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Meaning of admittance boundary condition explained on an analytical example of structure fluid coupling

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ABSTRACT

Simulation techniques in the linear acoustics of rooms with arbitrary geometry often lack sufficient knowledge about the dynamics of the surrounding walls. But the latter effect the sound distribution significantly. This is why boundary value problems (BVP) of fully coupled structure fluid systems should be solved. Unless one transforms the discretized form of this BVP into a system of only the sound pressure by means of the Schur complement. This produces a fully occupied coupling admittance matrix within this formulation. Out of sound pressure data it is certainly difficult to reproduce all entries of this matrix. Due to this fact the authors introduce an approximation for the coupling admittance by defining local admittance values on the boundary. This boundary condition type causes a simplification of the coupling admittance matrix. It is demonstrated on a simple structure fluid coupled system whose analytical equations are arranged in a matrix form matching a standard BEM-FEM formulation, followed by a short discussion about its applicability.

INTRODUCTION

In systems where fluid and structure is coupled at their interfaces, the system equations are formulated in terms of variables for the fluid, e.g. sound pressure, and variables for the structure, e.g. displacement. The variables for the structure can be substituted by evaluation of the Schur complement. In that case, part of the Schur complement can be understood as a coupling admittance with non-local boundary admittance entries. Herein, the authors try to explain and emphasize the effect on that coupling admittance by changing this non-local to a local definition.
A one-dimensional problem of a duct is introduced as a simple example in order to understand this issue by means of a boundary-element-method-like matrix formulation. Single degree of freedom systems are introducted at both ends of the duct. For the setup of an example with non-local boundary conditions, both ends are connected by two springs and a rigid block. For this system, the admittance matrix will be shown analytically. We will discuss the influence of different masses of that block onto local and non-local admittance condition within the formulation of the Schur complement, towards the acoustical medium. Throughout this contribution the authors try to emphasise the analogousness of the mathematics of this one-dimensional duct to discretised coupled systems of arbitrary geometry and size.

LOCAL ADMITTANCE

First of all we imagine a one-dimensional fluid domain bounded on one side by a flexible wall that may be considered as a simple mass-spring-damper system as shown in Figure 1.
For this system the balance of forces written in the time domain is

$$m_1\ddot{u}_1 + b_1\dot{u}_1 + c_1u_1 = F_1 - Ap_1 \tag{1}$$

with A being the cross-sectional area of the interface. If we confine ourselves to the stationary state we may transform this in

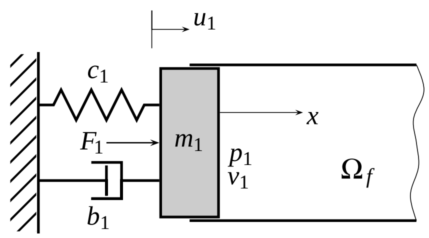

Figure 1: One-dimensional model of fluid bounded by mass-spring-damper system (MSD) as substitute for flexible wall.

terms of the circular frequency ω

$$-\omega^2 m_1u_1 - \mathrm{i}\omega b_1u_1 + c_1u_1 = F_1 - Ap_1\,. \tag{2}$$

Based on the Robin boundary condition formulation

$$v_1 - v_{1s} = Yp \tag{3}$$

with the structural velocity set to zero ($v_{1s} = 0$) and

$$v_1 = -\mathrm{i}\omega u_1 \tag{4}$$

we find the acoustical admittance

$$Y_1 = \frac{v_1}{p_1} = \frac{-\mathrm{i}\omega A}{-\omega^2 m_1 - \mathrm{i}\omega b_1 + c_1}\,. \tag{5}$$

During the following mathematics this parameter expresses the dynamics of the wall. It is a frequency dependend and complex parameter (Figure 2).

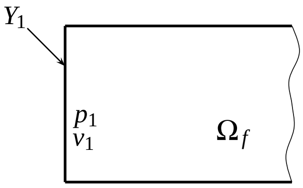

Figure 2: Acoustical admittance parameter Y_1 as equivalent for the MSD.

DISCRETE MATRIX FORMULATION

The boundary value problem for a stationary vibrating system consists of the Lame-Navier equation for the structure, of the Helmholtz equation for the fluid and of the full coupling condition $v_f = v_s$ along the fluid structure interaction (FSI).
Through discetisation we obtain this matrix formulation for a finite number of degrees of freedom in the frequency domain:

$$\begin{bmatrix} \boldsymbol{H} & -\boldsymbol{G}\boldsymbol{C}_{fs} \\ -\boldsymbol{C}_{sf} & \boldsymbol{A} \end{bmatrix} \begin{bmatrix} \boldsymbol{p} \\ \boldsymbol{u} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{f} \end{bmatrix}. \qquad (6)$$

In this context $\boldsymbol{H}$ and $\boldsymbol{G}$ are system matrices from the boundary element formulation (BEM), matrix $\boldsymbol{A}$ derives from a finite element formulation (FEM), as in [2]. The matrices $\boldsymbol{C}_{fs}$ and $\boldsymbol{C}_{sf}$ allow to convert structural displacement and fluid velocity back and forth.
Column matrix $\boldsymbol{p}$ denotes the nodal sound pressure amplitude, $\boldsymbol{u}$ the displacement of the structure.
Applying Schur complement eliminates $\boldsymbol{u}$

$$\left(\boldsymbol{H} - \boldsymbol{G}\boldsymbol{C}_{fs}\boldsymbol{A}^{-1}\boldsymbol{C}_{sf}\right)\boldsymbol{p} = \boldsymbol{G}\boldsymbol{C}_{fs}\boldsymbol{A}^{-1}\boldsymbol{f}. \qquad (7)$$

Here we may introduces the coupling admittance matrix

$$\boldsymbol{Y}_c = \boldsymbol{C}_{fs}\boldsymbol{A}^{-1}\boldsymbol{C}_{sf}. \qquad (8)$$

Thus we obtain a matrix equation reduced in size to give the sound pressure as solution:

$$(\boldsymbol{H} - \boldsymbol{G}\boldsymbol{Y}_c)\boldsymbol{p} = \boldsymbol{G}\boldsymbol{v} \qquad \text{with} \qquad \boldsymbol{v} = \boldsymbol{C}_{fs}\boldsymbol{A}^{-1}\boldsymbol{f}. \qquad (9)$$

$\boldsymbol{Y}_c$ is generally densely occupied due to full coupling.

MEANING OF NON-LOCAL ADMITTANCE

In this section the one-dimensional, analytical problem of a coupled structure fluid system is put into a particular form of a system of equation that matches equation (7) and equation (9) respectively.
Figure 3 shows this structural-fluid system with mass spring damper systems enclosing the fluid on either side. The two systems are themselves connected dynamically by an additional mass m_b. Therewith the structure acts in a non-local fashion towards the fluid.
As shown in Figure 3 we introduced a couple of degrees of freedom on the interfaces between the fluid and the two masses m_1 and m_2: the sound pressures p_1 and p_2 as well as the normal fluid velocities v_1 and v_2.
In the following we shall neglect a detailed description of the necessary mathematical steps. Instead, they shall be accounted for verbally.
Firstly, we assemble the balances of forces on the three masses m_1, m_2 and m_b for their displacements u_1, u_2 and u_b. The balance on m_b may then be used to remove u_b. Applying relation (4) in the frequency domain, the other two displacements are replaced by their interface velocities v_0 and v_l.
The analytical one-dimensional solution of the Helmholtz equation provides us with two equations for the sound pressures p_1 and p_2.
Coupling structure and fluid implies merging the four remaining equations to this system of equation:

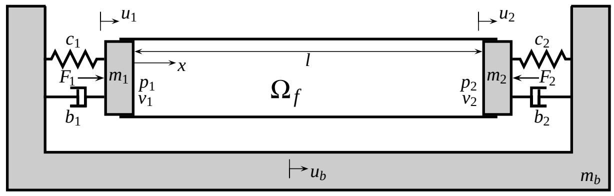

Figure 3: One-dimensional fluid with MSDs on either side and an additional mass m_b.

$$\begin{bmatrix} h_{11} & h_{12} & -\mathrm{i}\rho c\omega & 0 \\ h_{21} & h_{22} & 0 & \mathrm{i}\rho c\omega \\ A & 0 & d_1 - \frac{w_1^2}{d_b} & -\frac{w_1 w_2}{d_b} \\ 0 & -A & -\frac{w_1 w_2}{d_b} & d_l - \frac{w_2^2}{d_b} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ F_1 \\ -F_2 \end{bmatrix} \qquad (10)$$

Applying the Schur complement again breaks this system down to an equation that matches equation (9).
Now, here in particular the coupling admittance matrix looks like this:

$$\boldsymbol{Y}_c = \frac{-\mathrm{i}\omega A}{N} \begin{bmatrix} d_2 - \frac{w_2^2}{d_b} & -\frac{w_1 w_2}{d_b} \\ -\frac{w_1 w_2}{d_b} & d_1 - \frac{w_1^2}{d_b} \end{bmatrix}. \qquad (11)$$

The constants h_{ij}, d_1, d_2, d_b, w_1, w_2 und N appearing in equation (10) and (11) are as follows:

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{e^{\mathrm{i}kl} + e^{-\mathrm{i}kl}}{e^{\mathrm{i}kl} - e^{-\mathrm{i}kl}} & -\frac{2}{e^{\mathrm{i}kl} - e^{-\mathrm{i}kl}} \\ -\frac{2}{e^{\mathrm{i}kl} - e^{-\mathrm{i}kl}} & \frac{e^{\mathrm{i}kl} + e^{-\mathrm{i}kl}}{e^{\mathrm{i}kl} - e^{-\mathrm{i}kl}} \end{bmatrix} \qquad (12)$$

and

$$\begin{aligned} d_1 &= -\omega^2 m_1 - \mathrm{i}\omega b_1 + c_1 \\ d_2 &= -\omega^2 m_2 - \mathrm{i}\omega b_2 + c_2 \\ d_b &= -\omega^2 m_b - \mathrm{i}\omega(b_1 + b_2) + (c_1 + c_2) \\ w_1 &= -\mathrm{i}\omega b_1 + c_1 \\ w_2 &= -\mathrm{i}\omega b_2 + c_2 \\ N &= d_1 d_2 - \frac{d_1 w_2^2}{d_b} - \frac{d_2 w_1^2}{d_b}. \end{aligned} \qquad (13)$$

As with a coupling admittance of any complex model discretized using FEM and BEM, $\boldsymbol{Y}_c$ appears to be fully occupied. This fact conforms with general movement of the boundary that is connected through the structure itself. Thus, $\boldsymbol{Y}_c$ stands for a non-local boundary admittance towards the fluid.

STRUCTURAL DECOUPLING

In order to understand the effect of a local structural behavior on the equations we imagine an increasing mass m_b. If m_b reaches infinity, the movement of the two masses m_1 and m_2 become then

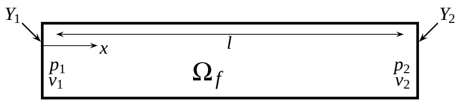

Figure 4: The same one-dimensional fluid but with local admittance parameters at its boundaries.

uncoupled, i.e. the dynamics of the structural path is cut.
Looking at the coupling admittance in (11) we realize that the matrix becomes diagonal:

$$\boldsymbol{Y}_c = \frac{-\mathrm{i}\omega A}{d_1\, d_2} \begin{bmatrix} d_2 & 0 \\ 0 & d_1 \end{bmatrix} = \begin{bmatrix} Y_1 & 0 \\ 0 & Y_2 \end{bmatrix}, \qquad (14)$$

with Y_i showing values according to the local admittance formulation of equation (5). The model in Figure 4 correspondes to equation (9) for the presented one-dimensional problem. It is based on the one-sided structure-fluid coupling utilising these local admittances Y_1 and Y_2 as part of the Robin boundary conditions.

CONCLUSION

The essence of this contribution is: a local definition of admittance boundary condition within acoustical calculations leads to a diagonalization of the coupling admittance matrix.
It is important for any simulation applications to investigate whether this fact may turn out advantageous or disadvantageous. As an example, the local formulation is suitable for reconstructing local admittance values out of sound pressure measurements using inverse techniques, [3, 4, 1]. One may rest assured that it is practically impossible to reconstruct structural interaction out of sound pressure data. Thus, it is of practical interest for which applications one may approximate the usually densely occupied coupling admittances by such diagonal matrices.

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