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Acoustic waves in quasi-periodic multi-layered media

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ABSTRACT

The issue of sound attenuation in multi-layered structures leads to find optimal geometrical stacking of layers in order to minimize vibrations and transmission of waves across the material. Various orientations of unidirectional layers are studied and their acoustic properties are presented using a geometrical simplification. In a first approximation, only bulk waves are considered. They propagate in a multi-layered medium where the celerity changes from one layer to another. This simple model of propagation in homogeneous medium authorizes a fast computation of the transmission coefficient thanks to the transfer matrix formalism. The effect of different geometrical stackings can thus be analyzed. Particularly, self-similar or fractal structures possess topological characteristics combining periodic and disordered ones. Therefore, they present very interesting acoustic properties : resonance and band gaps. Transmission coefficient is studied for classical (simple, periodic, random) and two alternative stackings based on the Cantor set.

INTRODUCTION

Increasing sound attenuation in multi-layered structures without enhancement of thickness or quantity of mass may lead to focus on special geometrical properties as viable solutions. Indeed, propagation of acoustic or elastic waves in inhomogeneous media depends on the medium geometry and the wavelength. Due to their topology, periodic or disordered media reduce the transmission of sound : the first ones reflecting acoustic wave in the same way as photonic crystals do with light, and the others trapping waves in random labyrinths. Eusebio Sempere's sculpture in Madrid that stops frequencies in certain directions is a phononic crystal [2]. While periodic media possess evanescent modes that define bad gap or "stop band" [3], in finite intervals of frequencies, disordered materials are characterized by exponentially localized modes [4].

Fractals are a third kind of geometry : the "disorder" is distributed in the same manner at each scale. Mandelbrot defined them by

> a rough or fragmented geometric shape that can be split into parts, each of which is (at least approximately) a reduced-size copy of the whole [1].

As they combine division of space and iteration process, fractals combine both periodicity and randomness properties. This process has to be driven to infinity to build a mathematical fractal. Since the beginning of the 80's, vibrational properties of self-similar structures have been widely studied [5-7]. Fractal structures are characterized by the fractal dimension D, related to the geometry. Their spectral properties show eigenmodes called "fracton" : they are neither extended in the usual sense nor exponentially localized. For real structures, different sort of modes exists because the pre-fractal order is finite : for wavelengths larger than the correlation length the medium seems homogeneous, for wavelength smaller than the smallest particle dimensions, the vibrations are those of the smallest components of the structure. Between this two regimes, eigenmodes are fractons and thus, crossover frequencies appear.

On the experimental point of view, Bernard Sapoval created a fractal noise-reducing wall [8] and achieved to increase significantly road noise absorption [9]. Then, evidence of fracton existence, was demonstrated by investigations on silica aerogels [10], ultrasound attenuation [11], propagation of sound in one-dimensional Cantor Composites [6] or prefractal waveguide [12]-[13].

After a brief theoretical presentation of sound propagation in 1D multi-layered structure, different self-similar media are described and the effects of diverse geometrical stackings (periodic, random and fractal) are analyzed.

SCALAR MODEL FOR SOUND PROPAGATION IN A HALF-PERIODIC MULTI-LAYERED STRUCTURE

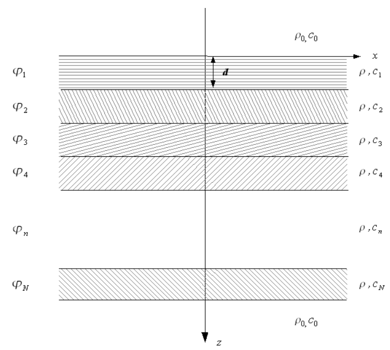

FIGURE 1: Multi-layered structure

First, a simplified 1D model for sound propagation in multi-layered media is presented. Acoustic waves propagate in a medium constituted by a succession of N semi-infinite layers (figure 1), with the same thickness d_n, characterized by their density ρ and the sound celerity c_n.

c_n changes from one layer to another, and is a function of layers orientation φ_n (figure 2) :

$$c_n = f(\cos\varphi_n, \sin\varphi_n). \quad (1)$$

The transmission of scalar waves in the structure can be obtained by a transfer matrix method. Pressure (p_n, p_{n-1}) and

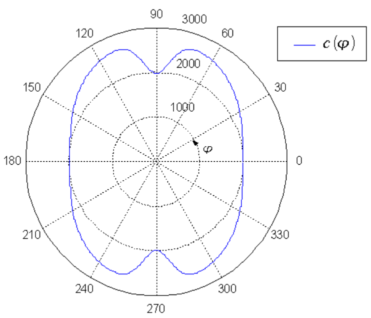

FIGURE 2: Polar plot of celerity, $c(\varphi)$

particle velocity (v_n, v_{n-1}) at both sides of the nth layer are connected by a transfer matrix operator M_n :

$$\begin{pmatrix} p \\ v \end{pmatrix}_n = \begin{pmatrix} \cos(k_n d_n) & jZ_n \sin(k_n d_n) \\ \frac{j}{Z_n} \sin(k_n d_n) & \cos(k_n d_n) \end{pmatrix} \begin{pmatrix} p \\ v \end{pmatrix}_{n-1}, \quad (2)$$

where $k_n(\varphi_n)$ is the wave number and

$$Z_n(\varphi_n) = \frac{\rho_n}{\sqrt{\frac{1}{c_n(\varphi)^2} - \frac{sin(\theta_{inc})}{c_0^2}}}. \quad (3)$$

consequently, on φ_n. The product of elementary matrices yields to the value of p and v at one end of the system as a function of the value at the other end. The transmission coefficient :

$$T(\omega) = \left| \frac{p_{N+1}}{p_0} \right|^2 \quad (4)$$

of the stacking can thus be computed. This scalar wave model enables to correlate peaks of transmission coefficient to the structure resonance modes.

PRE-FRACTAL GEOMETRIES

Physical fractals are not real mathematical fractals : they are only self-similar or pseudo-periodic. In order to understand the propagation of acoustic waves in such geometries, different stackings are studied : a simple one, constituted of one 0° oriented layer, a periodic one, a random one, and finally, two based on the Cantor's set. The periodic stacking consists in a succession of a unit cell composed by two layers with different orientation, thus different speed of sound, but similar density and thickness. The random stacking is formed by random oriented layers. The triadic Cantor set is created by repeatedly deleting the open middle thirds of a set of line segments. This process is iterated ad infinitum. It is self-similar, because it is equal to two copies of itself, if each copy is shrunk by a factor of 3 and translated. Two types of Cantor-like stackings are examined. The first is constructed by a sequence of two different oriented media A and B [7] :

$$\begin{array}{c} A \\ ABA \\ ABA\ BBB\ ABA \\ ABA\ BBB\ ABA\ BBBBBBBBB\ ABA\ BBB\ ABA \\ \dots \end{array} \quad (5)$$

The second is a Cantor set for the angle layers orientation φ. When the pre-fractal sequence is up to the Nth generation, and the central layer has an orientation φ_0, the structure is a succession of $2^{N+1}+3$ layers :

$$\left[0\ \frac{\varphi_0}{3^N}\ 0 \dots 0\ \frac{\varphi_0}{3^j}\ 0 \dots 0\ \frac{\varphi_0}{3}\ 0 \dots 0\ \frac{\varphi_0}{3^j}\ 0 \dots \frac{\varphi_0}{3^N}\ 0 \right] \quad (6)$$

RESULTS : PERIODICITY, RANDOMNESS AND SELF-SIMILARITY

Waves propagating through the structures are influenced by the self-similar geometry. What is the impact of such a self-similarity ? This issue is treated by studying the transmission coefficient of pre-fractal stakings and comparing it to simple, periodic and random ones.

Transmission

Acoustic waves are modified by acoustic impedance contrast between media, and therefore by difference of phase velocity or density. The more important the difference of impedance encountered is, the more the wave is reflected. The shape of $c_n(\varphi)$ (figure 2) indicates that a 72° oriented layer maximizes the difference of velocity, thus impedance, with a 0° oriented one. The largest angle of Cantor-like structure is then chosen to be 72°.

It is well known that periodic structures present frequency bad gaps due to interferences of waves reflected at interfaces. For a succession of $2d$ period composed by two layers characterized by a sound velocity v_i and a density ρ, central frequencies of gap can be worked out theoretically [14]. For an incident normal wave, there are two types of gaps centered respectively on :

$$f_m^b = \frac{2m-1}{2(d/v_1 + d/v_2)}, \quad (7)$$

$$f_m^c = \frac{m}{d/v_1 + d/v_2}, \quad (8)$$

where $m \in \mathbb{N}$. The more layers there are, the deeper the gap is. Considering a superposition of unit cells constituted by two 0° and 72° oriented layers, with $\rho = 1577\ \mathrm{kg \cdot m^{-3}}$, $d = 0.15$ mm, $v_1 = 1982$ m/s and $v_2 = 2643$ m/s, band gaps due to periodicity are centered on multiple of $f_1^b = 3.77$ MHz and $f_1^c = 7.55$ MHz.

The stacking is constituted by M layers, thus M coupled resonators, each possessing $f_m = c_m/2d_m$ as eigenfrequency. Coupling of underlayers makes difficult an interpretation of spectrum (transmission peaks) because other modes of vibration are raised.

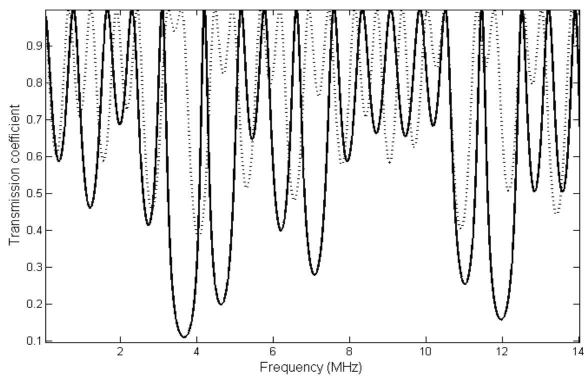

FIGURE 3: Transmission coefficient for a normal incident wave for Cantor-like stacking on thickness (5) — and on angle (6) ů ·ů ·ů · up to the third generation.

For an incident normal wave, the transmission coefficient presents peaks of resonance (figure 3). The first resonance occurs when the whole structure vibrates in phase, the others are associated with vibration of each or connected layers. While an unidirectional structure resonates for multiples of the eigenfrequency (depending on media thickness and sound velocity), Cantor-like stackings modes are not regularly distributed. This is due to the multiple scales of pseudo-periodic structures.

Thickness Cantor (5) and angle Cantor (6) stackings possess multiple scales. The first one has different sizes of similar cells : B, ABA, BBB, AB, each vibrating at it own eigenfrequency and harmonics. Sound velocity of sound for the second changes from one layer to another because they are oriented differently. This raises many eigenfrequencies. Thus both stackings possess many characteristic frequencies interacting and generating the transmission coefficient (figure 3).

It is very interesting to note that self-similar structures present drop of transmission almost in the band gap zone of periodic media : 3.6 MHz to 4.7 MHz for the angle Cantor structure, 3.1 MHz to 5.15 MHz and 5.8 MHz to 7.6 MHz. However, for the thickness Cantor structure, there is a resonance around the gap central frequency. This could be caused by the self-similarity of structures, they are nearly periodic, so it seems logical that a gap appears in the same zone as for periodic systems. Fractons are distributed in and out the gap. For structures with many layers, the density of states could determine whether they are extended or localized.

Transmitted Power

Moreover, stackings can be compared in term of transmitted power per frequency band comparatively with a total transmission. The band is chosen in order to give the maximum of information, without showing too many oscillations (figure 4). For random stacking, the calculated transmitted power is the average of transmitted power for aleatory oriented stackings (orientation angles are chosen among angles composing the Cantor-angle stacking : $[0\ \ \varphi_0\ \ \varphi_0/3\ \varphi_0/9]$.

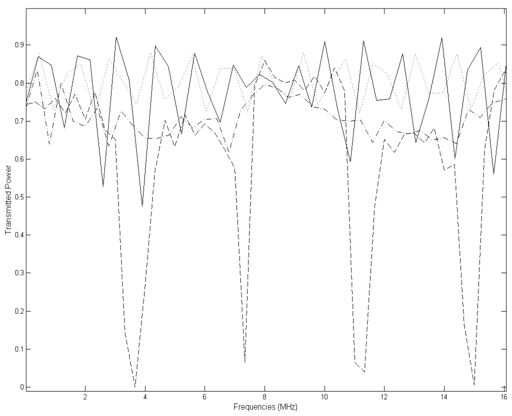

FIGURE 4: Transmitted power for unidirectional ů ·ů ·ů · periodic - - -, angle cantor — and random stackings -ů · -

The unidirectional medium transmits 80% of incident energy. When at least one layer oriented differently , transmitted power changes. Due to depth of gaps, the periodic stacking blocks the most energy, only 63% of the power is transmitted. On average, the Cantor-angle stacking transmits more energy than random ones, 78% versus 71%. On the whole frequencies, coupling of modes compensates the presence of gap. Nevertheless, for given frequency bands, Cantor is more efficient than random systems : half of energy is attenuated. Due to self-similarity, its behaviour is halfway between periodic and random ones.

Pre-fractal order

Dealing with self-similarity, it seems essential to analyze the pre-fractal order influence (figure 5).

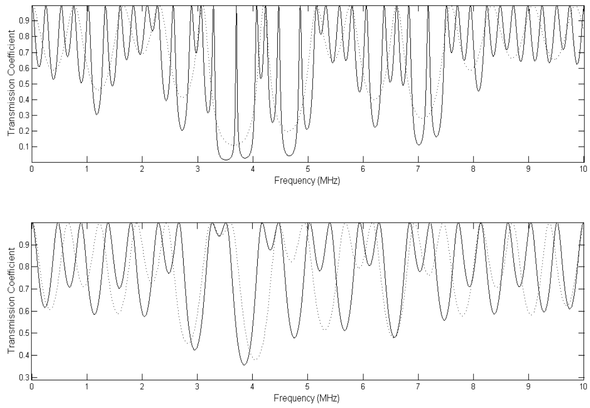

FIGURE 5: Transmission Coefficient for thickness Cantor (up) and angle Cantor (down) : (order 3 ů ·ů ·ů · and order 4 —)

Increasing pre-fractal order for the thickness Cantor induces a deepening of gaps. Transmission coefficient of the first gap falls from 0.11 to 0.018. While order of fractal increases, multiple characteristic thickness, thus frequencies, appear. The wave is scattered by the structure, and transmission drops. The angle Cantor stacking is less affected by fractal order : the main gap decreases from 0.38 to 0.35. Actually, this can be explained noticing that the more important the order is, the closer to 0° the layers are, and the less reflected they are. The scale difference introduced by impedance contrast is not significant.

CONCLUSION

Finally, waves propagation in 1D self-similar and multi-layered structures can be described by a transfer matrix formalism leading to the media transmission coefficient. Unidirectional, peridoric, random and self-similar stackings properties are analyzed and compared. Self-similar (or pseudo-periodic) and periodic stackings band gaps are located around the same frequencies. They transmit a little more than periodic stackings but in certain frequency band they are more efficient than random ones. Unfortunately, a coupling of modes for angle Cantor structure limits is efficiency. The investigations will be deepened studying density of states and localization properties of the two self-similar structures.

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