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Active vibration control of clamped beams using PPF controllers with piezoceramic actuators

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ABSTRACT

This paper reports active vibration control of clamped beams using positive position feedback (PPF) controllers. The control actuator is considered to be a piezoceramic patch (PZT). We then considered PPF control to overcome the limitations of instability. We first implement a single mode PPF controller and obtain a significient reduction of vibration at the tuned mode. We also implement a multi-mode PPF controller under single channel control scheme. It follows that a good reduction performance can be obtained at the first and third modes. The presented multimode PPF controller can be suggested for active vibration feedback controller having a large gain margin.

INTRODUCTION

Mechanical developments recently require the design of structure excluding unintended vibration. When mechanical structures are in activating, noise and unwanted disturbances follow inevitably. These conditions can be led some weakness of the structural reliability, inaccuracy of the function or structural damages. To solve these problems vibration control should be incorporated to satisfy the requirements. There are two classical methods to reduce structural vibrations. One is a redesign of the structure. This method leads high expenses. The other method is to apply passive damping materials to the structures. This passive treatment works more effectively at higher frequencies. The damping material treatment has practical limitations to control low frequency vibrations which dominantly affect the structural weakness and damage. Alternative method to control structural vibration is active vibration control (AVC) techniques emerged as viable technologies to control in low frequency range[1]. There is a representative active vibration control method, positive position feedback (PPF).

Positive position feedback (PPF) control method is used to suppress the vibration of large flexible structures presented by Fanson and Caughey[2] and Goh and Caughey[3]. PPF controller has advantages as compared with widely used velocity feedback control laws. PPF control is insensitive from unmodeled modes[2, 4]. Since the system response of the PPF controller quickly reduces at high frequencies due to its feature of a second order low pass filter. In addition, PPF controller is easy to implement. Because of these advantages, PPF controller along with smart materials, in particular PZT, has been applied to many flexible systems to do active control.

The disadvantage of PPF controller is that one channel of PPF can control only one mode. So the number of channel is required as much as that of mode which we want to control. To control multi-mode effectively, the multi-input and multi-output (MIMO) PPF controller may need. This can cause the cost to be high. Hence, the study of PPF control is kept going on by many researchers. Friswell studied PPF controller of SISO and MIMO cases[5]. Kwak applied PPF controller to grid structure such as a solar panel commonly mounted on satelites[6]. He utilized a digital signal processor for MIMO PPF controller. Rew presented adaptive positive position feedback controller(APPF)[7]. APPF controller effectively suppressed the target modes under variation of the structure. APPF had some limitation. No significant performance reduction had been observed with respect to approximately 10% frequency changes of the corresponding modes. The PPF controller has been widely researched. The feedback control system must have own open loop transfer function. The open loop transfer function shows the characteristics of the control system. To understand the control system, the study of the open loop transfer function must be needed.

In this paper a clamped-clamped beam is considered to be actively controlled using PPF controller with a sensor/actuator pair. Firstly, equation of motion of a clamped beam with lumped masses of sensors/actuators was explained. Secondly, PPF control method was rearranged for the study. Then, study of PPF controller design parameters was followed using open loop transfer fuction. Although SISO control, multi-mode control was tried. Finally, the controlled result by designed controller will be shown.

EQUATION OF MOTION OF CLAMPED BEAMS WITH LUMPED MASSES OF SENSORS AND ACTUATORS

Consider a clamped-clamped beam having lumped masses as shown in Figure 1. The lumped masses are modelled to cope with the experimental setup in the further study. The lumped masses represent the masses of force transducer, PZT actuator, and accelerometer.

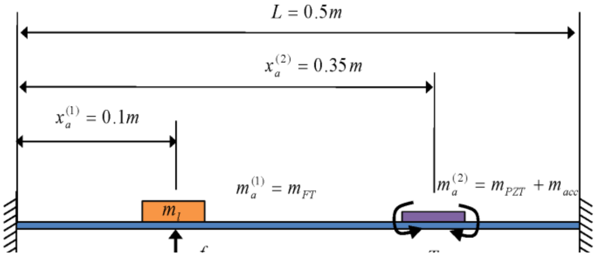

Figure 1. Simulation model for active feedback control system for a clamped beam with a sensor/actuator pair system.

External disturbances along the beam make structural vibrations, through the force transducer the structural vibration signal can be measured. PZT actuator generates moment pair to the beam. Because the sensors and actuators are attached to the beam, they need to take into account their mechanical influences of them on dynamics of the beam[8]. In this study, a force transducer is modeled by a lumped mass which is attached to the beam. Also, PZT actuator and accelerometer are modeled by lumped masses. Beacuse their behaviors like as a concentrated mass in the low frequency range. Using these mechanical properties, the total kinetic energy, the total potential energy, and the generalized non-consertive force can be obtained by Lagrange's equation [9].

$$\mathrm{KE} = \frac{1}{2}\int_0^L \rho A\left(\frac{\partial y}{\partial t}\right)^2 dx + \frac{1}{2}m_a^{(1)}\left\{\frac{\partial y}{\partial t}\delta\left(x - x_a^{(1)}\right)\right\}^2 + \frac{1}{2}m_a^{(2)}\left\{\frac{\partial y}{\partial t}\delta\left(x - x_a^{(2)}\right)\right\}^2 , \quad (1)$$

$$\mathrm{PE} = \frac{1}{2}\int_0^L EI\left(\frac{\partial^2 y}{\partial x^2}\right)^2 dx , \quad (2)$$

$$\mathrm{Q} = f_p(x,t) + \frac{\partial T_s(x,t)}{\partial x} - j\eta EI\frac{\partial^4 y}{\partial x^4} \quad (3)$$

where KE is the total kinetic energy, PE is the total potential energy and Q is the generalized non-conservative forces including the internal dissipative forces of the beam and external forces and moments applied to the beam. L is the length of the beam, ρ is mass density, A is cross sectional area, $m_a^{(1)}$ is the mass of a force transducer on the beam and $m_a^{(2)}$ is the mass which is a sum of a PZT patch mass and an accelerometer mass on the beam. E is Young's modulus, I is the moment of inertia of the beam. $f_p(x,t)$ and $T_s(x,t)$ are the applied external force and control moment which are generated by the actuators and η is loss factor.

Substituting equation (1), (2) and (3) into Lagrange's equation, the equation of motion can be written as

$$EI(1 + j\eta)\frac{\partial^4 y}{\partial x^4} + \rho A\frac{\partial^2 y}{\partial t^2} = f_p(x,t) + \frac{\partial T_s(x,t)}{\partial x} - m_a^{(1)}\frac{\partial^2 y}{\partial t^2}\delta\left(x - x_a^{(1)}\right) - m_a^{(2)}\frac{\partial^2 y}{\partial t^2}\delta\left(x - x_a^{(2)}\right) \quad (4)$$

For harmonic motions, y can assume that the response can be expressed as

$$y(x,\omega,t) = Y(x,\omega)e^{j\omega t} \quad (5)$$

Also, the general solution, $Y(x,\omega)$, can be expressed as a sum of theweighted modal function as

$$Y(x,\omega) = \sum_{n=1}^{N} w_n(\omega)\phi_n(x) , \quad (6)$$

where ϕ_n is the modal function and w_n is the modal displacement. Substituting equation (5) and (6) into (4) , and utilizing the orthogonality lead to matrix equation

$$\left[-\omega^2\mathbf{M} + j\omega\mathbf{D} + \mathbf{K}\right]\{w\} = \{f\} \quad (7)$$

where

$$\mathbf{M} = \rho AL\mathbf{I} + \sum_{i=1}^{2} m_a^{(i)}\left\{\phi\left(x_a^{(i)}\right)\right\}\left\{\phi\left(x_a^{(i)}\right)\right\}^T , \quad (8)$$

$$\mathbf{D} = 2\sqrt{\rho A\frac{EI}{L^2}}\mathrm{diag}\left(\zeta_n(\beta_n L)^2\right) , \quad (9)$$

$$\mathbf{K} = \frac{EI}{L^3}\mathrm{diag}\left((\beta_n L)^4\right) , \quad (10)$$

$$\{w\} = \{w_1 \quad w_2 \quad \cdots \quad w_n\}^T . \quad (11)$$

β_n in equation(9) and (10) can be evaluated from the frequency equation of clamped-clamped beams as

$$\cos\beta_n L\cosh\beta_n L = 1 . \quad (12)$$

Defining the modal vector as

$$\{\phi\} = \{\phi_1 \quad \phi_2 \quad \cdots \quad \phi_n\}^T , \quad (13)$$

the modal force vector, $\{f\}$, can be obtained

$$\{f\} = \{F_n\} = \int_0^L \left(F_p(x) + \frac{\partial T_s(x)}{\partial x} \right) \{\phi(x)\} dx \quad . \quad (14)$$

When a force and a moment are concentrated at $x_p = x_a^{(1)}$ and $x_s = x_a^{(2)}$, respectively, the modal force vector of equation (14) can be expressed as

$$\{f\} = F_p \{\phi(x_p)\} - T_s \left. \frac{\partial \phi(x)}{\partial x} \right|_{x=x_x} \quad . \quad (15)$$

Figure 2 shows the calculated receptance at 0.7*L* of mass-loaded beam excited by a force at 0.2*L*. The lumped mass at $x_a^{(1)}$ is the mass of the upper part of the force transducer. Also, the mass of accelerometer and PZT at $x_a^{(2)}$ is taken into account. The mechanical properties of the beam and masses are liseted in Table 1.

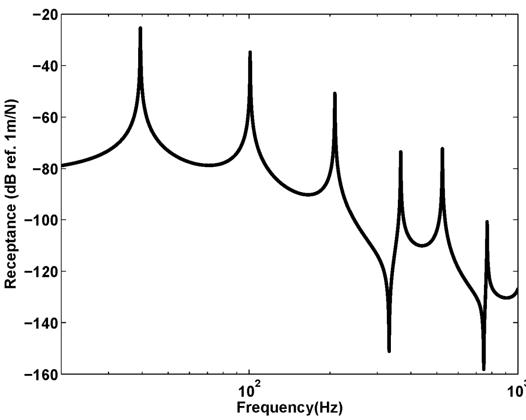

Figure 2. The calculated receptance of the beam at 0.7*L* of mass-loaded beam which is excited by a force at 0.2*L*.

Table 1. Mechanical properties and masses.

	symbol	value	unit	Descriptions
Beam	E	78	GPA	Young's modulus
	ρ	2850	kg/m^3	Density
	υ	0.3		Poisson ratio
	L	0.5	m	Length
	b	0.03	m	Width
	h	0.002	m	Thickness
	η	0.002		Loss factor
Mass	m_{FT}	0.006	kg	Mass of upper part of force transducer
	m_{PZT}	0.009	kg	Mass of PZT
	m_{acc}	0.0024	kg	Mass of accelerometer

Positive Position Feedback Control

The PPF control has some advantages relative to other control techniques. We can increase the damping of a specific frequency band with the PPF control. Hence, we may tackle the target mode which needs to be suppressed. The realization of the PPF control is easy since its function is same as the low-pass filter. However, one PPF controller can suppress only one mode at a time.

To operate the PPF controller, it needs structure modal displacement information. Using structural modal displacements, the compensators tuned at specific modes make control signals of designed modes. Control signals go to actuators. Finally, the disturbances of structure are controlled. The equations describing PPF operation are given as [2, 4]

$$\text{Structure}: \quad \ddot{q} + 2\zeta_s \omega_s \dot{q} + \omega_s^2 q = g \omega_s^2 p \quad (16)$$

$$\text{Compensator}: \quad \ddot{p} + 2\zeta_c \omega_c \dot{p} + \omega_c^2 p = \omega_c^2 q \quad , \quad (17)$$

where q is a modal coordinate describing displacement of he structure to control, ζ_s and ω_s are damping ratio and natural frequency of the structure, respectively. g is feedback gain, p is the compensator coordinate, ζ_c and ω_c are the damping ratio and natural frequency of the compensator, respectively. From equation (16), the transfer function of compensator can be written as

$$H(\omega) = \frac{\omega_c^2}{\omega_c^2 - \omega^2 + j2\zeta_c \omega_c} \quad . \quad (18)$$

Figure 3 shows the block diagram for active vibration control using PPF controller. $\vec{X}_p$ is modal displacement of structure excited by external forces, $\mathbf{B}_s$ is the participation matrix for sensors, $\mathbf{E}_c$ is control mode extraction matrix, $\mathbf{H}(\omega)$ is the transfer function matrix of the PPF controller, $\mathbf{E}_a$ is the mode rearrange matrix and $\mathbf{B}_a$ is the participation matrix for actuators. Plant response, $\mathbf{G}(\omega)$, and controller transfer function , $\mathbf{H}_{PPF}(\omega)$, can be rewritten as

$$\mathbf{G}(\omega) = \mathbf{B}_s \mathbf{\Phi} \left[\mathbf{K} - \omega^2 \mathbf{M} + j\omega \mathbf{D} \right]^{-1} \left[\frac{\partial \mathbf{\Phi}}{\partial x} \right]^T \mathbf{B}_a^T \quad (19)$$

$$\mathbf{H}_{PPF}(\omega) = \mathrm{diag}(g\omega_s^2) \mathbf{H}(\omega) \quad (20)$$

Measured sensing voltage is the result of the sum of disturbances by the external force and the structure responses by the control actuation. This can be written as

$$\vec{V}_s(\omega) = \mathbf{B}_s \mathbf{\Phi} \vec{X}_p(\omega) + \mathbf{G}(\omega) \vec{V}_a(\omega) \quad (21)$$

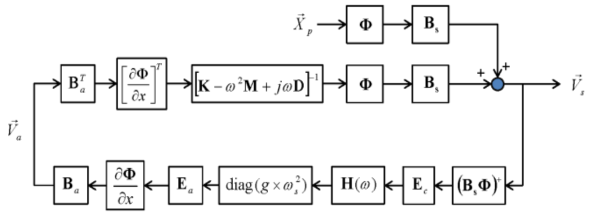

Figure 3. Block diagram for active vibration control using PPF controllers.

$\vec{V}_a(\omega)$ can be defined as

$$\vec{V}_a(\omega) = \mathbf{B}_a \frac{\partial \mathbf{\Phi}}{\partial x} \mathbf{E}_a \mathbf{H}_{PPF} \mathbf{E}_c [\mathbf{B}_s \mathbf{\Phi}]^+ \vec{V}_s(\omega) \quad , \quad (22)$$

where $^+$ denotes the pseudo inverse. Using equation (21) and (22), final controlled sensing voltage and actuator voltage for actuator activity by external force can be written as

$$\vec{V}_s(\omega) = \left[I - \mathbf{G}\mathbf{B}_a \frac{\partial \mathbf{\Phi}}{\partial x} \mathbf{E}_a \mathbf{H}_{PPF} \mathbf{E}_c [\mathbf{B}_s \mathbf{\Phi}]^+ \right]^{-1} \times \mathbf{B}_s \mathbf{\Phi} \vec{X}_p(\omega) \quad , \quad (23)$$

and

$$\vec{V}_a(\omega) = \mathbf{B}_a \frac{\partial \mathbf{\Phi}}{\partial x} \mathbf{E}_a \mathbf{H}_{PPF} \mathbf{E}_c [\mathbf{B}_s \mathbf{\Phi}]^+ \times \left[I - \mathbf{G}\mathbf{B}_a \frac{\partial \mathbf{\Phi}}{\partial x} \mathbf{E}_a \mathbf{H}_{PPF} \mathbf{E}_c [\mathbf{B}_s \mathbf{\Phi}]^+ \right]^{-1} \times \mathbf{B}_s \mathbf{\Phi} \vec{X}_p(\omega) \quad . \quad (24)$$

Effects of the design parameters

Equation (23) represents the steady state response in the PPF control system using a collocated sensor/actuator pair. To ensure the reduction of the steady-state response system, open loop transfer function can be written as

$$\mathrm{OLTF}(\omega) = -\mathbf{G}\mathbf{B}_a \frac{\partial \mathbf{\Phi}}{\partial x} \mathbf{E}_a \mathbf{H}_{PPF} \mathbf{E}_c [\mathbf{B}_s \mathbf{\Phi}]^+ \quad (25)$$

If the Nyquist plot of open loop transfer function encircle the (*-1,j0*) point, the control system goes to unstable state. For effective control, the phase of open loop transfer function must be in ±90◦. The responses of out of phase in ±90◦ lead to enhance the structural responses. The PPF controller design parameters are controller's damping ratio and gain. To verify the effects of parameters, consider a SISO control for the 1st and 3rd modes. Figure 4 shows the open loop transfer function effects by the 1st mode controller damping ratio change.

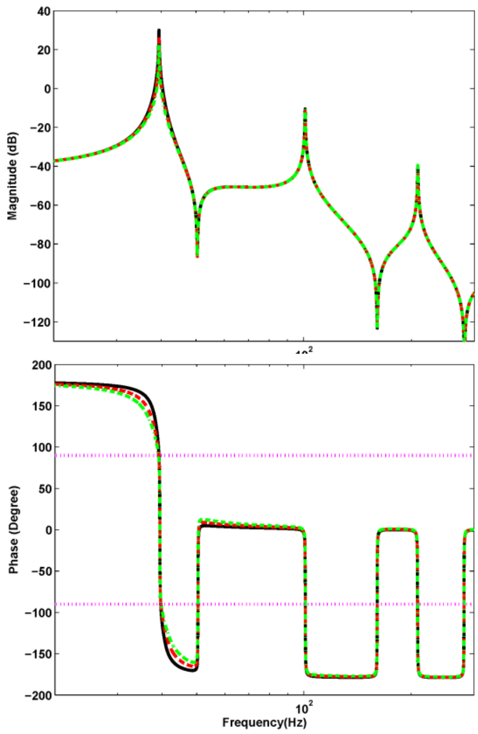

Figure 4. The effect of magnitude as the 1st mode controller damping ratio change of open loop transfer function of the clamped beam at 0.7*L* which is excited by a force at 0.2*L*, and set the 1st mode controller gain, 4×10^{-4}, damping ratio, 3×10^{-2}(solid line), 5×10^{-2}(dashed line) and 7×10^{-2}(dash-dotted line).

Set the 1st mode controller damping ratios are 3×10^{-2}(solid line), 5×10^{-2}(dashed line) and 7×10^{-2}(dash-dotted line). The controller gain was 4×10^{-4}. The damping ratio of the 1st mode controller effects on the designed mode. By increasing the damping ratio of the 1st mode controller, the amplitude of the control mode was decreased. The region of controller effect is under ±90◦ in the phase of open loop transfer function responses. The responses of out of range from -90◦ and +90◦ lead to reduce the performance of the controllers. Also, the region of 360◦ phase inverse is the effective zone by controller. By increasing the damping ratio, the region of phase change around target mode was widen. Figure 5 shows the open loop transfer function effects by the 1st mode controller gain change. Set the 1st mode controller gains are 5×10^{-5}(solid line), 1×10^{-4}(dashed line), 2×10^{-4}(dash-dotted line) and 4×10^{-4}(dotted line). The 1st mode damping ratio was fixed as 5×10^{-2}. The gain change of the 1st mode controller effects on all modes. By increasing the gain of the 1st mode controller, all responses were increased, proportionally. Although the 1st mode gain was changed, the phases of all modes were not changed. Figure 6 shows the open loop transfer function effects by the 3rd mode controller damping ratio change. Set the 3rd mode controller damping ratios are 3×10^{-2}(solid line), 5×10^{-2}(dashed line) and 7×10^{-2}(dash-dotted line). The 3rd mode gain was fixed as 4×10^{-5}. The damping ratio change of the 3rd mode controller effects on the only 3rd mode response. By increasing the damping ratio of the 3rd mode controller, the magnitude of the open loop transfer function was decreased. The 360◦ phase inverse appeared at only target mode. By increasing the damping ratio of the 3rd mode controller, the region of phase change around target mode was widen. Figure 7 shows the open loop transfer function effects by the 3rd mode controller gain change.

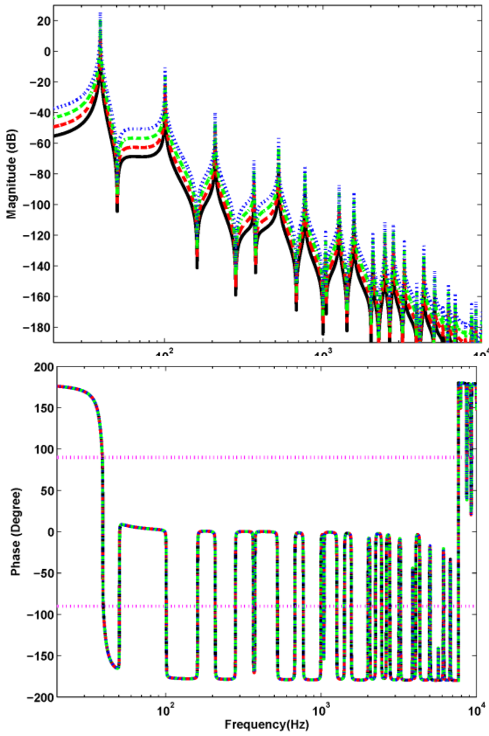

Figure 5. The effect of magnitude as the 1st mode controller gain change of open loop transfer function of the clamped beam at 0.7*L* which is excited by a force at 0.2*L*, and set the 1st mode controller damping ratio, 5×10^{-5}, gain, 5×10^{-5}(solid line), 1×10^{-4}(dashed line), 2×10^{-4}(dash-dotted) and 4×10^{-4}(dotted line).

Set the 3rd mode controller gains are 5×10^{-6}(solid line), 1×10^{-5}(dashed line), 2×10^{-5}(dash-dotted line) and 4×10^{-5}(dotted line). The 3rd mode damping ratio was fixed as 5×10^{-2}. By increasing the gain of the 3rd mode controller, all responses were increased, proportionally. And the resonance magnitudes under the target mode show high amplitudes. This phenomenon shows that the function of PPF control is similar to the that of low pass filter. By increasing the gain of the 3rd mode controller, the phases of all modes were not changed. In a nutshell, the higher mode PPF controllers effect on the lower modes. The damping ratios of target mode controllers have influence on the designed modes. The gains of target mode controllers can affect all modes. Figure 8 shows final designed controller of open loop tranfer function by try and error. Designed parameters are followed. The 1st mode controller gain was 4×10^{-4} and 3rd mode controller gain, 2×10^{-5}. And both controller damping ratios are 5×10^{-2}. The open loop transfer function amplitudes of tuned modes are almost 25dB. Also, the phase responses of final tuned open loop transfer function show the phase inverse in $\pm90\circ$ at target modes. It is expected to work well by designed controller.

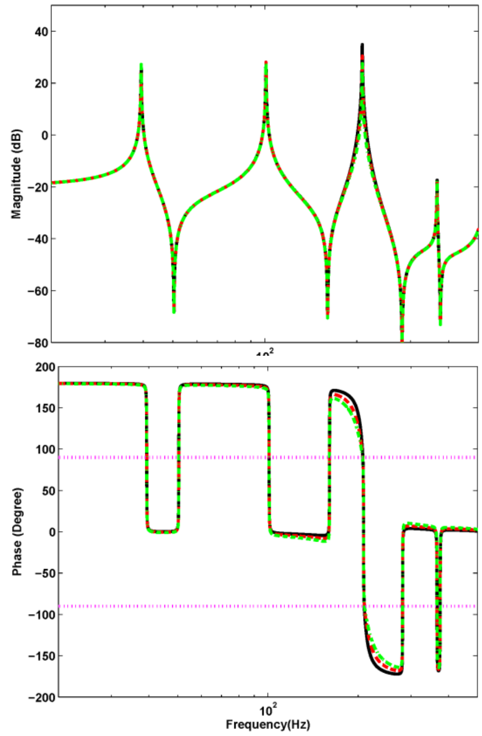

Figure 6. The effect of magnitude as the 3rd mode controller damping ratio change of open loop transfer finction of the clamped beam at 0.7*L* which is excited by a force at 0.2*L*, and set the 3rd mode controller, gain, 4×10^{-5}, damping ratio, 3×10^{-2}(solid line), 5×10^{-2}(dashed line) and 7×10^{-2}(dash-dotted line).

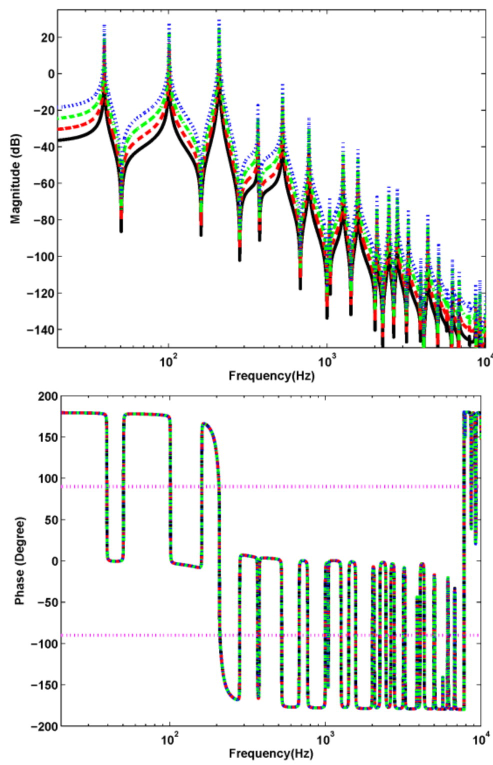

Figure 7. The effect of magnitude as the 3rd mode controller gain change of open loop transfer finction of the clamped beam at 0.7*L* which is excited by a force at 0.2*L*, and set the 3rd mode controller damping ratio, 5×10^{-2}, gain, 5×10^{-6}(solid line), 1×10^{-5}(dashed line), 2×10^{-5}(dash-dotted line) and 4×10^{-5}(dotted line).

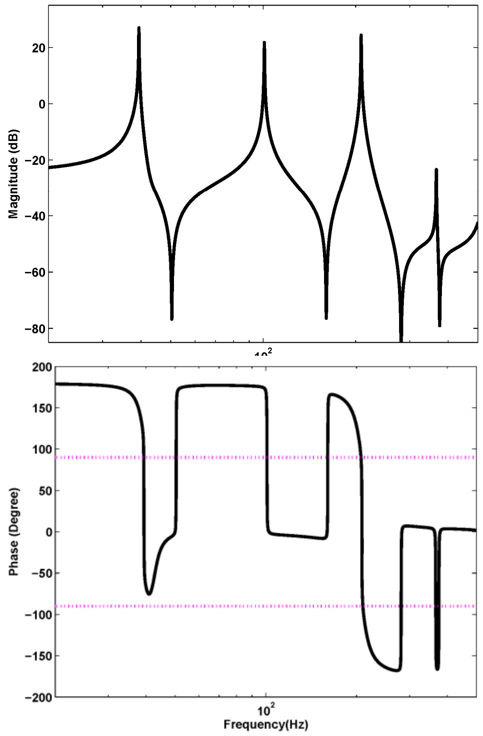

Figure 8. The final tuned open loop transfer function response of the clamped beam at 0.7*L* which is excited by a force at 0.2*L*, and set the 1st mode controller gain, 4×10^{-4}, 3rd mode controller gain, 2×10^{-5} and all controller damping ratios are 5×10^{-2}.

Implementation of SISO PPF controller

Designed PPF controllers were implemented to structure. To express the performances of controller, the total kinetic energy (TKE) of the beam is used. They can be expressed by

$$\mathrm{TKE} = 10\log_{10}\int_{\omega_1}^{\omega_2}\frac{\rho A}{2L}\vec{v}^{H}\vec{v}\,d\omega \quad , \quad (25)$$

where the superscript H indicates the Hermiltion transpose, ω_1 and ω_2 are the lower and upper frequency of the frequency range of interest. Figure 9 shows the SISO PPF control performance of the clamped beam excited by concentrated force only at 0.2L(solid line), and when subjected to the control moment pair at 0.7L(dashed line). The total kinetic energy of the beam without control was 16.56dB and those under feedback control was 5.43dB. The total kinetic energy along the clamped beam using the PPF controller was reduced 11.13dB.

Conclusion

This paper considered a clamped beam with sensor/actuator by theoretical method. In theoretical analysis, mass-loaded effect was regarded to the clamped beam. To control the structure, PPF controller was applied. As controller design parameters, damping ratio and gain of controller were considered by open loop transfer function.

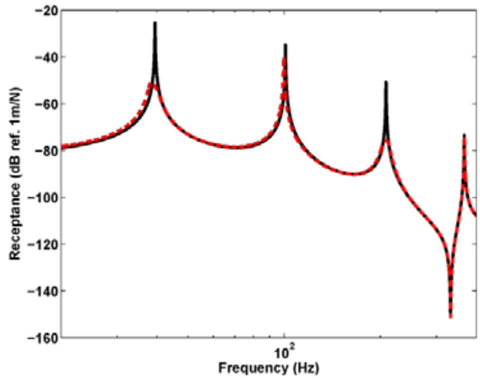

Figure 9. SISO PPF control performance of the clamped beam excited by the moment pair at 0.7*L*, and uncontrolled response(solid line) and controlled response(dashed line).

The damping ratios of target mode controllers have influence on the designed modes. The gain of target mode controllers can affect on all modes. According to the open loop transfer function, the function of PPF control was silimar to the that of low pass filter. PPF controller was designed reflecting on the PPF controller parameters properties to control the 1st mode and the 3rd mode. As a result of this study, the total kinetic energy along the clamped beam using the PPF controller was reduced 11.13dB than without controller

REFERENCES

1 S.O.Reza Moheimani and Dunant Halim, *Spatial control of vibration theory and experiments* (World Scientific 2003)

2 J L Fanson and Caughey T K, *Positive position feedback control for large space structures*. AIAA. **28**, 717-724 (1990)

3 C J Goh and Caughey T K, *On the stability problem caused by finite actuator dynamics in the collocated control of large space structures*. International Journal of Control. **41**, 787-802 (1985)

4 G Song and S P Schmidt, *Experimental robustness study of positive position feedback control for active vibration suppression*. Journal of Guidance. **25**, 179-182 (2002)

5 Michael I. Friswell and Daniel J.Imman, *The relationship between positive position feedback and output feedback controllers,* Smart Materials and Structures. **8**, 285-291 (1999)

6 Moon K.Kwak and Seok Heo, *Active vibration control of smart grid structure by multiinput and multioutput position feedback controller,* Journal of Sound and Vibration. **304**, 230-245 (2007)

7 Kun-Ho Rew and Jae-Hung Han, *Multi-modal vibration control using adaptive positive position feedback,* Journal of Intelligent Material System. **13**, 13-22 (2002)

8 Chinsuk Hong, *Active control of resiliently-mounted flexible structure* (Institute of sound and vibration research, 2005)

9 L. Meirovitch, *Analysis methods in vibration* (The Macmillan Company, 1967