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# Origin of the broad-band noise in acoustic cavitation

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## ABSTRACT

Acoustic emission from acoustic cavitation is called acoustic cavitation noise. In the present study, numerical simulations of acoustic cavitation noise have been performed under the experimental condition of Ashokkumar et al.[J.Am.Chem.Soc. 129, 2250-2258 (2007)] at 515 kHz taking into account the temporal fluctuation in the number of bubbles. It is shown that the temporal fluctuation in the number of bubbles results in the braod-band noise. As the temporal fluctuation in the number of bubbles is manly caused by fragmentation of bubbles, transient cavitation bubbles which have short lifetimes such as one or a few acoustic cycles results in the broad-band noise. On the other hand, stable cavitation bubbles do not cause the broad-band noise even if they emit shock waves.

## INTRODUCTION

Acoustic cavitation is the formation and collapse of bubbles in liquid irradiated by an intense acoustic wave such as ultrasound [1]. Bubbles pulsate according to the pressure oscillation of an acoustic wave. Pulsating bubbles emit acoustic waves called acoustic cavitation noise. The frequency spectra of acoustic cavitation noise consist of peaks at the driving frequency, its harmonics, subharmonics and the broad-band noise which is a continuum part of the frequency spectrum [2].

The intensity of the broad-bnd noise has been measured as an indicator of the intensity of acoustic cavitation [3]. However, the origin of the broad-band noise is still under debate. One hypothesis is that it originates from non-periodic pulsation of bubbles called chaotic pulsations [4]. An acoustic wave radiated from a chaotically pulsating bubble is temporally non-periodic and results in the broad-band noise. Another hypothesis is that it originates from shock waves emitted from collapsing bubbles [5, 6]. A bubble emits a shock wave at the end of its collapse and the emitted shock wave hits a hydrophone as a strong pulse. A single strong pulse like a delta function results in the broad-band noise. In the present study, it will be shown that shock waves emitted from stably pulsating bubbles do not result in the broad-band noise because a frequency spectrum of a temporally periodic function only consists of peaks at the fundamental (driving) frequency and its harmonics. Instead it will be shown that the temporal fluctuation in the number of bubbles results in the broad-band noise.

Ashokkumar et al. [7] reported that in low concentration surfactant solutions the frequency spectra of acoustic cavitation noise consisted only of peaks at the driving frequency (515 kHz) and its harmonics. The broad-band noise was very weak. On the other hand, in pure water, there was a strong broad-band noise. In the present study, numerical simulations of acoustic cavitation noise have been performed under the experimental condition of Ashokkumar et al. [7]

## MODEL

We consider a bubble cloud in which bubbles are spatially uniformly distributed. The ambient bubble radius, which is the radius of a bubble when an acoustic wave is absent, is assumed to be the same for all the bubbles. Then, the pressure ($P$) of acoustic waves radiated from bubbles is given by Eq. (1) [8].

$$P = \rho\left(R^2\ddot{R} + 2R\dot{R}^2\right)\left(\sum_{i=1}^{N}\frac{1}{r_i}\right) \tag{1}$$

where $\rho$ is the liquid density, $R$ is the instantaneous bubble radius, the dot denotes the time derivative, $r_i$ is the distance from a bubble numberd $i$ to an observation point (hydrophone), and the summation is for all the bubbles.

In the expeiment of Ashokkumar et al. [7], a hydrophone is a hollow, open-ended cylindrical cavity. The inner cylindrical surface of the cavitation sensor is formed from 110-μ piezoelectric film. Then the sum in the right hand side of Eq. (1) is estimated as follows.

$$\begin{aligned} S &= \sum_{i=1}^{N}\frac{1}{r_i} = \frac{\lambda}{4}\int_0^{a-\Delta}\frac{2\pi n r}{a-r}dr \\ &= \frac{\pi\lambda n}{2}\left(-a + \Delta - a\log_e\Delta + a\log_e a\right) \end{aligned} \tag{2}$$

where the sum is named as $S$ , $\lambda$ is the acoustic wavelength, $a$ is the inner radius of the cylindrical cavity, $\Delta$ is the mean distance from the inner surface of the cylindrical cavity to a nearest bubble, $n$ is the number density of bubbles, and $r$ is the radius from the center of the cylindrical cavity. In Eq. (2), only a single antinodal region of thickness $\lambda/4$ in a standing ultrasonic field is considered. The mean distance from the surface of the cylindrical cavity to a nearest bubble is estimated by $\Delta = 1/\sqrt[3]{n}$.

The total number ($N$) of bubbles in the cylindrical anti-nodal region is estimated by Eq. (3).

$$N = \frac{\pi \lambda n a^2}{4} \qquad (3)$$

Thus $S$ monotonically increases as $N$ increases [8].

With regard to the bubble pulsation, the modified Keller equation is used [9].

$$\begin{aligned}
&\left(1-\frac{\dot R}{c_\infty}+\frac{\dot m}{c_\infty \rho_{L,i}}\right)R\ddot R+\frac{3}{2}\dot R^2\left(1-\frac{\dot R}{3c_\infty}+\frac{2\dot m}{3c_\infty\rho_{L,i}}\right)\\
&=\frac{1}{\rho_{L,\infty}}\left(1+\frac{\dot R}{c_\infty}\right)\left[p_B-p_S\left(t+\frac{R}{c_\infty}\right)-p_\infty\right]+\frac{\ddot m R}{\rho_{L,i}}\left(1-\frac{\dot R}{c_\infty}+\frac{\dot m}{c_\infty\rho_{L,i}}\right)\\
&+\frac{\dot m}{\rho_{L,i}}\left(\dot R+\frac{\dot m}{2\rho_{L,i}}+\frac{\dot m \dot R}{2c_\infty\rho_{L,i}}-\frac{R}{\rho_{L,i}}\frac{d\rho_{L,i}}{dt}-\frac{\dot m R}{c_\infty\rho_{L,i}^2}\frac{d\rho_{L,i}}{dt}\right)+\frac{R}{c_\infty\rho_{L,\infty}}\frac{dp_B}{dt}\\
&-S\left(2\dot R^2 R+R^2\ddot R\right)
\end{aligned} \qquad (4)$$

where $c_\infty$ is the sound velocity in liquid, $\dot m$ is the evaporation rate of water vapour at the bubble wall, $\rho_{L,i}$ is the liquid density at the bubble wall, $p_B$ is the liquid pressure at the bubble wall, $p_s(t)$ is the instantaneous pressure of ultrasound at time $t$, $p_s(t) = -p_a \sin \omega t$, $p_a$ is the pressure amplitude of ultrasound (acoustic amplitude), $\omega$ is the angular frequency of ultrasound, $p_\infty$ is the static pressure, and $\ddot m = \frac{d\dot m}{dt}$.The last term of Eq. (4) indicates the effect of surrounding bubbles which radiate acoustic waves and modify the pressure fileld. The effect is called the bubble-bubble interaction. In the present study, however, the effect is negligible.

In the present model, the amplitude of non-spherical component of the bubble shape is numerically calculated as a function of time. The equations for the calculations have been given in Ref. [8, 10, 11]. It is assumed that a bubble disintegrates into daughter bubbles when the amplitude of non-spherical component exceeds the mean bubble radius. In this way, a lifetime ($L$) of a bubble is numerically calculated. When a bubble disintegrates into daughter bubbles, the number of bubbles changes. Accordingly, $S$ in Eq. (1) changes. In the present model, the temporal variation of $S$ is calculated by Eq. (5).

$$S(t+T) = S(t) + \frac{S_0}{L} r_n \qquad (5)$$

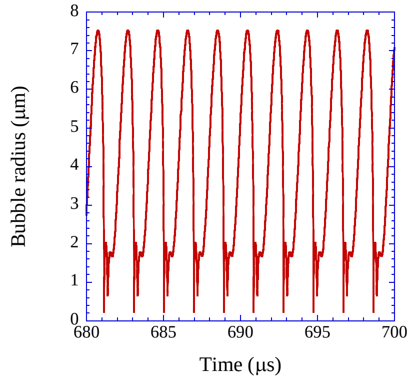
where $T$ is the acoustic period, $S_0$ is the initial value of $S$, and $r_n$ is a random number generated by a computer beween -1 and 1. The change of $S$ by Eq. (5) is once per acoustic cycle. By the temporal variation of $S$, the acoustic cavitation noise in Eq. (1) temporally varies.

In the present study, the response of the hydrophone is modeled as Eq. (6).

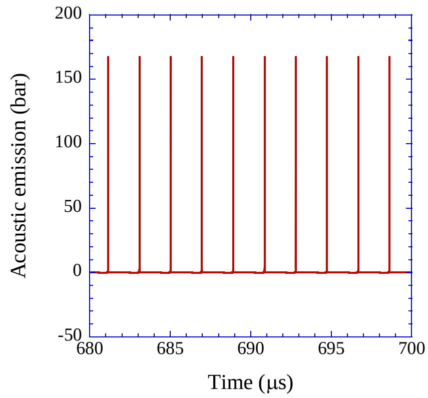
$$\ddot U + 2\gamma\pi f_c \dot U + 4\pi^2 f_c^2 U = P(t) + p_S(t) \qquad (6)$$

where $U$ is the hydrohone signal, the dot denotes the time derivative, $\gamma$ is the coefficient for damping, $f_c$ is the characteristic frequency of the hydrophone, $P(t)$ is the instantaneous pressure of acoustic waves radiated from bubbles given by Eq. (1), and $p_s$(t) is the instantaneous pressure of the driving ultrasound. The characteristic frequency ($f_c$) of hydrophone is related to a cut-off frequency and assumed as 5 MHz in the present study. $\gamma$=1 is also assumed.

## RESULTS AND DISCUSSIONS




(a)

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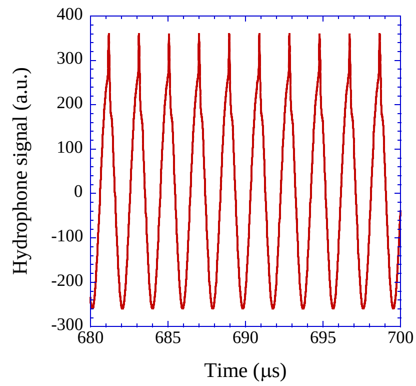


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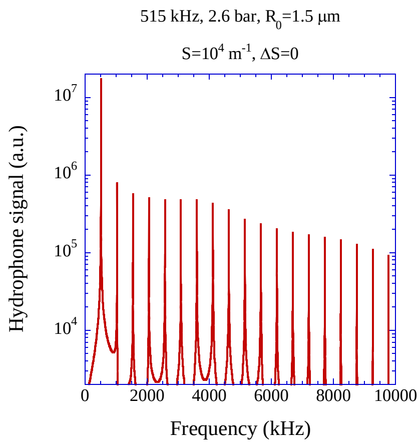
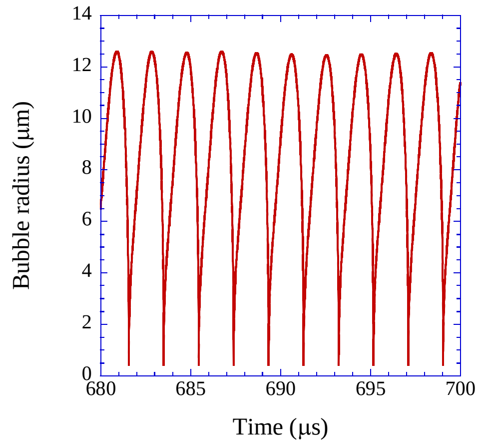
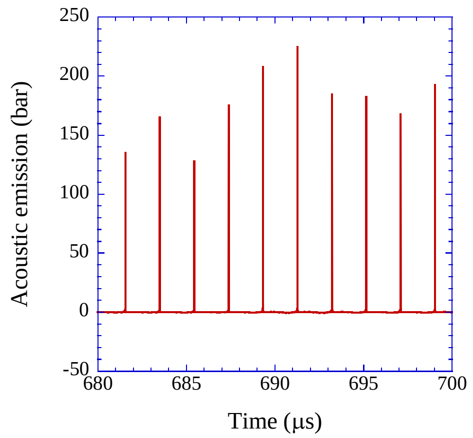



**Figure 1**. The result of the numerical simulation for the ambient radius of 1.5 µm. A bubble is shape stable and there is no temporal fluctuation in the number of bubbles and *S*. The frequency and pressure amplitude of ultrasound are 515 kHz and 2.6 bar, respectively. $S=10^4$ m$^{-1}$. (a) The bubble radius as a function of time. (b) The pressure of acoustic waves radiated from bubbles as a function of time. (c) The hydrophone signal as a function of time. (d) The frequency spectrum of the hydrophone signal. Reprinted from Ultrasonics Sonochemistry, vol. 17, K.Yasui, T.Tuziuti, J.Lee, T.Kozuka, A.Towata, and Y.Iida, Numerical simulations of acoustic cavitation noise with the temporal fluctuation in the number of bubbles, pp.460-472, Copyright (2010), with permission from Elsevier.

The numerical simulatins have been performed under the experimental condition of Ashokkumar et al. [7]. The ultrasonic frequency and pressure amplitude are 515 kHz and 2.6 bar, respectively. The ambient bubble radius is assumed as 3 µm and 1.5 µm in pure water and 1.5 mM SDS (sodium dodecyl sulfate: surfactant) solution, respectively.

In Fig. 1, the results of the numerical simulation for the ambient radius of 1.5 µm are shown. A bubble pulsates temporally periodically (Fig. 1 (a)). Furthermore, a buble is shape stable and there is no temporal fluctuation in the number of bubbles and *S*. As a result, pressure of acoustic waves (mostly shock waves) radiated from bubbles is a periodic function of time with acoustic period (Fig. 1 (b)). Consequently, the hydrophone signal is also a periodic function of time (Fig. 1 (c)). Thus, the frequency spectrum of the hydrophone signal consists of peaks at driving ultrasonic frequency and its harmonics because this is the nature of any periodic function (Fig. 1 (d)). Then, the broad-band noise is absent in the frequency spectrum. Thus, even with the emission of shock waves, the broad-band noise is absent if the bubble pulsation is temporally periodic and the number of bubbles is constant. This is why there was very weak broad-band noise in 1.5 mM SDS solution in the experiment of Ashokkumar et al. [7].



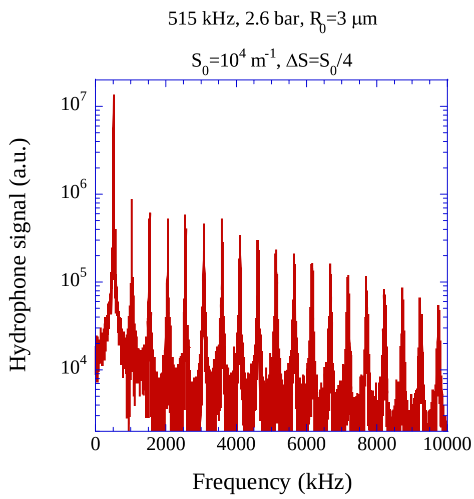




**Figure 2** The result of the numerical simulation for the ambient radius of 3µm. The lifetime of a bubble is 4 acoustic cycles and there is temporal fluctuation in the number of bubbles and *S*. The frequency and pressure amplitude of ultrasound are 515 kHz and 2.6 bar, respectively. $S_0=10^4$ m$^{-1}$. (a) The bubble radius as a function of time. (b) The pressure of acoustic waves radiated from bubbles as a function of time. (c) The frequency spectrum of the hydrophone signal. Reprinted from Ultrasonics Sonochemistry, vol. 17, K.Yasui, T.Tuziuti, J.Lee, T.Kozuka, A.Towata, and Y.Iida, Numerical

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In Fig. 2, the results of the numerical simulation for the ambient radius of 3 μm are shown. Although the bubble pulsation is nearly temporally periodic (Fig. 2 (a)), a bubble disintegrates into daughter bubbles after the $4^{th}$ collapse (the lifetime is 4 acoustic cycles). Thus, in this case, there is the temporal fluctuation in the number of bubbles and $S$ according to Eq. (5). As a result, the pressure of shock waves randomly varies with time (Fig. 2 (b)). Thus the frequency spectrum of the hydrophone signal consists not only of peaks at driving frequency and its harmonics but also of the broad-band noise (Fig. 2 (c)). It is concluded that the temporal fluctuation in the number of bubbles results in the broad-band noise. In other words, transient cavitation bubbles which are defined as bubbles with a lifetime of one or a few acoustic cycles results in the broad-band noise. On the other hand, stable bubbles do not cause the broad-band noise as seen in Fig. 1.

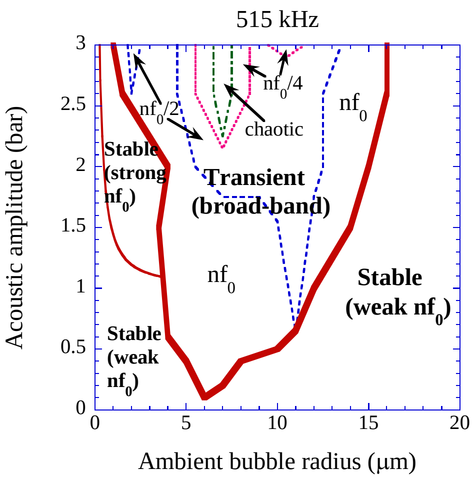


**Figure 3** The result of the numerical simulation for various ambient radii and acoustic amplitudes at 515 kHz. Above the thickest line, a bubble is shape unstable (transient cavitation). Below the thickest line, a bubble is shape stable (stable cavitation). The type of the frequency spectrum of acoustic cavitation noise is also indicated. Reprinted from Ultrasonics Sonochemistry, vol. 17, K.Yasui, T.Tuziuti, J.Lee, T.Kozuka, A.Towata, and Y.Iida, Numerical simulations of acoustic cavitation noise with the temporal fluctuation in the number of bubbles, pp.460-472, Copyright (2010), with permission from Elsevier.

Numerical simulations have been performed for various ambient radii and acoustic pressures as shown in Fig. 3. Above the thickest line, a bubble is shape unstable and has a finite lifetime. It is called transient cavitation bubbles which always result in the broad-band noise as shown above. Below the thickest line, a bubble is shape stable and called stable cavitation bubbles. For these bubbles, there is no broad-band noise. The region indicated by $nf_0$ is that for which a bubble pulsate temporally periodically with the acoustic period. For these bubbles, the frequency specta of acoustic cavitation noise contain peaks at driving frequency and its harmonics. For the regions indicated by $nf_0/2$ and $nf_0/4$, the bubble pulsation is temporally periodic with doubled and quadrupled acosutic period, respectively. The frequency spectra contain peaks at $nf_0/2$ and $nf_0/4$, respectively. The region indicated by "chaotic" means that the bubble pulsation is non-periodic which results in the broad-band noise. In the experimental data of Ashokkumar et al. [7], there is no peaks at $(2n+1)f_0/4$ in the frequency spectra of acoustic cavitation noise. It suggests that the number of bubbles in the region for $nf_0/4$ is negligible. It further suggests that the number of bubbles in the region for "chaotic" is also negligible because the area for the region is even smaller than that for $nf_0/4$. Thus it is suggested that the contribution of non-periodically pulsating bubbles to the broad-band noise is negligible. Thus the main origin of the broad-band noise is the temporal fluctuation in the number of bubbles for transient cavitation bubbles at least under the present condition.

## CONCLUSION

Numerical simulations of acoustic cavitation noise have been performed under the experimental condition of Ashokkumar et al. [7] It is shown that the temporal fluctuation in the number of bubbles results in the broad-band noise. In other words, transient cavitation bubbes results in the broad-band noise.


## ACKNOWLEDGEMENT

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