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Investigation of the damping effect on the energy response of a structure-cavity system in the case of a heavy fluid

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ABSTRACT

The damping effect on the energy flow between an excited structure and a receiving acoustic cavity filled with a heavy fluid will be studied in this paper. In particular, in the high frequency domain, classical Statistical Energy Analysis (SEA) relation describing the energy transmitted between two subsystems indicates that the energy ratio of the two subsystems is independent of the damping loss factor of the excited subsystem. However, this relation is based on a weak coupling assumption which does not hold in the case of a heavy fluid. Then, we will study the consequence of the non respect of this assumption on the energy flow and the damping effect. The Dual Modal Formulation (DMF) is used to describe the fluid-structure coupling from the modes of each uncoupled subsystem. This formulation allows us to study the convergence of the modal series, to determine easily the modal energy of each subsystem and to compare these results with the classical SEA assumptions. We observe that the fluid added mass effect and the non-resonant coupling have a strong effect on the energy flow between the structure and the cavity for frequencies below the critical frequency. As a result, the energy ratio of the two subsystems is not always independent of the damping loss factor of the excited subsystem as it will be shown on an example.

1 INTRODUCTION

The interaction between a vibrating structure and an acoustic cavity is a vibroacoustic problem which concerns many industrials. The fluid-structure interaction between the two closed domains is a complicated process which had been investigated theoretically and experimentally by many researchers ([1-5]). However, these studies consider generally a cavity filled with light fluid (air). The coupling between the structure and the cavity is then weak (i.e. the modes of the structure and the modes of the cavity are not greatly influenced by the coupling) and some simplifying assumptions can be introduced in the model. It is the case for the classical Statistical Energy Analysis (SEA) [5] model which could be used to analyse the structure-cavity coupling in the high frequency domain. The model expresses globally the energy exchanged by the resonant modes of the two subsystems (i.e. the structure and the cavity). When the structure is excited, SEA indicates that the energy ratio between the two subsystems is independent of the structural damping. As the energy of the structure decreases when the structural damping increases, the energy of the cavity decreases also in the same order of magnitude. It is an important result for noise control. It says that the noise level inside the cavity can be controlled by the damping of the excited structure.

In this paper, we are interested by studying the behaviour of a flexural structure coupled with a cavity filled with a heavy fluid (water for example). This case shows among others some applications in the nuclear and the submarine industries. The coupling between the structure and the cavity is strong; the classical SEA assumption of weak coupling is not respected. We are especially interested by analysing the consequence of the strong coupling on the structural damping effect on the acoustic field in the cavity. In other words, we would like study if the energy ratio is always independent of the structural damping if the cavity is filled with a heavy fluid. To do that, we will consider the case shown on figure 1 and composed of a rectangular flat plate coupled with a parallelepiped water-filled cavity. The plate is supposed simply-supported and excited by a normal point force. For this academic case, the Dual Modal Formulation (DMF) [6] could be used to describe the fluid-structure coupling from the modes of each uncoupled subsystem. The energy of the resonant modes and the non resonant modes of the two subsystems could be estimated without a weak coupling assumption. DMF will permit to compare the energy ratio with different structural damping in the case of a heavy fluid.

This paper is organized as follows. We propose to remember some classical SEA results before to describe the Dual Modal Formulation for the structure-cavity coupling. Some results are then presented in term of modal energy distributions. The fluid added mass effect and the non resonant transmission are identified. The results are then compared for different damping.

2 TEST CASE DESCRIPTION

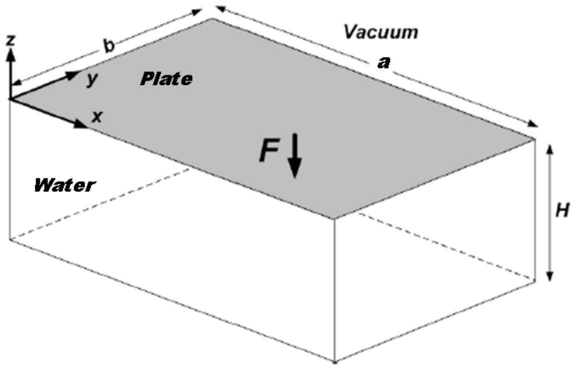

Figure 1. A rectangular simply-supported plate excited by a point force and coupled to a parallelepiped water-filled cavity.

We consider the plate-cavity system described on figure 1. It is composed of a rectangular simply-supported plate coupled with a parallelepiped cavity. The cavity is filled with water (mass density ρ_0=1000 kg/m^3, celerity c_0=1500 m/s, damping loss factor η_2=0.01). The plate is made of steel (mass density ρ=7800 kg/m^3, Young modulus E=2.10^{11}Pa). It is supposed to be excited by a random point force with a white spectrum in the frequency band of central frequency ω_c and of band-width $\Delta\omega$. The behaviour of the plate will be described by Kirchhoff equation whereas the Helmholtz equation is supposed to be respected into the fluid domain.
Two different test cases will be considered in this paper. The parameter values describing them are given on table 1. They have been chosen in order to have enough resonant modes in the frequency bands of interest (i.e. third octave bands centred on 1000 Hz, 2000 Hz and 3150 Hz). One can notice that the frequencies of interest are well below the critical frequency of the 8mm thick plate immerged in water (around 28kHz).

Geometric parameters	Test case 1	Test case 2
Plate length	a=2 m	a=1.2 m
Plate width	b=1.8 m	b=1 m
Plate thickness	e=8 mm	e=8 mm
Cavity depth	H=1.4 m	H=2 m
Force coordinates	(0.3m,0.5m)	(0.3m,0.5m)

Table 1. Geometric parameters of the two test cases.

3 STATISTICAL ENERGY ANALYSIS

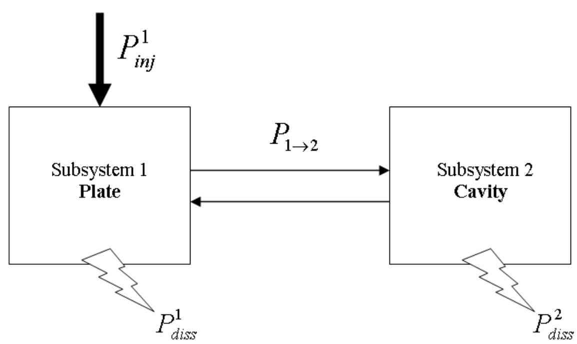

Figure 2. SEA model of the plate coupled with the cavity.

The basic SEA schema of the plate coupled with the cavity is given on figure 2. It indicates that the injected power by the external force, P^1_{inj} is either dissipated by the plate P^1_{diss} or exchanged with the cavity $P_{1\to 2}$. The cavity being not directly excited $\left(P^2_{inj}=0\right)$, it dissipates the energy received from the plate.

The energy conservation for the plate and the cavity writes ([5]):

$$\begin{cases} P^1_{inj} = P^1_{diss} + P_{1\to 2}, \\ 0 = P^2_{diss} - P_{1\to 2}. \end{cases} \tag{1}$$

The injected power by the external force is given by [5]:

$$P^1_{inj} = \frac{\pi n}{2M} < F^2 >, \tag{2}$$

where n and M are respectively the modal density and the mass of the plate, and $< F^2 >$ is the time averaged of the quadratic force in the frequency band of interest.
One can notice that this quantity is independent of the plate damping factor.

Using simplifying assumptions (weak coupling, resonant transmission,…), the classical SEA model links the power exchanged by two subsystems to their total energies:

$$P_{1\to 2} = \omega_c \left(\eta_{12}E_1 - \eta_{21}E_2\right), \tag{3}$$

whre η_{12} and η_{21} are the coupling loss factor and E_1, E_2 , the total energy of the plate and the cavity, respectively.

The dissipated powers in the frequency band $\Delta\omega$ are:

$$\begin{aligned} P^1_{diss} &= \omega_c\eta_1 E_1 \text{, for the plate, and,} \\ P^2_{diss} &= \omega_c\eta_2 E_2 \text{, for the cavity,} \end{aligned} \tag{4}$$

where η_1 and η_2 are the damping loss factors of the plate and the cavity.

Injecting (3) and (4) in (1), and assuming that $P^1_{inj} >> P_{1\to 2}$, one can write:

$$E_1 \approx \frac{P^1_{inj}}{\omega_c\eta_1} \text{, and, } E_2 \approx \frac{\eta_{21}P^1_{inj}}{\omega_c\eta_1\left(\eta_2+\eta_{12}\right)}. \tag{5}$$

The ratio of the two energies is then independent of the plate damping factor, η_1:

$$\frac{E_2}{E_1} \approx \frac{\eta_{21}}{\eta_2+\eta_{12}}. \tag{6}$$

Moreover, in the classical SEA process [5], the total energy of the plate and the cavity are deduced from a physical quantity:

$$E_1 \approx M < V^2 >, \tag{7}$$

$$E_2 \approx \frac{abH}{\rho_0 c_0^{\,2}} < p^2 >. \tag{8}$$

where $< V^2 >$ is the space averaged of the quadratic plate velocity and $< p^2 >$ is the space averaged of the quadratic pressure in the cavity. These relations are established on the assumption that the kinetic energy and the potential energy of each subsystem have almost the same magnitude.

Introducing (7), (8) in (5), (6), one can write:

$$< V^2 > \approx \frac{\pi n < F^2 >}{2M^2\eta_1}, \tag{9}$$

$$< p^2 > \approx \frac{\pi n \rho_0 c_0^{\ 2} < F^2 >}{2abHM} \frac{\eta_{21}}{(\eta_2 + \eta_{12})\eta_1}. \qquad (10)$$

The space average quadratic velocity of the plate and the space average quadratic pressure of the cavity are then inversely proportional to the damping loss factor of the plate. Then if the damping of the plate is multiplied by 10, the vibratory level and the pressure level should decrease of 10dB. This result has been obtained by using the classical SEA assumption. We will verify, with the Dual Modal Formulation described in the next section, if its result hold in the case of a heavy fluid.

4 DUAL MODAL FORMULATION (DMF)

DMF permits to calculate the force response of two coupled subsystem from the knowledge of the modes of each uncoupled subsystem. A modal schema is obtained which is in accordance with the schema assumed in the SEA formulation [5]. This approach is then well adapted to investigate the effect of a heavy fluid on the SEA development of the previous section. DMF has been described in [6] for the general case of the coupling of two elastic continuous mechanic system. The application to the plate-cavity system is straightforward. The DMF for this particular case is well know in the literature (see [1-2]).

4.1 Calculation of the forced response

As the cavity-plate system is excited by a random force of white spectrum S_{ff} in the frequency band $\Delta\omega$, the time average of the response ϕ at a receiving point, $< \phi >_{\Delta\omega}$ is given by:

$$< \phi >_{\Delta\omega} = S_{ff} \int_{\Delta\omega} \left| H_{f\phi}(\omega) \right|^2 d\omega, \qquad (11)$$

where $H_{f\phi}(\omega)$ is the frequency transfer function between the excited point and the receiving point. It corresponds to the frequency response at the receiving point when the system is excited by a unit harmonic force. It will be estimated in this section with the dual modal formulation.

DMF is based on modal expansions with the modes of each uncoupled subsystem [6]. The plate is described by a displacement field (i.e. normal displacement) and its uncoupled-free modes (i.e. modes in-vacuo of the simply-supported plate) whereas the cavity is described by a stress field (i.e. acoustic pressure) and its uncoupled-blocked modes (i.e. modes of the parallelepiped cavity with rigid boundaries on the 6 faces). The uncoupled modes of the plate and of the cavity can be easily calculated on this academic case (see [2]).

The modal expansions of the plate displacements, *W* and the acoustic pressure, *p* may be written:

$$W(x, y, t) = \sum_{p=1}^{\infty} \chi_p(t) \widetilde{W}_p(x, y), \qquad (12)$$

$$p(x, y, z, t) = \sum_{q=1}^{\infty} \xi_q(t) \widetilde{p}_q(x, y, z), \qquad (13)$$

where: - $\widetilde{W}_p$ and χ_p, are, respectively, the displacement shape and the amplitude of the p^{th} mode of the plate;

- $\widetilde{p}_q$ and ξ_q, are, respectively, the pressure shape and the amplitude of the q^{th} mode of the cavity;

Thereafter, the space and time dependencies are suppressed of the notation whereas they are always considered. DMF establishes the response of the plate coupled with the cavity from a weak formulation of the problem and using expansions (12-13). With the change of variable,

$$\xi_q = \dot{\zeta}_q, \qquad (14)$$

the following modal equation system is finally obtained [6]:

$$\begin{cases} M_p\left(\ddot{\chi}_p + \omega_p \eta_1 \dot{\chi}_p + \omega_p^{\ 2} \chi_p\right) + \sum_{q=1}^{\infty} W_{pq} \dot{\zeta}_q = F_p, \\ K_q\left(\ddot{\zeta}_q + \omega_q \eta_2 \dot{\zeta}_q + \omega_q^{\ 2} \zeta_q\right) - \sum_{p=1}^{\infty} W_{pq} \dot{\chi}_p = 0, \end{cases} \qquad (15)$$

where: - M_p, K_q, F_p are the generalised masses, stiffnesses and forces given by:

$$M_p = \rho e \int_{S_{Plate}} \widetilde{W}_p^{\ 2} dS, \quad K_q = \frac{1}{\rho_0 c_0^{\ 2}} \int_{V_{Cavity}} \widetilde{p}_q^{\ 2} dV \qquad (16)$$

- W_{pq} are the modal interaction works:

$$W_{pq} = \int_{S_{Coupling}} \widetilde{W}_p \widetilde{p}_q dS. \qquad (17)$$

One can notice that the equation system (15) can be interpreted as the coupling between a set of oscillators associated to the plate with another set of oscillators associated to cavity. It corresponds to the modal coupling shema suggested in the classical SEA formulation ([5-6]).

Now, considering the time-harmonic dependency $e^{j\omega t}$ and normalising the generalised mass and stiffness to unit; the system can be rewritten on a matrix form:

$$\begin{bmatrix} Y_{11} & -j\omega W_{12} \\ +j\omega W_{12}^{\ T} & Y_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ Z_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ 0 \end{bmatrix}, \qquad (18)$$

with the matrixes:

$$X_1 = \begin{bmatrix} \vdots \\ \xi_p \\ \vdots \end{bmatrix}_{P\times 1}, \quad Z_2 = \begin{bmatrix} \vdots \\ \zeta_q \\ \vdots \end{bmatrix}_{Q\times 1}, \quad F_1 = \begin{bmatrix} \vdots \\ F_p \\ \vdots \end{bmatrix}_{P\times 1}, \qquad (19)$$

$$Y_{11} = diag\left(-\omega^2 + j\omega\omega_p\eta_1 + \omega_p^{\ 2}\right)_{P\times P}, \qquad (20)$$

$$Y_{22} = diag\left(-\omega^2 + j\omega\omega_q\eta_2 + \omega_q^{\ 2}\right)_{Q\times Q}, \qquad (21)$$

$$W_{12} = \left\lfloor W_{pq} \right\rfloor_{P\times Q}, \qquad (22)$$

and T indicates the transpose of the matrix.

The solution of this system is given by;

$$X_1 = \left(Y_{11} - \omega^2 W_{12} Y_{22}^{\ -1} W_{12}^{\ T}\right)^{-1} F_1, \text{ and,} \qquad (23)$$

$$Z_2 = -j\omega Y_{22}^{\ -1} W_{12}^{\ T} X_1. \qquad (24)$$

As the matrix Y_{22} is diagonal, it is easy to calculate its inverse. The relation (23) requires to inverse a square matrix $P \times P$ (i.e. matrix of dimensions equal to the number of the plate modes). The time necessaries for this calculation do not depend of the number of modes *Q* considered for the cavity. Then, we could consider a large number of resonant and non-resonant modes for the cavity. In contrary, the computing time increases highly when the number of modes of the plate increases. It is, however, not an obstacle for the considered cases.

With the modal amplitudes obtained with Eq. (23,-24), it is easy to obtain the harmonic response at any point of the system using the modal expansions (12,13). The response in the frequency band is then obtained discretizing the integral in (11).

4.2 Modal energy and total energy

As we have notice previously, Eq. (15) can be interpreted as the coupling of two oscillators sets. Each set of oscillators represents the set of the uncoupled modes of a subsystem. A kinetic and a potential energy can be estimated for each oscillator (i.e. mode). The sum of these two energies gives the total energy of the mode, which is called here the modal energy.

In this section, we will establish the relations between the modal energies and the total energy of each subsystem. For the sake of clarity, we consider the case of a harmonic force at the angular frequency ω.

The time averaged of the kinetic energy of the plate is given by:

$$< E_1^K >= \frac{1}{4}\rho e\omega^2 \int_{S_{Plate}} W^2 dS. \quad (25)$$

Introducing the modal expansion of the plate displacement and taking the modal orthogonality property into account, one obtains:

$$< E_1^K >= \frac{1}{4}\omega^2 \left(\sum_{p=1}^{P} M_p \chi_p{}^2 \right), \quad (26)$$

which can be rewritten:

$$< E_1^K >= \sum_{p=1}^{P} < E_p^K > . \quad (27)$$

with the kinetic energy of the p^{th} oscillator (i.e. mode), $< E_p^K >$:

$$< E_p^K >= \frac{1}{4}\omega^2 M_p \chi_p{}^2 . \quad (28)$$

By the same way, the potential energy of the plate may be expressed as:

$$< E_1^P >= \sum_{p=1}^{P} < E_p^P >, \quad (29)$$

with the potential energy of the p^{th} oscillator:

$$< E_p^P >= \frac{1}{4}\omega_p{}^2 M_p \chi_p{}^2 = \frac{1}{4} K_p \chi_p{}^2 . \quad (30)$$

The total energy of the plate which is defined as the sum of the kinetic energy and the potential energy may be written:

$$< E_1^T >= \sum_{p=1}^{P} \left(< E_p^K > + < E_p^P > \right) = \sum_{p=1}^{P} < E_p^T >, \quad (31)$$

where $< E_p^T >$ represents the total energy of the p^{th} oscillator and it is called the modal energy.

For the cavity, the potential energy is given by:

$$< E_2^P >= \frac{1}{4\rho_0 c_0{}^2} \int_{S_{Plate}} p^2 dV. \quad (32)$$

Introducing the modal expansion of the acoustic pressure and taking the modal orthogonality property into account, one obtains:

$$< E_2^P >= \sum_{q=1}^{Q} < E_q^K >, \quad (33)$$

where

$$< E_q^K >= \frac{1}{4}\omega^2 K_q \zeta_q{}^2 \quad (34)$$

represents the kinetic energy of the q^{th} oscillator (of mass K_q).

The potential energy of the cavity is then related to the kinetic energy of the modes. This result which could be surprising at first sight is due to the fact that the oscillator displacement is the modal pressure.

Similarly, the kinetic energy of the cavity is related to the potential energy of the oscillators:

$$< E_2^K >= \sum_{q=1}^{Q} < E_q^P >, \quad (35)$$

with

$$< E_q^P >= \frac{1}{4}\omega_q{}^2 K_q \zeta_q{}^2 = \frac{1}{4} M_q \zeta_q{}^2 . \quad (36)$$

The total energy of the cavity is given by:

$$< E_2^T >= \sum_{q=1}^{Q} \left(< E_q^K > + < E_q^P > \right) = \sum_{q=1}^{Q} < E_q^T >, \quad (37)$$

where $< E_q^T >$ represents the total energy of the q^{th} oscillator.

One can notice that for a resonant mode (i.e. $\omega_q \approx \omega$), one has:

$$< E_q^K > \approx < E_q^S > . \quad (38)$$

This equivalence between the kinetic energy and the potential energy for a resonant mode is used in the classical SEA formulation to estimate the total energy of a subsystem from twice its kinetic energy (for example to obtain Eq. 7). This statement holds as long as the response of the system is dominated by the resonant modes. We will see in the next section that this equivalent between the kinetic and potential energies do not hold when the non resonant contributions are significant.

5 ANALYSIS OF RESULTS

The calculations developed in the previous section are achieved on the two test cases described in section 2. Three third octave bands are considered: 1000Hz, 2000Hz, 3150Hz. In order to study the convergence of DMF, different calculations had been made with different numbers of modes P and Q taken into account in the modal expansions. They show that it is not necessary to take modes of the plate over 4500 Hz to have converge results until the third octave 3150 Hz. In contrary, for the cavity, the convergence is slow and necessitates taking a large number of non resonant modes. The modal parameters taken into account for the results shown in this section and the numbers of resonant modes for each third octave are given on table 2.

Subsystem	Modal parameters	1/3 Octave	Test case 1	Test case 2
Plate	Maximum eigenfrequency	All	4500 Hz	4500 Hz
	Total number of modes	All	P=618	P=200
	Number of resonant modes	1000 Hz	36	11
	Number of resonant modes	2000 Hz	68	20
	Number of resonant modes	3150 Hz	95	33
Cavity	Max. eigenfrequency	All	14000 Hz	20000 Hz
	Number of modes	All	Q=18409	Q=25427
	Number of resonant modes	1000 Hz	8	5
	Number of resonant modes	2000 Hz	50	23
	Number of resonant modes	3150 Hz	168	81

Table 2. Modal parameters of the DMF calculation.

5.1 Fluid added mass effect and non resonant modes

Figure 2 shows the modal energy distribution (i.e. modal energy versus eigenfrequency) of the plate for the third octave 1000 Hz. The limits of the frequency band have been symbolised with vertical dash line. It can be observed that the modes having highest energy levels are not contained in the excited third octave band. Their natural frequencies are just above this band. This could be explained by the effect of the heavy fluid of the cavity as it will be shown in this section. Indeed, the modal energy distribution of the cavity shown on figure 3 indicates that a great number of non resonant modes (having natural frequencies above 5000 Hz) have a significant energy level. These non resonant modes are not coincidence in frequency with the resonant modes of the plate but are coincidence in space with these modes.

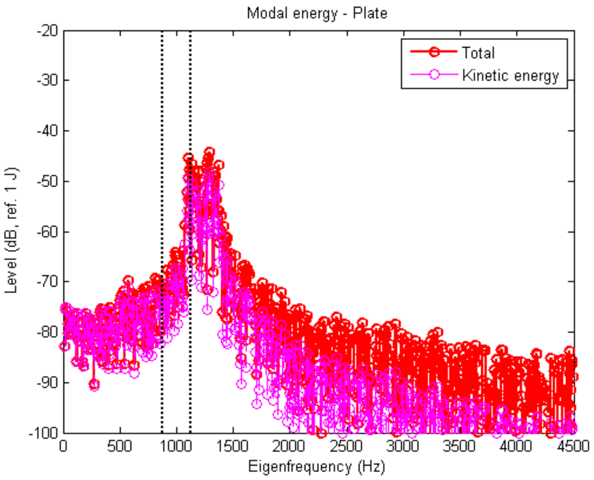

Figure 2. Modal energy distribution of the plate. Excitation of the plate in the third octave band 1000 Hz (band limits symbolised with dashed lines). Test case 1

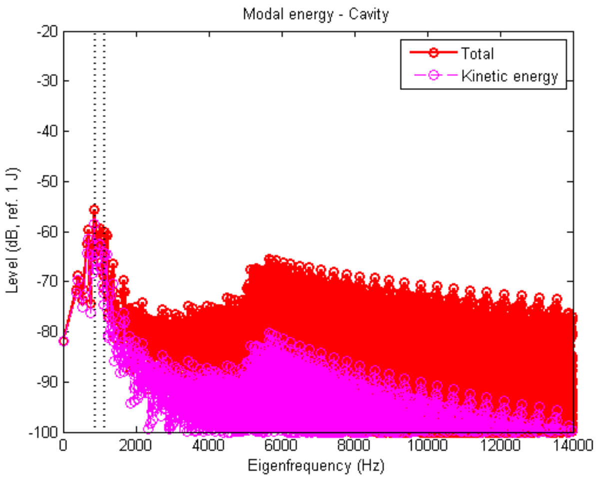

Figure 3. Modal energy distribution of the cavity. Same case than figure 2.

These space coincidences are illustrated on figure 4 by plotting the modal interaction works W_{pq} (see Eq. (17)) between the resonant modes of the plate and the modes of the cavity. We can observe that the modal interaction works are lower for the resonant modes of the cavity than for the non resonant modes of the cavity having natural frequencies above 5000Hz. The resonant modes of the cavity are not coincidence in space because the frequencies of interest are well below the critical frequency of the plate. The modes of the cavity which are coincidence in space with the resonant modes of the plate have higher natural frequencies (i.e. above 5000 Hz).

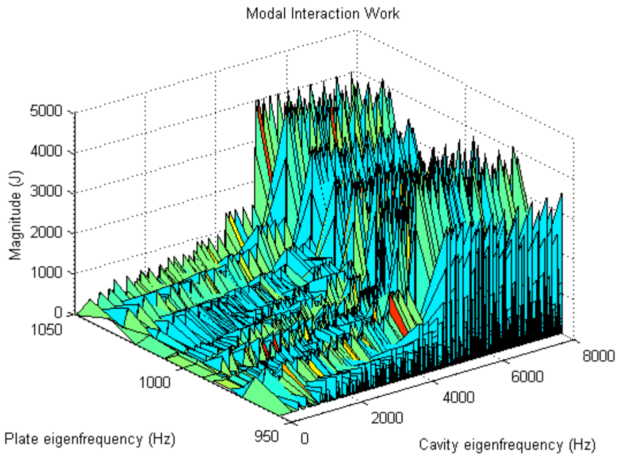

Figure 4. Modal interaction works, W_{pq} for the resonant modes of the plate in the frequency band [950 Hz -1050 Hz].

To analyse the effect of the non resonant modes of the cavity on the resonant modes of the plate, we can suppose:

$$\omega^2 << \omega_q{}^2, \forall q \in [Q_{nr}, Q], \tag{39}$$

when Q_{nr} is the modal order of the first non resonant mode which is considered (for example, the first mode in frequency above 1800 Hz).

If one breaks down the Y_{22} matrix with the set of the non resonant modes $\forall q \in [Q_{nr}, Q]$ and the set of the other modes $\forall q \in [1, Q_{nr} - 1]$,

$$Y_{22} = \begin{bmatrix} Y_{22}^r & 0 \\ 0 & Y_{22}^{nr} \end{bmatrix}, \tag{40}$$

one can approximate Y_{22}^{nr} by:

$$Y_{22}^{nr} \approx diag\left(\omega_q{}^2\right)_{(Q-Q_{nr}+1)\times(Q-Q_{nr}+1)}. \tag{41}$$

whereas as

$$Y_{22}^r = diag\left(-\omega^2 + j\omega\omega_q\eta_2 + \omega_q{}^2\right)_{(Q_{nr}-1)\times(Q_{nr}-1)}. \tag{42}$$

Eq. (23) is given by:

$$X_1 = \left[Y_{11} - \omega^2\left(W_{12}^{nr} Y_{22}^{nr\,-1} W_{12}^{nr\,T} + W_{12}^r Y_{22}^{r\,-1} W_{12}^{r\,T}\right)\right]^{-1} F_1 \tag{43}$$

This equation may be rewritten on the form:

$$X_1 = \left[\overline{Y}_{11} - \omega^2 W_{12}^r Y_{22}^{r\,-1} W_{12}^{r\,T}\right]^{-1} F_1, \tag{44}$$

with $\overline{Y}_{11} = diag\left(M_p'\left(-\omega^2 + j\omega\omega_p\eta_1 + \overline{\omega}_p{}^2\right)\right)_{P\times P}$ (45)

where M_p' represents the modified generalised mass,

$$M_p' = 1 + \sum_{q=Q_{nr}}^{Q} \left(\frac{W_{pq}}{\omega_q}\right)^2, \tag{46}$$

and $\overline{\omega}_p$ the modified eigenvalue.

$$\overline{\omega}_p = \omega_p \sqrt{\frac{1}{M_p'}}. \tag{47}$$

Then, the non resonant modes of the cavity which contribute significantly at the cavity response have an added mass effect on the modes of the plate. This effect is quantified by Eq.

(46,47). An illustration of the added mass contribution of the cavity non resonant modes on the 164th modes of the plate is given on figure 5. This contribution corresponds to the square term in the summation of Eq. 46 and can be compared to the unit value of the generalised mass. One can see that the cavity modes around 6000 Hz add the highest mass on this plate mode. The eigenfrequency of this mode is around 1238 Hz. By taking into account the added mass effect with Eq. (47), one obtains a modified eigenfrequency of 1001 Hz. On figure 6, one proposes to observe the modal energies of the plate in function of the modified eigenfrequencies. The modal energies of highest values are then contained in the frequency band of excitation (contrary to the figure 2). This shows that the added mass on the plate modes is well due to the non resonant modes of the cavity which are coincidence in space. This added mass can be easily estimated with Eq. (47).

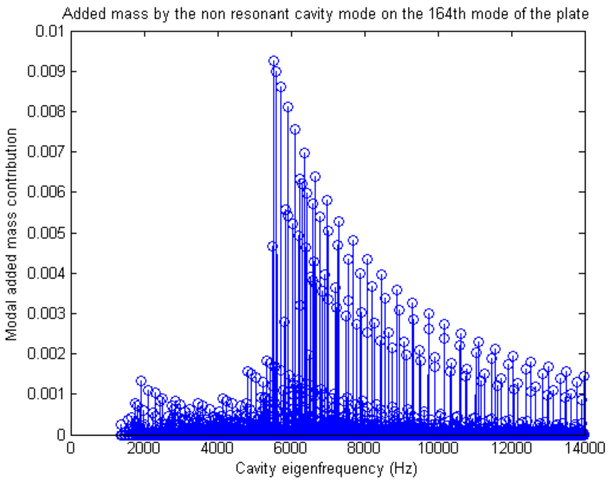

Figure 5. Added mass contribution of the non resonant modes of the cavity on the 164th mode of the plate. Test case 1.

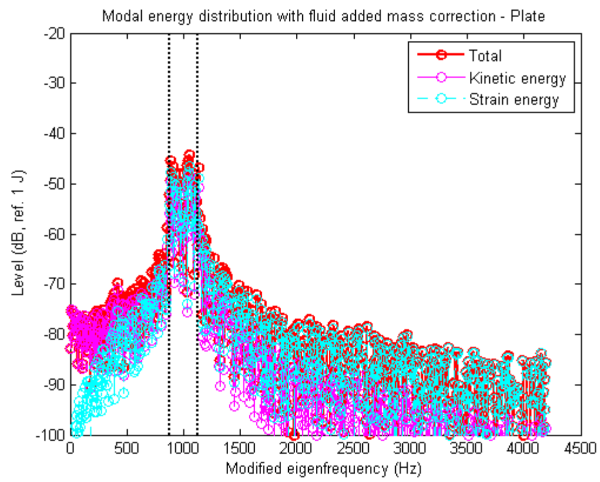

Figure 6. Modal energy distribution of the plate with correction on the plate eigenfrequency. Excitation of the plate in the third octave band 1000 Hz. Test case 1.

On another point, one can notice on figure 3 that the equivalence between the kinetic energy and the potential energy does not hold for the non resonant oscillator. The potential energies of the non resonant modes in space coincidence with the plate modes are very higher than their kinetic energies. By consequence, the kinetic energy of the cavity is well higher than its potential energy as shown on table 3. This focus on a matter in a SEA context when the total energy of the cavity should be estimated from the space averaged of the quadratic pressure of the cavity (see Eq. (8)). Indeed, to illustrate this matter, one has given in table 1 the total energies of the plate and the cavity estimate with Eq. (7-8) and the point responses at 100 positions in each subsystem (i.e. space average estimate from 100 points results). The total energy of the plate is correctly estimated by this approach but the total energy of the cavity is underestimated about 10 dB. Expressions (7-8) make the assumption that the kinetic and potential energies are equivalent whereas it is not the case for the cavity due to the significant non resonant mode contribution.

	Kinetic Energy	Strain Energy	Total Energy	Estimated Total Energy
E_1, Plate	-38.1 dB	-36.3 dB	-34.1 dB	-35.8 dB
E_2, Cavity	-40.5 dB	-50.6 dB	-40.1 dB	-49.1 dB
E_2/E_1			-6.0 dB	-13.4 dB

Table 3. Values of the kinectic, potential and total energies of the two subsystems. Excitation of the plate in the third octave band 1000 Hz. Test case 1.

5.2 Energy ratio versus damping loss factor

We are now interesting on the effect of the structural damping on the energy levels. This is of practical interest for noise control in the cavity.

On figure 7, one compares, for two plate damping values (η_1=0.01, η_1=0.1), the spectrum of the space average of the quadratic velocity of the plate and the spectrum of the space average of the quadratic pressure of the cavity. These space averages have been estimated from the mean about 100 points inside each subsystem. One can observe that the multiplication by 10 of the damping lead to a division by 10 of the quadratic velocity as it could be expected in reference to Eq. (9). On another hand, the quadratic pressure is not divided by 10; this is in contradiction with Eq. (10).

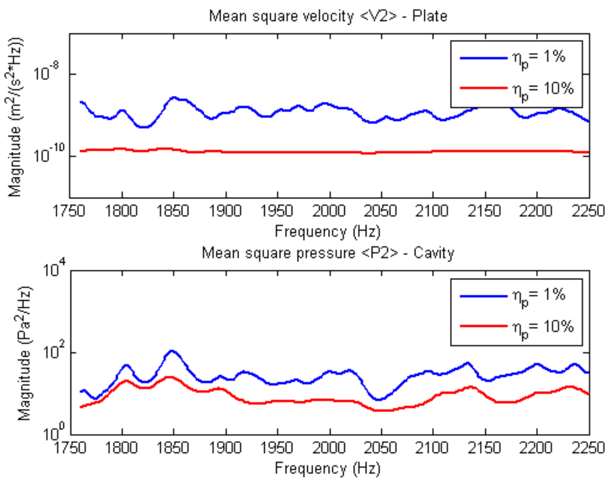

Figure 7. Comparison for two plate damping values (η_1=0.01, η_1=0.1) of the space averaged of the quadratic velocity of the plate (upper) and the quadratic pressure of the cavity (lower). Excitation of the plate in the third octave band 2000 Hz. Test case 1.

To study this point, one proposes for the same case, the modal energy distributions of the plate and the cavity on figure 8 and 9, respectively. The energy of the resonant modes of the plate decreases of 10 dB when the plate damping value is multiplied by 10 whereas the energy of the non resonant modes of the plate is practically unaffected by the damping changes. For the cavity, the energy of the non resonant modes which are coincidence in space with the resonant plate modes decreases of 10 dB whereas the energy of the other modes (resonant or non resonant) decreases only of 5 dB. The 10 dB energy decrease of the resonant modes of the plate leads to the 10 dB energy decrease of the non resonant modes of the cavity which are in space coincidence but it does not lead to the 10 dB energy decrease of the modes of the cavity which are in frequency coincidence. The consequence is that the total energy of the cavity does not decrease of 10 dB as it is emphasized on table 1 and 2 for the two test cases. Whatever the frequency band and the test case, a decrease of about 10 dB can be observed for the plate. For the cavity, the decrease varies between 6.5 dB and 10 dB. The variations are even more important if one considers the “pseudo” energies estimated from the space average of the sound pressure (i.e. between 5 dB and 10 dB). The role of the plate damping is then more or less important in function of the frequency band and the test case. The parameters which control the efficiency of the plate damping on the cavity response filled with a heavy fluid have not yet be identified. It is an important point if we would like to maximaze the damping effect on the noise reduction inside the cavity.

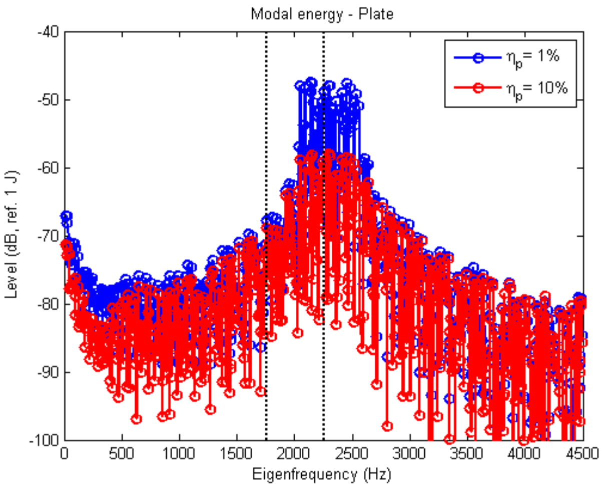

Figure 8. Comparison for two plate damping values (η_1=0.01, η_1=0.1) of the modal energy distribution of the plate. Excitation of the plate in the third octave band 2000 Hz (band limits symbolised with dashed lines). Test case 1.

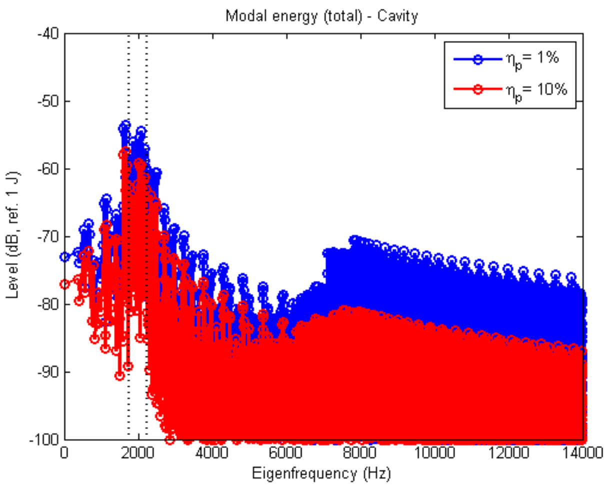

Figure 9. Comparison for two plate damping values (η_1=0.01, η_1=0.1) of the modal energy distribution of the cavity. Excitation of the plate in the third octave band 2000 Hz (band limits symbolised with dashed lines). Test case 1.

Central frequency	η_p	E_1 (dB)	EE_1 (dB)	E_2 (dB)	EE_2 (dB)	E_2/E_1 (dB)	EE_2/EE_1 (dB)
1000 Hz	0.01	-34.1	-35.8	-40.1	-49.1	-6.0	-13.4
	0.1	-43.6	-43.6	-49.5	-58.6	-5.8	-13.1
2000 Hz	0.01	-34.0	-38.6	-38.6	-44.9	-4.6	-6.3
	0.1	-43.3	-48.5	-45.1	-49.9	-6.3	-1.4
3150 Hz	0.01	-34.4	-40.2	-38.7	-46.0	-4.3	-5.8
	0.1	-43.8	-50.2	-44.8	-51.7	-1.0	-1.5

Table 4. Comparison for different plate damping values and different third octave band of the total energy of the two subsystem: (a), E_i, total energy calculated with the modal energy of subsystem *i* (see Eq. 31,37); (b), EE_i, total energy estimated from the space averaged of the physical field of subsystem *i* (see Eq. 7,8). Test case 1.

Central frequency	η_p	E_1 (dB)	EE_1 (dB)	E_2 (dB)	EE_2 (dB)	E_2/E_1 (dB)	EE_2/EE_1 (dB)
1000 Hz	0.01	-35.7	-37.2	-38.6	-43.1	-2.9	-5.9
	0.1	-44.2	-46.3	-47.3	-51.8	-3.1	-5.5
2000 Hz	0.01	-34.7	-39.4	-38.4	-44.7	-3.6	-5.3
	0.1	-43.4	-48.2	-44.9	-50.5	-1.5	-2.3
3150 Hz	0.01	-34.9	-40.8	-37.6	-44.0	-2.7	-3.2
	0.1	-43.9	-49.9	-44.1	-50.2	-0.2	-0.3

Table 5. Idem than table 4 for test case 2.

6 CONCLUSIONS

The vibro-acoustic behaviour of an excited structure coupled with a receiving acoustic cavity filled with a heavy fluid has been studied using the Dual Modal Formulation. By analysing the modal energy distributions of the plate and the cavity, one has observed that the fluid added mass and the non-resonant coupling have a strong effect on the energy flow between the structure and the cavity for frequencies below the critical frequency. The presence of a heavy fluid induces interaction between the plate and the cavity which is more difficult to analyse than for a light fluid. Comparisons with different plate damping values have shown than the plate damping is less efficient on the cavity response than on the plate one. This is in violation of the classical SEA model which can be explained by the non respect of the weak coupling assumption for a cavity filled with a heavy fluid. The role of the plate damping is then more or less important in function of the frequency band and the considered case. The parameters which control the damping efficiency on the cavity response have not been identified. It is an important point which should be addressed in the future in order to maximaze the damping effect on the noise reduction inside the cavity.

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