Effect of reflecting surfaces on the performance of active noise control

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ABSTRACT

This paper deals with the evaluation of power radiated by two monopoles located near a reflective surface, especially in the case when one monopole-strength is actively controlled for the minimization of total radiated power. The radiated power and the optimal input for the controlled monopole are derived analytically. It is found that the power output of an active noise control system consisting of two point sources and located near a reflective surface is dependent upon the system orientation angle. Based on the mechanism that the reflecting surface can convert a dipole vertical to the surface into a longitudinal quadrupole, we demonstrate that the introduction of a reflecting surface to the active noise control system could further enhance the total power reduction. Experimental results are also included to demonstrate that the effect of a reflecting surface on the sound radiation of a dipole is still significant even though the size of the surface is much smaller than the wavelength of the sound radiation.

INTRODUCTION

The power output of a sound source is affected by the surface near it. If a point source with constant volume velocity q_a is located at a distance $d/2$ from a rigid surface, the radiated sound power is expressed as

$$W_1 = W_m(1 + \frac{\sin kd}{kd}) \tag{1}$$

where $W_m = \frac{\rho_o c_o k^2}{8\pi} q_a^2$ is the power radiated by the point source in the free field and k is the wavenumber. Due to the reflection of the rigid surface, the point source actually faces an impedance $\mathrm{Re}\{Z_1\} = \frac{\rho_o c_o k^2}{4\pi}(1 + \frac{\sin kd}{kd})$ which is different from that ($\mathrm{Re}\{Z_m\} = \frac{\rho_o c_o k^2}{4\pi}$) in the free field by a factor of $(1 + \frac{\sin kd}{kd})$, so that the radiated sound power also differs by this factor. The superposition of direct and reflected fields also differs from the free field sound pressure from the same source by a factor of $2\cos(\frac{kd}{2}\sin\varphi)$ in the far field, where φ is a directional angle between the field position vector and that normal to the surface. Integrating the sound intensity in the far field over a semi-hemisphere also gives rise to the radiated sound power. The portion of the integration associated with the factor $\int_0^{\pi/2} 4\cos^2(\frac{kd}{2}\sin\varphi)\cos\varphi d\varphi$ contributes to the term $(1 + \frac{\sin kd}{kd})$ in Equation (1).

Ingard and Lamb (1957) considered a monopole, vertical/horizontal dipoles and vertical/ longitudinal quadrupoles affected by a rigid reflecting surface of infinite size. Bies (1961) generalized the consideration of source configurations of Ingard and Lamb by including any orientation of either a dipole or any kind of quadrupole. Their analytical expressions of total power were obtained by integrating the approximate expression of far field sound intensity and assuming that source size (eg. the source size of a dipole is described by the dipole distance l as shown in Figure 1) is much less than the source height $d/2$.

In this paper, we use the energy-carrying sound pressure (the part in phase with the volume velocity of the source) and the volume velocities at the source locations for the sound power calculation. As a result, the effect of dipole distance is included in the expression of the total power and the assumption of $l << d/2$ is relaxed. Thus, the expressions of Ingard and Lamb, and Bies become limiting cases when the dipole distance is much smaller than the source height. Using the analytical expression for the total power, a significant change of the sound radiation mechanism from the source and its image is illustrated as the orientation of the dipole with respect to the surface varies. Typically, the vertical dipole and its image form a longitudinal quadrupole that radiates much less sound power than the dipole does in the free field. On the other hand the horizontal dipole and its image just form two parallel dipoles with a power radiation greater than that of the same dipole in the free field. Thus, the understanding of the change in sound radiation mechanism indicates a possibility of improving the performance of an active control system by suitably placing a reflecting surface near it. This possibility is confirmed by the derived analytical expressions of optimal source strength for the controlled monopole, minimum value of total power and simulation results. Evidence of using a finite reflecting surface (less than the wavelength) to enhance the performance of an active control system is also provided.

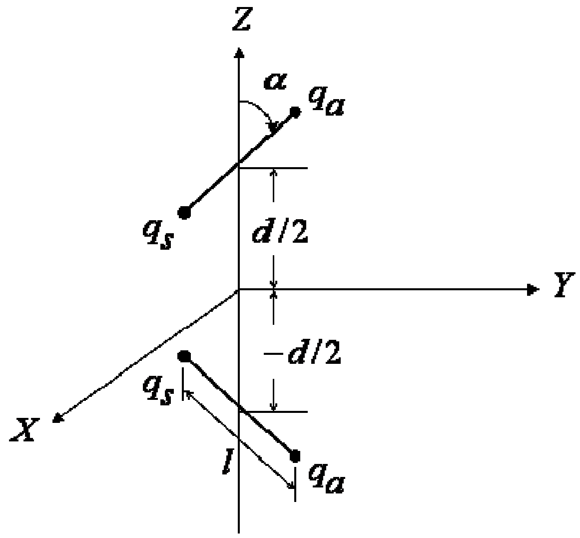

Figure 1. The active control system consisting of two point sources and a rigid surface. When $\alpha = 0^\circ$ and $q_s = -q_a$, it represents a vertical dipole above the rigid surface, while $\alpha = 90^\circ$ and $q_s = -q_a$ corresponds to a horizontal dipole above the surface.

POWER OF A DIPOLE NEAR A RELECTING SURFACE

We first consider two monopoles located near a reflecting surface of infinite size as shown in Figure 1. When the source strengths satisfy $q_s = -q_a$, and the distance, l, between them is much less than the wavelength, these two monopoles are described as a dipole. For this case, α in Figure 1 represents the orientation angle of the dipole axis with respect to the normal direction of the surface. l is defined as the dipole distance, and d the distance between the dipole and its mirror image ($d \ge l$). Using the sound pressure and volume velocity at each monopole and its image, the total power radiated from an arbitrarily oriented dipole near a rigid surface is obtained as

$$W_2 = 2W_m[1 - \frac{\sin kl}{kl}] - 2W_m[\frac{\sin k\sqrt{d^2 + (l\sin\alpha)^2}}{k\sqrt{d^2 + (l\sin\alpha)^2}} - \frac{\sin k(d + l\cos\alpha)}{2k(d + l\cos\alpha)} - \frac{\sin k(d - l\cos\alpha)}{2k(d - l\cos\alpha)}] \quad (2)$$

When $\alpha = 0^\circ$, Equation (2) gives rise to the power of the vertical dipole near a rigid surface:

$$W_{2V} = 2W_m[1 - \frac{\sin kl}{kl}] - 2W_m[\frac{\sin kd}{kd} - \frac{1}{2}(\frac{\sin k(d-l)}{k(d-l)} + \frac{\sin k(d+l)}{k(d+l)})] . \quad (3)$$

It is clear from Figure 1 that the vertical dipole and its image form a longitudinal quadrupole. Indeed by using the compact source assumption $k(d + l) < 1$ and by selecting $d = 2l$, the sound power in Equation (3) is approximated as:

$$\frac{W_{2V}}{W_m} \approx \frac{2}{5}(kl)^4 \quad (4)$$

which is the typical power ratio between the free-field quadrupole and monopole.

When $\alpha = 90^\circ$, Equation (2) describes the sound power due to a horizontal dipole:

$$W_{2H} = 2W_m[1 - \frac{\sin kl}{kl}] + 2W_m[\frac{\sin kd}{kd} - \frac{\sin k\sqrt{d^2 + l^2}}{k\sqrt{d^2 + l^2}}] . \quad (5)$$

The properties of the sinc function suggest that $[\frac{\sin kd}{kd} - \frac{\sin k\sqrt{d^2 + l^2}}{k\sqrt{d^2 + l^2}}] > 0$ when $k\sqrt{d^2 + l^2}$ is small, which indicates the possibility of the sound power radiated from a horizontal dipole near a reflecting surface being greater than that from a free-field dipole radiator ($2W_m[1 - \frac{\sin kl}{kl}]$). For example, if $d = l$ and $k(d + l) < 1$ Equation (5) is approximated as

$$\frac{W_{2H}}{W_m} \approx \frac{2}{3}(kl)^2 . \quad (6)$$

This ratio is twice the power ratio between a free-field dipole and monopole.

The above observation confirms that the effect of a rigid surface on the sound power radiation from dipole sources is dependent upon the dipole orientation angle. In particular, the vertical dipole and its image form a longitudinal quadrupole that radiates much less sound power than the dipole does in the free field. On the other hand the horizontal dipole and its image form two parallel dipoles with a power radiation greater than that of the same dipole in the free field.

The above discussion also motivated a study of the effect of a rigid surface on the performance of an active noise control system located near the surface. Since the rigid surface is capable of converting a dipole into a longitudinal quadrupole, this property might be used to further improve the performance of an active control system.

DESCRIPTION OF THE ACTIVE CONTROL SYSTEM

We consider an active control system consisting of two point sources and a rigid surface, shown in Figure 1. The volume velocity of the primary source is defined as q_a and that of the secondary source is q_s.

The total sound power is used to describe the performance of the active control system. It is expressed as a standard quadratic function of q_s:

$$W_3 = q_s^* A q_s + q_s^* B + B^* q_s + C \quad (7)$$

where:

$$A = \frac{W_m}{q_a^2}[1 + \frac{\sin k(d - l\cos\alpha)}{k(d - l\cos\alpha)}],$$

$$B = \frac{W_m}{q_a^2}(\frac{\sin kl}{kl} + \frac{\sin k\sqrt{d^2 + (l\sin\alpha)^2}}{k\sqrt{d^2 + (l\sin\alpha)^2}})q_a;$$

and:

$$C = W_m[1 + \frac{\sin k(d + l\cos\alpha)}{k(d + l\cos\alpha)}]$$

is the uncontrolled power of the primary monopole source $d/2$ away from the rigid surface. The solution of Equation (7) for the optimal secondary source strength and the corresponding minimum sound power radiation are respectively:

$$q_{so} = -\frac{B}{A} = -\frac{(\frac{\sin kl}{kl} + \frac{\sin k\sqrt{d^2 + (l\sin\alpha)^2}}{k\sqrt{d^2 + (l\sin\alpha)^2}})}{[1 + \frac{\sin k(d - l\cos\alpha)}{k(d - l\cos\alpha)}]} q_a \quad (8)$$

and:

$$W_{3\min} = W_m[1 + \frac{\sin k(d + l\cos\alpha)}{k(d + l\cos\alpha)}] - W_m\frac{[\frac{\sin kl}{kl} + \frac{\sin k\sqrt{d^2 + (l\sin\alpha)^2}}{k\sqrt{d^2 + (l\sin\alpha)^2}}]^2}{[1 + \frac{\sin k(d - l\cos\alpha)}{k(d - l\cos\alpha)}]}. \quad (9)$$

Similar to the power of a dipole located near a reflecting surface (Equation (2)), the total power of the active control system, $W_{3\min}$, is also significantly dependent upon the orientation angle α. This property of angular dependence and the mechanism behind it become the basis of discussion of the performance in controlling the power radiated by a monopole using another monopole with adjustable source strength. Quantitatively, the performance is described by the power reduction defined by the level difference between the power of the primary monopole, W_m, in free field and that of the control system consisting of the primary monopole and a secondary monopole.

RESULTS AND DISCUSSION

A straightforward means of reduction of the power from the primary source is to use the dipole arrangement. In free field, this power reduction is well known:

$$\Delta L_d = -10\log_{10}\left[\frac{W_d}{W_m}\right] = -10\log_{10}\left\{2[1 - \frac{\sin kl}{kl}]\right\}. \quad (10)$$

Equation (2) indicates that more power reduction in the dipole arrangement is achievable by introducing a reflecting surface to the dipole and arranging $\alpha = 0^\circ$. For this case, the reduction level is calculated by

$$\Delta L_2 = -10\log_{10}\left[\frac{W_2}{W_m}\right]. \quad (11)$$

The difference in power reduction of the dipole arrangement due to the effect of a reflecting surface is illustrated by the ratios of power radiated from horizontal and vertical dipoles to the free field dipole power (illustrated in Figure 2 by the dashed curves, and we note

$$\Delta L_2 - \Delta L_d = -10\log_{10}\left[\frac{W_2}{W_d}\right]).$$

The characteristics of power ratios for horizontal and vertical dipoles are the same as that obtained by Ingard and Lamb (1957) and Bies (1960). As $d/2 \to 0$, the horizontal dipole radiates twice the amount of power that the free field dipole does. At $kd = 5.7$, the horizontal dipole power has a minimum which is less than the free-field dipole power by a factor of 0.91. For the vertical dipole the total power becomes much less than the free-field dipole power as $kd \to kl/2$. It is noted that $kl/2$ is the lower limit for kd for the vertical dipole case as the dipole always has non-zero dipole distance.

When the secondary source strength is generated by Equation (8), further power reduction becomes possible and the reduction level (maximum achievable power reduction) is written as:

$$\Delta L_{3\min} = -10\log_{10}\left[\frac{W_{3\min}}{W_m}\right]. \quad (12)$$

The ratio of the minimum sound power of the active control system to the free field power output of a dipole with strength of $q_a l$ is shown by the solid curves in Figure 2 as a function of the source height. In contrast with the power ratios of horizontal and vertical dipoles (dashed curves in Figure 2), the control system with orientations at $\alpha = 90^\circ$ and $\alpha = 0^\circ$ always radiates less power because it is optimized for minimum power output.

The dependence of the power reduction on the orientation angle α is demonstrated in Figure 3, where the system parameters are $d = l$ and $kl = \pi/4$ for all the cases. The dash-dotted line is the maximum achievable power reduction ($\Delta L_{W\min} = -10\log_{10}[1 - (\frac{\sin kl}{kl})^2]$) of a two point source active noise control system in the free field. This reduction is independent of the angle and the reduction level is 7.2dB. A significant variation in the maximum achievable power reduction with the angle is observed when the primary monopole is actively controlled by the optimal secondary source and they are placed near a reflecting surface. The maximum achievable power reduction (solid curve) varies from 18.1dB to 4.3dB as α increases from 0° to 90°. It then returns to 17.2dB as α further increases to 180°. Also shown in Figure 3 is the dipole power reduction affected by the reflecting surface.

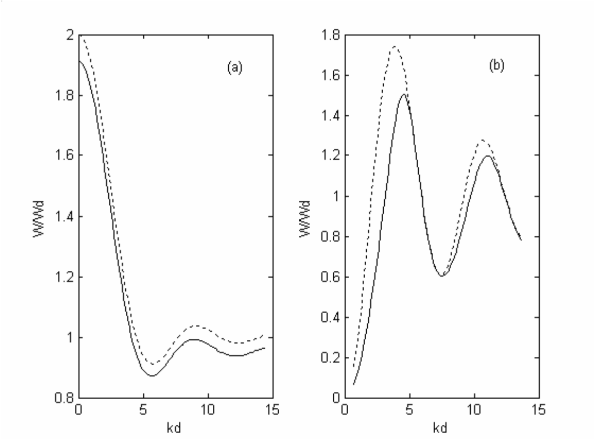

Figure 2. The power radiation with respect to free field dipole power radiation as a function of source height. $k = 1.8265(\mathrm{m}^{-1})$ and $l = 0.4(\mathrm{m})$. (a) Horizontal dipole (dashed curve) and optimal control system (solid curve) at $\alpha = 90^\circ$; (b) Vertical dipole (dashed curve) and optimal control system (solid curve) at $\alpha = 0^\circ$.

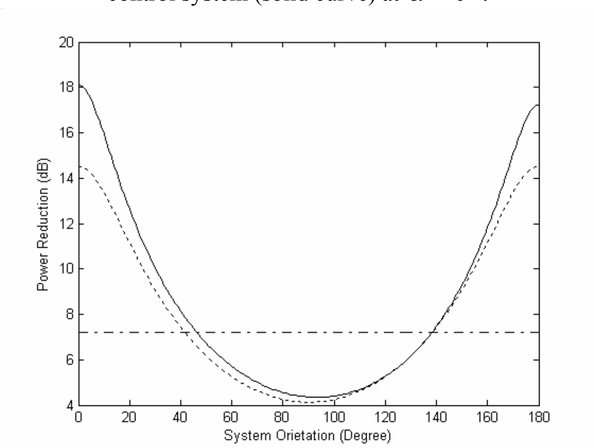

Figure 3. Reduction in total sound power radiated (referenced to the power from the primary source in free field). Solid curve: for the active control system near the rigid surface; Dash-dotted line: for the active control system in the free field ($kl = \pi / 4$); Dashed curve: for a dipole near the rigid surface.

Figure 3 shows both the negative and positive effects of a reflecting surface on the power reduction. When α is approximately within 40° to 140° , the radiation mechanism of the dipole and the active control system is dominated by that of a parallel dipole. For this case, the reflecting surface increases the power radiated, which is higher than that of the active control system in the free field. However when α is in the vicinity of 0° or 180° , a significant increase in the power reduction is observed. It is because the radiation mechanism of the dipole and the active control system is now characterized by a vertical dipole, which has weak radiation efficiency.

For example, if the primary source radiates 100dB sound power into the free field, the active control using one secondary source without using the surface reflection and at $kl = \pi / 4$ only gives rise to a 7.2dB reduction. Thus the controlled total power for this case is 92.8dB. By introducing a reflecting surface at $d = l$, the active control system is capable of reducing the total power to 81.9dB ($= 100 - 18.1$ dB) for $\alpha = 0^\circ$, 82.8dB for $\alpha = 180^\circ$ and 95.7dB for $\alpha = 90^\circ$. Therefore, we observe more than 10dB improvement in the sound power reduction when a reflecting surface is introduced to the active control system at $\alpha = 0^\circ$. With the reflecting surface, a huge difference in the power reduction exists (14dB!) between vertical and parallel arrangements of the active control system.

Finally the ratio of the optimal secondary source strength to the primary source strength is shown in Figure 4. The solid curve is for the control system with a reflective surface while the dash-dotted line is for that without. The optimal source strength of the secondary source varies around -1 with the system orientation. The small variation indicates that configuration of the control system has the general features of a dipole, and such variation is necessary to achieve the maximum power reduction. This result confirms the mechanism of changing the dipole radiation into quadrupole radiation while the control system orientation is vertical to the rigid surface.

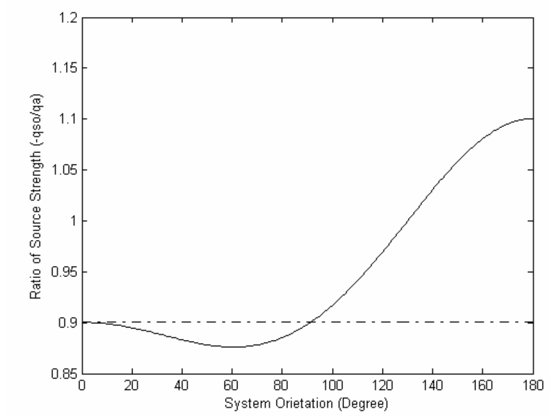

Figure 4. Ratio of optimal secondary source strength to primary source strength. Solid curve: with rigid surface, dash-dotted line: without rigid surface. $d = l$ and $kl = \pi / 4$.

EXPERIMENTAL RESULTS

To experimentally confirm the effect of a reflection surface on the power radiation of vertical and horizontal dipoles, a measurement of sound pressure from a dipole source was conducted when the source is placed in front of reflection surfaces of different size. The other purpose of the experimental work is to investigate if the reflective surface can still affect the source radiation when the surface size becomes smaller than the wavelength.

Figure 5 shows the experimental set-up in an anechoic chamber. The dipole sound source consists of a small loudspeaker with its back cavity removed. The reflecting surface is made

Figure 5. Experimental set-up in an anechoic room with a dipole source (vertical dipole) in front of the reflecting panels.

from 20mm thick honeycomb panels. The height of the surface is 2.1m, and the width can be adjusted for the effect of surface size on the sound radiation. Figure 6 is the set-up for a horizontal dipole arrangement.

Figure 6. Experimental set-up in an anechoic room with a horizontal dipole in front of the reflecting panels.

The directivity of the dipole was checked in a horizontal plane where the source is located before the reflecting surface was introduced. As shown in Figure 7, the open loudspeaker produces a reasonable dipole radiation pattern. When a reflecting surface is introduced to the sound field, the vertical dipole is constructed if the loudspeaker's diaphragm is parallel to the surface; and the horizontal dipole is constructed if the diaphragm is perpendicular to the surface. For all the cases, the distance from the centre of the loudspeaker to the surface is 85mm.

To study the effect of a reflecting surface on the sound radiation from the dipole, the spatial averaged sound pressure is measured along a circle of the horizontal plane containing the loudspeaker. The angular increment of the measurement is 15 degrees. Different surface sizes are used for the same measurement of averaged sound pressure level of both vertical and horizontal dipoles. The results are shown in Figure 8, where the sound pressure levels are normalized with the free field dipole (where no reflecting surface is used) sound pressure level. Experimental results demonstrate that the reflecting panel indeed reduces the total sound radiated and the effect gradually decreases with a reduction of the surface width. The effect is still significant when the width is much smaller than the wavelength.

For the case of a horizontal dipole the reflecting panel increases the radiated sound in front of the panel and the effect of different size of panel is relatively smaller than in the case of the vertical dipole as shown in Figure 8.

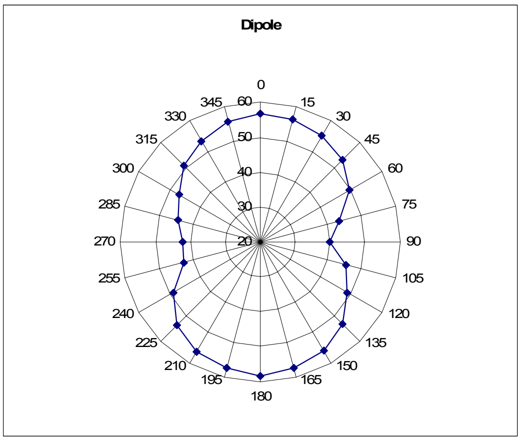

Figure 7. Sound radiation directivity from an open loudspeaker. The operating frequency is 230Hz.

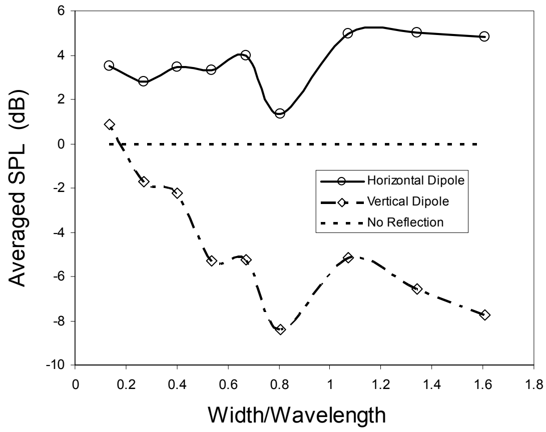

Figure 8. Spatial averaged sound pressure level of horizontal, vertical and free field dipoles.

CONCLUDING REMARKS

One of the important roles acousticians may play in the development of active noise control systems is to use the acoustical features of the systems to improve the control performance. Examples of such include installation of a short duct at the outlet of an axial fan so that the control of sound radiation into a three dimensional space is converted to the control of plane wave propagation in a one dimensional duct (Wong et al 2003). The work on active cancellation of sound pressure in a pure tone diffuse field by Garcia-Bonito et al (1997) also suggested that introduction of reflecting surfaces restricts the possible directions of random contributions and broadens the actively generated zone of quiet. The work presented in this paper is another demonstration of effort in this direction of research. We have shown that a reflecting surface may convert dipole sound radiation into quadrupole radiation and such a feature may be used to improve the performance of active noise control systems. Useful application of such findings may be found in the following two cases:

1. If a monopole type of primary source is already located near a reflecting surface, to achieve improved reduction in total sound power, the secondary control source should be arranged so that the control system is vertically oriented with respect to the surface.
2. Reflective surfaces may be purposely introduced to the primary source of monopole type so that the secondary control source could also use the surface reflection to further reduce total sound radiation.

In reference 6 (Lin et al 2004), the performance of an active control system located near a rigid sphere is investigated. A small piston acting as a primary source is located on the surface of the sphere (where the elevation angle is zero) and an optimal secondary control source is located at a fixed distance away from the primary source. The cost function in their analysis is the averaged sound pressure measured by 201 error microphones in the far field of the sphere, which is proportional to the total power radiated. The level difference between the uncontrolled and controlled sound pressure is defined as the noise reduction and used to describe the performance of the control system. The elevation angle of the secondary source is used as a varying parameter. From their simulated noise reduction (see Figure 9(a) in Reference 6), the maximum noise reduction is achieved at zero elevation angle (dominated by the vertical dipole mechanism) and the noise reduction level gradually decreases as the elevation angle increases. A typical example was the control of a 500Hz sound radiation, where the ratio between the diameter of the sphere and the wavelength is $\frac{0.175m}{0.688m} = 0.254$, and the distance between the two sources is $0.0125m$. At zero elevation, the noise reduction is 33dB. At the maximum elevation angle where the secondary source is located on the surface of the sphere, the noise reduction becomes 22dB.

Although the reflecting surface is modelled as a rigid baffle of infinite size in this paper, the evidence provided in reference 6 indicates that the above conclusions can be extended to the cases even where the reflecting surface is finite and less than the wavelength of the sound radiated. Experimental work on sound radiation of dipoles in front of finite surfaces confirms this.

ACKNOWLEDGEMENT

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