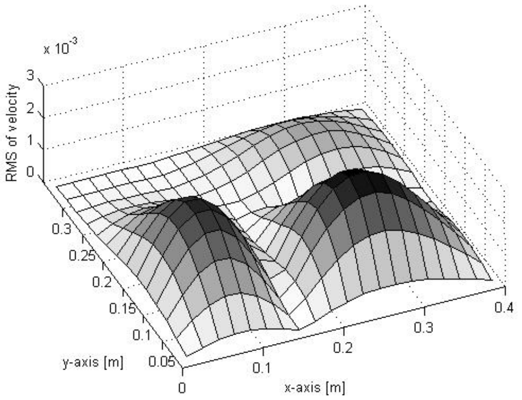

Figure 14. RMS vibration level over the panel with adaptive control for the second spatial weighting: tonal case for mode (2,2).

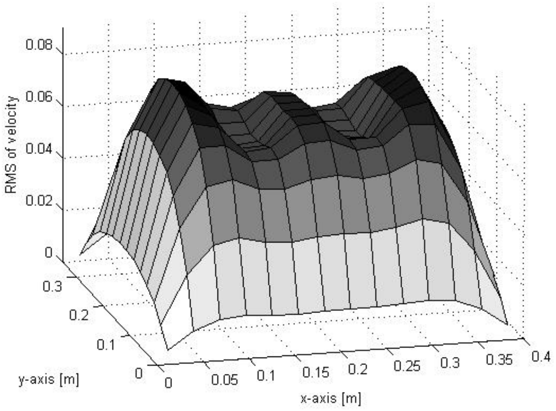

Figure 15. RMS vibration level over the panel with adaptive control for the second spatial weighting: broadband case.

Figure 15 shows the broadband control result where the adaptive spatial controller attempts to minimise the vibration level at the region of interest. Note that the symmetry in the results occurs because of the symmetry in the mode shapes for this simply-supported panel. The simulation results have demonstrated the control performance in controlling the spatially-weighted vibration. The performance depends on the continuous spatial weighting used which can be varied depending on the desired structural regions that need to be controlled.

CONCLUSIONS

The implementation of an adaptive control strategy for spatially controlling structural vibration has been presented. It is shown that by utilising the developed spatial filter, the instantaneous spatially-weighted vibration energy can be achieved and an FX-LMS adaptive control algorithm can be used effectively for tonal and broadband control cases. Since the control method does not rely on the dynamic model of the structure, it can be used for practical applications in conjunction with the adaptive control strategy presented here.

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