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Analysis of the radiation impedance with effect of reflected wave from sonar-dome in a cylindrical array

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ABSTRACT

Radiation impedance is one of the most important factors in designing an underwater sonar system. In such a system, transducer array is surrounded by a dome to protect its elements from the underwater environment and then the effect of a reflected wave on radiation impedance cannot be ignored. To analyze the effect in a cylindrical array, a simplified model was introduced and radiation impedance was investigated by considering the parameters, such as the separation distance from the reflector, the interval between elements, element size, and reflection coefficient. It is clarified that the effect of a reflected wave increases with ka and the reflection coefficient, with decreasing the distance and the interval of elements. The radiation impedance of the array with 25 elements mounted on the cylindrical surface is analyzed experimentally when the array has a dome reflector. To verify the effect of the reflector, the input impedance of the array with the reflector was investigated with theoretically and experimentally. The experimental results show a similar tendency to the theoretical results. It is noted that the equivalent circuit model for theoretical analysis is useful for estimating the radiation impedance change caused by the reflected wave.

INTRODUCTION

In designing an underwater sonar system, such as a Hull-Mounted sonar, radiation impedance is one of the very important design factors because it is related to the radiation power of the system and the mutual interference force among vibrating elements[1]. The numerical calculation of radiation impedance with good precision for a cylindrical array is not easy because it requires a very long computation time. The previous work demonstrated that the radiation impedance of a cylindrical array can be calculated efficiently using the calculation algorithm proposed by our team[2]. However, in a practical sonar system, the array is surrounded by a dome to protect its elements from the underwater environment, such as flow resistance and shock pressure. A radiated acoustic wave from the elements is then reflected from the surface of the dome and the reflected wave has an significant effect on not only a vibrating element itself but also the mutual interference among the elements. These effects, therefore, cannot be ignored in the calculation of radiation impedance. Although Ikeda's group studied these problems in 1970, it has not been sufficiently investigated yet[3].

In this study, to analyze the effect of a reflected wave on radiation impedance, we introduced a simplified model in which two vibrating elements are mounted on an infinite planar rigid baffle and a plane reflector exists in front of the baffle. Using this model, the variations in radiation impedance with wave number and element size (ka), interval between elements (d), separation distance from the reflector (z), and reflection coefficient (Γ) were investigated. In the calculation, the ring function is introduced to evaluate the acoustic pressure distribution by the reflector[4]. To verify the effect of the radiation impedance, input impedance for the small array with 5×5 vibrators is also investigated experimentally. The input impedance is analyzed using an equivalent circuit with the radiation impedance.

THEORETICAL MODEL

Calculation of radiation impedance

If a transducer array consists of N elements on a rigid baffle, the mutual radiation impedance between the mth and nth elements is given by

$$Z_{mn} = \frac{1}{u_m} \int_{S_n} p_{mn} dS_n. \tag{1}$$

where u_m is the vibration velocity of the mth element, S_n is the area of the nth element, and dS_n is infinitesimal area of the area; p_{mn} is the affected pressure on the nth element due to the mth vibrating element. The total radiation impedance of the nth element is expressed as[5]

$$Z_n = \sum_{m=1}^{N} Z_{mn} \frac{u_m}{u_n}, \tag{2}$$

where u_n is the vibration velocity of the nth element and Z_{mn} is the mutual radiation impedance between the two elements. If an infinite plane reflector is placed in front of the transducer array with an arbitrary distance, then the total radiation impedance of the nth element can be expressed as

$$Z_n = \sum_{m=1}^{N} (Z_{mn} + Z'_{mn}) \frac{u_m}{u_n}, \tag{3}$$

where

$$Z'_{mn} = \frac{1}{u_m} \int_{S_n} p'_{mn} dS_n, \tag{4}$$

which is the mutual radiation impedance between the two elements due to the reflected pressure from the reflector. p'_{mn} is the sound pressure from the reflector due to the vibration of the mth element. In order to investigate the effect of a reflected wave on radiation impedance, we assume that a plane reflector that has a complex reflection coefficient Γ is placed in front

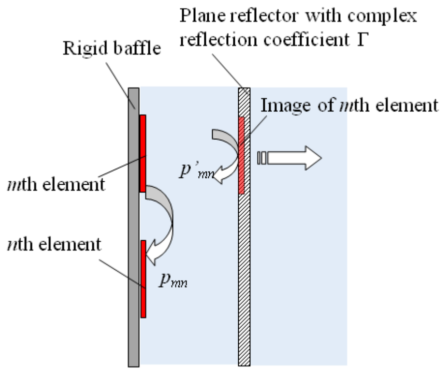

Figure 1: Geometry model of transducer array with a reflector.

of the two piston sources mounted on an infinite baffle[6], as shown in Figure 1. Reflection coefficient is determined by the thickness and characteristic acoustic impedance of the material. The sound pressure radiated by the mth element generates a reflected wave when the pressure strikes the reflector, and then the wave induces the vibration of the nth element. To analyze

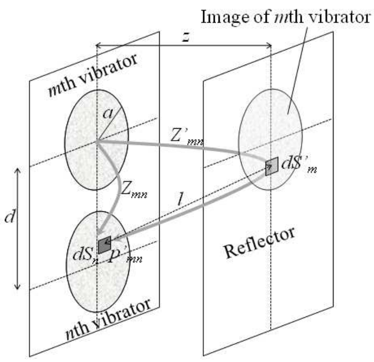

Figure 2: Equivalent model for calculation.

the effect of the reflected wave, we transformed the original configurations described above into an equivalent model that is obtained by substituting an image source for the reflector, as shown in Figure. 2. That is, it is considered that the reflector is removed and a image source exists on the image plane. Using the ring function[3], the sound pressure of a point on the circle of radius x in the baffle is given by

$$p'_{mn} = j\rho ck\Gamma \int_{l_N}^{l_F} R_1(l,x)\exp(-jkl)dl, \tag{5}$$

where

$$R_1(l,x) = \frac{1}{\pi}\cos^{-1}\left(\frac{x^2-a^2-z^2+l^2}{2x\sqrt{l^2-z^2}}\right), \tag{6}$$

$$l_F = \sqrt{(a+x)^2+z^2}, \tag{7}$$

$$l_N = \begin{cases} \sqrt{(a-x)^2+z^2}, & a < x \\ z, & 0 \le x \le a \end{cases}. \tag{8}$$

Here, ρ is the density, c is the sound velocity, k is the wave number of the medium, and l is the distance between the infinitesimal area dS'_m in the image source and dS_n. If $0 < x < a$ and $z < l < \sqrt{(a-x)^2+z^2}$, then $R_1(l,x) = 1$. Using eqs. (4)∼(7), the radiation impedance induced by the reflected wave Z'_{mn} can be rewritten as

$$Z'_{mn} = j2\pi\rho ck\Gamma \int_{x_N}^{x_F} xR_2(x)\left\{\int_{l_N}^{l_F} R_1(l,x)\exp(-jkl)dl\right\}dx, \tag{9}$$

where

$$R_2(x) = \frac{1}{\pi}\cos^{-1}\left(\frac{x^2+d^2-a^2}{2xd}\right), \tag{10}$$

$$x_F = a+d, \tag{11}$$

$$x_N = \begin{cases} d-a, & d > a \\ 0, & 0 \le d \le a \end{cases}. \tag{12}$$

If $0 < d < a$ and $0 < x < a-d$, then $R_2 = 1$.

Equivalent circuit for input impedance

On the basis of the equivalent circuit theory, the vibrator can be modeled by an RLC resonance circuit, as shown in Figure 3, and the relationship between the voltage and the current is expressed as follows:

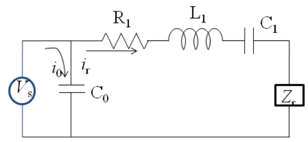

Figure 3: Equivalent circuit with acoustic radiation impedance.

$$\frac{1}{j\omega C_0}(i_0 - i_r) = V_s, \tag{13}$$

$$-\frac{1}{j\omega C_0}(i_0 - i_r) + \left\{R_1 + j\omega L_1 + \frac{1}{j\omega C_1} + Z_r\right\}i_r = 0. \tag{14}$$

From eqs. (13) and (14), an impedance matrix can be obtained as

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \times \begin{bmatrix} i_0 \\ i_r \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \end{bmatrix}. \tag{15}$$

Here,

$$z_{11} = \frac{1}{j\omega C_0}, z_{12} = z_{21} = -\frac{1}{j\omega C_0},$$
$$z_{22} = \frac{1}{j\omega C_0} + R_1 + j\omega L_1 + \frac{1}{j\omega C_1} + Z_r,$$

and Z_r is the acoustic radiation impedance, as given by

$$Z_r = \sum_{n=1}^{N} Z_n. \tag{16}$$

From the equivalent circuit and eq. (15), the input impedance of the vibrator is obtained using by the following equation for unit input voltage.

$$Z = z_{11} - \left(\frac{z_{12}z_{21}}{z_{22}}\right). \tag{17}$$

RESULTS AND DISCUSSIONS

To investigate the variation in radiation impedance induced by the reflected wave, the impedance for the variables z, d, ka and Γ are calculated using eqs. (3)∼(10). In this calculation, five materials were considered, and their thickness is 20 mm. The characteristic acoustic impedances of the reflectors and the reflection coefficients Γ are summarized in Table 1. Figure 4(a) and (b) show the total radiation resistance R_n and reactance X_n change with z. It can be seen from these results that radiation impedance was markedly affected by not only z but also Γ. In the case of $ka = 1.54$ and $z = 2.3\lambda$, the variations in the mutual radiation impedance with d and Γ are shown in Figure 5. In these figures, the d-axis is normalized by the wavelength λ. The effect from the reflectors on mutual radiation impedance decreased with d rapidly. In the case of reflector 1 whose Γ

is small, the effect was higher than that of without the reflector by 2% in resistance and 1% in reactance. However, in the case of reflector 5 whose Γ is large, the values changed to 10% in resistance and 9% in reactance. Figure 6 shows the effect of the reflected wave on radiation impedance according to ka for five different reflection coefficients of the reflector when $z = 250$ mm and $d = 2a$. The effect increased with ka and the Γ. The maximum differences in radiation impedance induced by the reflector were 15% in resistance and 14% in reactance for a given range of ka. The total radiation impedance with ka was calculated as shown in Figure 7. It can be confirmed that total radiation impedance fluctuates owing to the effect of the reflected wave. To verify the effect of the reflected wave on the radiation impedance, 25 tonpilz transducers are mounted on a cylindrical baffle, as shown in Figure 8. The thickness of the reflector with the acoustic impedance of 9.72 Mrayl is 2 cm, and the separation distance between the transducer array and the reflector is 30 cm. In the equivalent circuit, the mechanical characteristics of the tonpilz transducer (R_1, L_1, C_1) and the electric characteristic (C_0) of the piezoelectric vibrator are estimated with air load as follows: $R_1 = 212.8\ \Omega$, $L_1 = 133.2$ mH, $C_1 = 3.32$ nF, and $C_0 = 13.23$ nF. When only the 13th vibrator in Figure 8 is driven, the radiation impedance becomes to

$$Z_r = Z_{13} = Z_{1313} + Z'_{1313}. \tag{18}$$

The input impedance with the radiation impedance Z_r of the vibrator is calculated using eq. (17), and the results are shown in Figure 9 with the measured result. The measured input impedance with various ka is in good agreement with the theoretical one. For the five vibrators (11th, 12th, 13th, 14th and 15th) in Figure 8, the radiation impedance is represented as the following equation when all 25 vibrators are driven.

$$Z_n = \sum_{m=1}^{25} (Z_{mn} + Z'_{mn}) \frac{u_m}{u_n}. \tag{19}$$

Here $n = 11$, 12, 13, 14, or 15. Using the result of eq. (17) and eq. (18), the input impedance of each vibrator is calculated for various ka, and the results are shown in Figure 10 together with the measured result. The tendencies of the results change with the location of the vibrators because the mutual radiation impedance is strongly affected by the path difference between the vibrators. The pressure distribution of each vibrator surface is varied with the phase of the reflected wave because of constructive and destructive interference.

CONCLUSION

Radiation impedance including the effect of a reflected wave from a reflector was calculated. To analyze the effect on radiation impedance, not only the radiation variables such as d and ka, but also the reflection variables such as z and Γ are varied in the calculation. As results, radiation impedance markedly fluctuates according to the distance between the reflector and the transducer array. The amplitude of the fluctuation is proportional to the reflection coefficient of the reflector. For the distance d, radiation impedance rapidly decreased. With increasing ka, radiation impedance increases with fluctuation. Fluctuation amplitude also increases with Γ. The radiation impedance of the arrayed with 25 elements mounted on the cylindrical surface is analyzed experimentally when the array has a reflector. The radiation impedance is obtained more in the vibrator located at the center of the array because the symmetric phase of the reflection causes constructive interference. It can be confirmed that the 13th vibrator has the largest values in the experimental result as well as the theoretical result. From these results, it is not ed that the radiation impedance could be changed with not only the location of the vibrator but also the ka value. The estimated values in the equivalent circuit (R_1, L_1, C_0, and C_1) are obtained with the assumption of air load for single resonance. This results in the difference between the theoretical and experimental results in the minor peaks in Figure 10. It could be expected that the effect of the reflected wave from the sonar dome can be estimated by the suggested calculation method, although there is a small difference between the experimental and theoretical results. The result of this study would provide useful information for designing an underwater sonar system.

ACKNOWLEDGMENT

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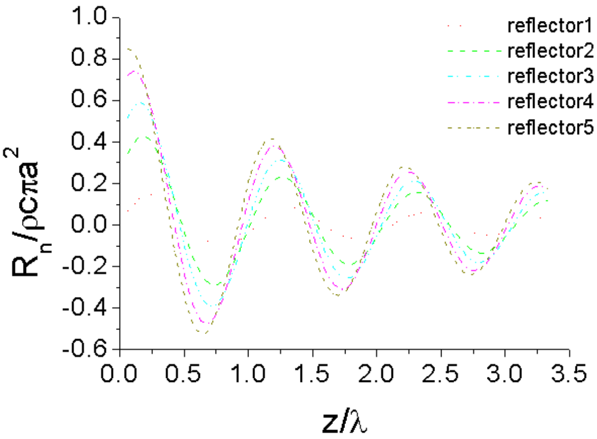
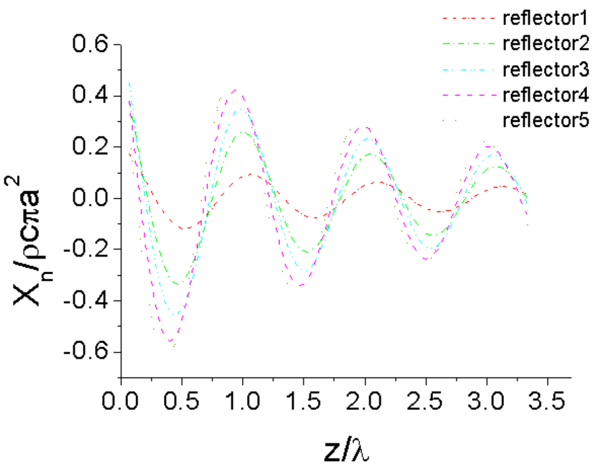
Table 1: Characteristic acoustic impedances and velocities of the reflectors used in the reflection coefficient calculation[6].

	reflector1	reflector2	reflector3
Acoustic impedance [Mrayl]	2.88	4.958	17.01
Sound velocity [m/s]	4000	2680	6300
Reflection coefficient $\Gamma = \dfrac{1}{A - jB\cot\{\frac{0.1257}{\lambda}\}}$	$A = 1.80$ $B = 1.50$	$A = 1.21$ $B = 0.69$	$A = 1.02$ $B = 0.18$

	reflector4	reflector5
Acoustic impedance [Mrayl]	27	46.97
Sound velocity [m/s]	6000	6100
Reflection coefficient $\Gamma = \dfrac{1}{A - jB\cot\{\frac{0.1257}{\lambda}\}}$	$A = 1.01$ $B = 0.11$	$A = 1.00$ $B = 0.066$

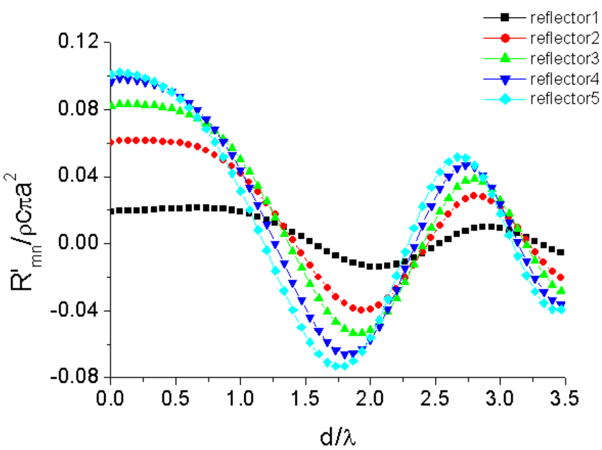


(a) Radiation resistance

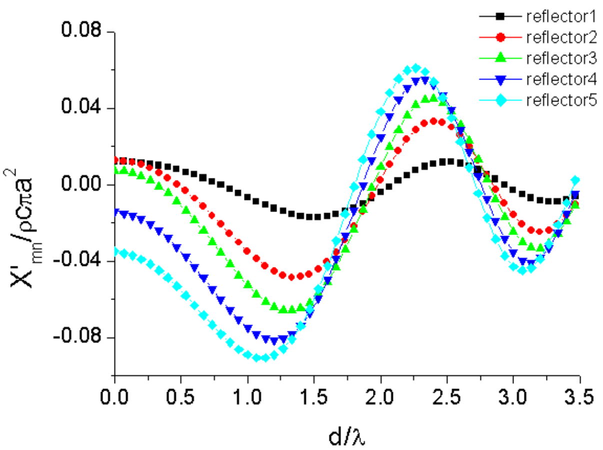


(b) Radiation reactance

Figure 4: Normalized total radiation impedance change with z.

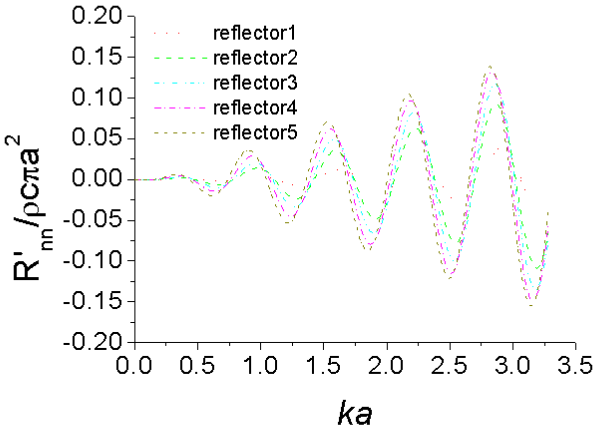


(a) Radiation resistance

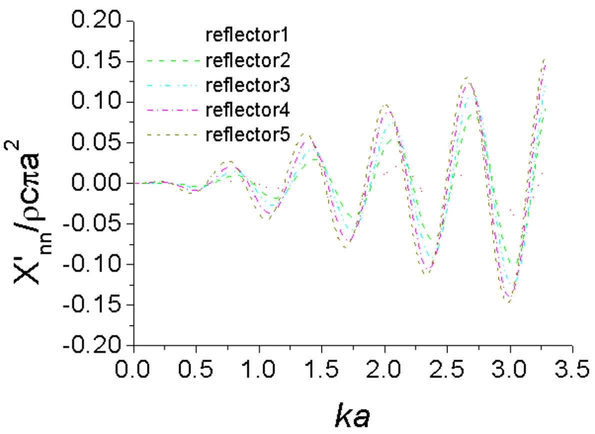


(b) Radiation reactance

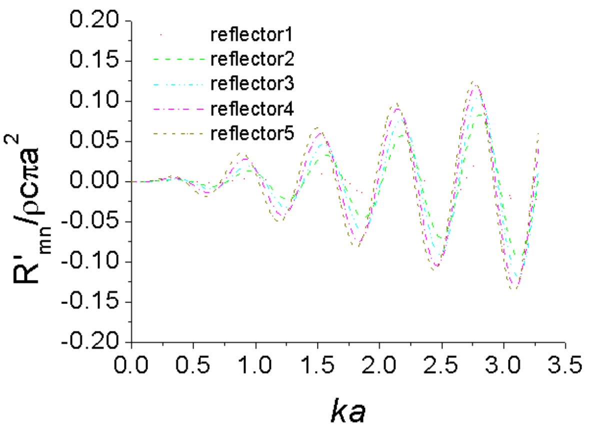
Figure 5: Effect of the reflectors with d.



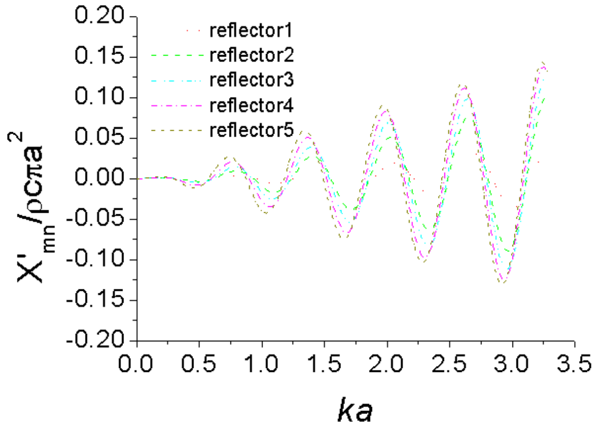
(a) Self-radiation resistance



(b) Self-radiation reactance



(c) Mutual-radiation resistance



(d) Mutual-radiation reactance

Figure 6: Effect of reflectors on radiation impedance according to *ka*.

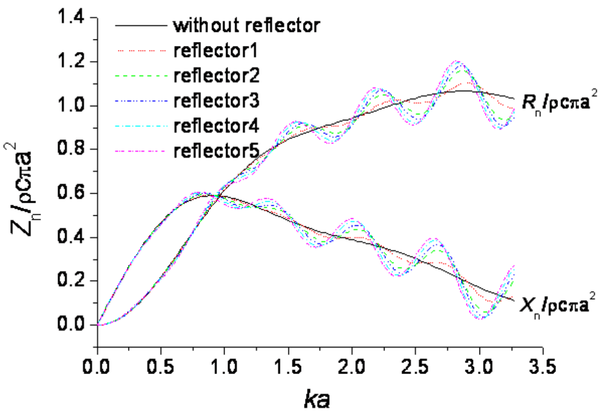


Figure 7: Normalized total radiation impedance change with *ka*.

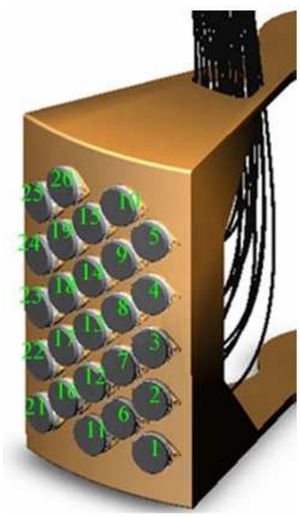


Figure 8: Structure of the 5×5 transducer array.

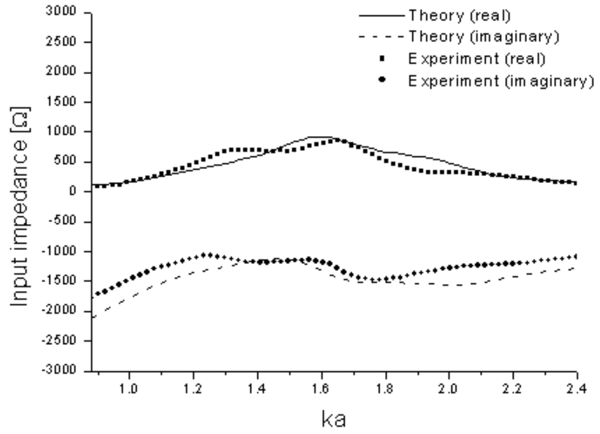
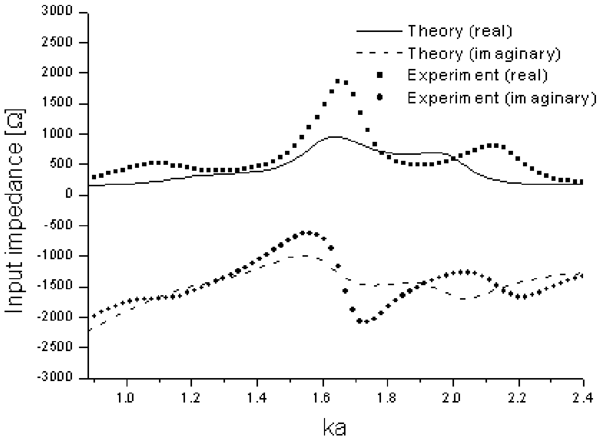
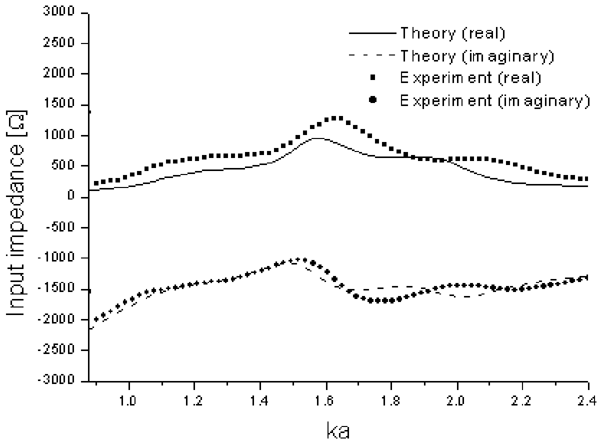


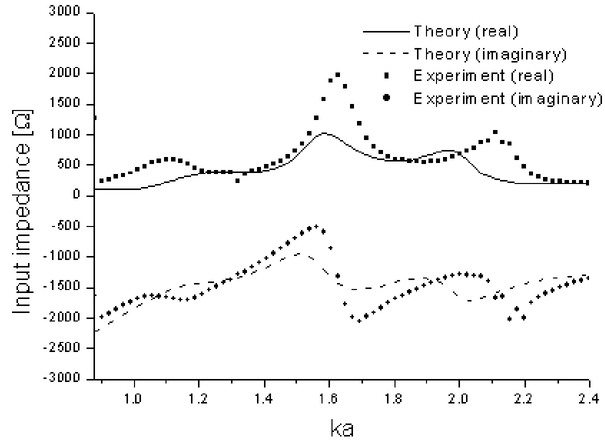
Figure 9: Input impedance of the 13th vibrator in the 5×5 transducer array with one driving vibrator.



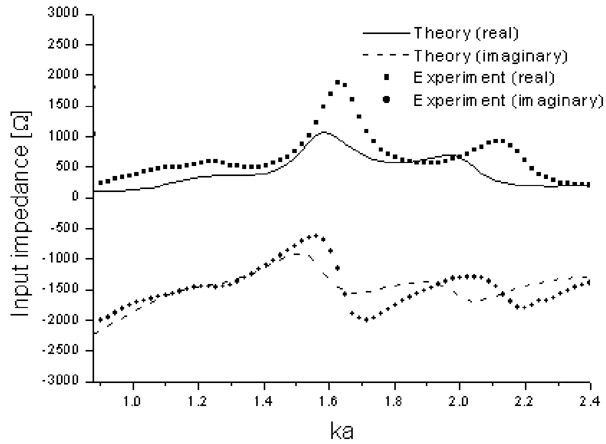
(a) 11th vibrator



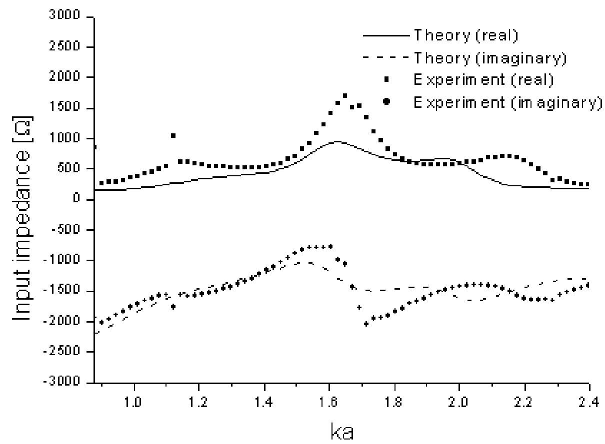
(b) 12th vibrator



(c) 13th vibrator



(d) 14th vibrator



(e) 15th vibrator

Figure 10: Input impedance of the five vibrators in the 5×5 transducer array when all 25 vibrators are driven.