Proceedings of 20th International Congress on Acoustics, ICA 2010

23-27 August 2010, Sydney, Australia

# Mathematics Summer Schools for Acoustics Research Training

**Keith Attenborough**

Department of Design, Development, Environment and Materials, The Open University, Milton Keynes, MK7 6AA, UK

**PACS:** 43.10.Sv

## ABSTRACT

Mathematical methods are important for research in many aspects of acoustics. Most researchers in acoustics do not have access to master level courses to broaden their postgraduate study. Currently, individual researchers advance their fundamental mathematical methodologies taught at undergraduate level through independent learning. They develop their mathematical skills as appropriate rather than being made aware of the potential of advanced mathematical tools at the onset of their research career. In the UK, attempts to improve this situation have been made through summer schools held in 2003, 2005 and 2007 at Southampton and Salford Universities. The background to these Summer Schools, their content, timetable planning, recruitment figures and student feedback on them are reported together with general conclusions about their performance and role.

## BACKGROUND

In the UK, the arrival of the new millennium was accompanied by considerable concern about the state of the mathematical education of engineers. This resulted in the funding for several projects intended to improve the situation. For example in 2002 'Funding for the Development of Teaching and Learning (FDTL Phase 4)' supported the 'Helping Engineers Learn Mathematics' (HELM) project based at Loughborough University [1]. This resulted in 50 workbooks, each approximately 50-pages long, containing learning materials, worked examples and case studies of applications of mathematics in engineering (including a few acoustical ones). Illustrative web-delivered Computer Aided Learning (CAL) courseware on many of the first 20 Workbooks are provided to support and enhance the content of the workbooks and either web-delivered or CD-based computer aided assessment.

Also a survey of the UK engineering community in autumn 2001 highlighted a need to increase the general level of Mathematical competencies of UK postgraduate engineers. In 2002, the UK Engineering and Physical Sciences Research Council initiated a 'Maths for Engineers Summer Schools' pilot study. Proposals were sought for Summer Schools that would impart a greater appreciation of contemporary mathematical techniques to PhD students in core areas of engineering research. Applicants were asked to identify an engineering theme, and to develop a Summer School programme suitable for the purpose and audience.

The mathematical demands of acoustics research are wide: ranging from solution of partial differential equations to the methods of digital signal processing. Some of the mathematical basics are covered by undergraduate mathematics for engineers but many important topics such as Green's function methods, asymptotic methods, integral transforms and variational calculus lie outside the typical engineering mathematics syllabus.

Given the importance of mathematical methods to research in many aspects of acoustics, the opportunity was taken, following a meeting of potential contributors, to submit a proposal for an EPSRC-funded pilot study to address the mathematical needs of UK post-graduate researchers in engineering acoustics and physical acoustics. The proposal was for an intensive but collegial residential week for up to 40 researchers, mainly graduate students drawn from across the UK, with a high staff/student ratio and with the aim of introducing important mathematical techniques in acoustical contexts.

Funds were requested to cover:

- Honoraria for the organiser, lecturers and tutors for the Summer School;
- The full subsistence and accommodation costs of at least 30 research students based at UK universities and up to 10 staff (using student accommodation).
- Travel costs of the organiser, lecturers and tutors;
- Hire of a Summer School Venue and technical equipment;
- Course administration costs including preparation of course material, posters and other publicity material and mailing.

The proposal was successful and the first Summer School 'Support Mathematics for Acoustical Research Training' (SMART) was held at the University of Southampton in July 2003. The only other pilot School funded by the EPSRC in 2002 was for Process Engineering.

As a result of the interest generated by SMART-1, second and third versions were proposed successfully in response to further EPSRC calls in 2004 and 2006. SMART-2 and SMART-3 were held in 2005 and 2007 at Southampton and

Salford Universities respectively, and were planned at meetings held earlier in each of these years.

## SYLLABUS AND STRUCTURE

The assumption was made that most students embarking on postgraduate studies in acoustics would welcome revision of some of the mathematical topics commonly found in engineering degree programmes. Specific topics included Ordinary Differential Equations, Vector and Matrix algebra, Calculus with Complex variables and Integral Transforms. These topics provided most of the content in the first two days. Three lectures per day were interspersed with appropriate tutorials and opportunities for individual one-to-one 'surgeries' with contributing and supporting lecturers. More advanced topics were chosen from the mathematics needed for general acoustics (waveguides), aeroacoustics (generalised functions, Green's functions) and signal processing. These more advanced sessions were organised by colleagues from the mathematics and acoustics departments at Cambridge, ISVR, Keele, Loughborough and Salford. Each contributing Department was responsible for a specific day or half day.

The summer school week occupied weekdays Monday to Friday preceded by registration on Sunday night. Table 1 shows the structure of the 2007 school week and Figure 1 shows a corresponding flier.

**Table 1**. The SMART-3 week

| *Day 1* | *Day 2* | *Day 3* | *Day 4* | *Day 5* |
|---|---|---|---|---|
| Vector Calculus | Wave-guides | Lecture | Finite elements 1 | Aero-acoustics 1 |
| Tutorial | | Signal processing principles | Tutorial | |
| Complex variables | Mode matching | Matlab Tutorial | Finite elements 2 | Aero-acoustics 2 |
| Tutorial | | Signal processing techniques | Tutorial | |
| Integral Transforms | Advanced techniques | Matlab tutorial | Boundary elements | Computing |
| Tutorial | | Iterative methods | FEMLAB | Lecture |
| Lecture | Green Functions | Lecture | Lecture | |

Source: (Author, 2010)


Source: (Author, 2010)

**Figure 1**. Publicity flier for SMART-3 in 2007

Figures 2, 3 and 4 show examples of tutorial problems for the sessions on vector calculus, complex variables and waveguides.

**Question 1.1** Suppose $\mathrm{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}} = \overrightarrow{OP}$ is the position vector of a general point $P$ relative to a given origin $O$. If $r = |\mathrm{r}| = \sqrt{x^2+y^2+z^2}$ confirm that

$$\nabla \cdot \mathrm{r} = 3, \qquad \nabla \times \mathrm{r} = 0,$$

Use the above results, together with particular vector identities, to evaluate

$$(i) \quad \nabla\, [f(r)], \qquad (ii) \quad \nabla \cdot [f(r)\mathrm{r}],$$
$$(iii) \quad \nabla \times [f(r)\mathrm{r}], \qquad (iv) \quad \nabla^2 [f(r)],$$

where $f$ is an arbitrary function of $r$. Hence, for $n$ integer, obtain

$$\nabla\, [r^n], \qquad \nabla\, [\ln r], \qquad \nabla \cdot [r^n \mathrm{r}], \qquad \nabla^2 [r^n], \qquad \nabla^2 [\ln r].$$

**Question 1.2** Using cylindrical polar coordinates, namely

$$x = R\cos\phi, \quad y = R\sin\phi, \quad z = z,$$

find, starting from their (multiple) integral definitions,

(i) the curved surface area $A$ of the cylinder $S$: $x^2+y^2=a^2$, $0 \le z \le H$;

(ii) the volume $V$ of the region $\Sigma$: $x^2+y^2 \le a^2$, $0 \le z \le H$ within the cylinder.

Confirm that the evaluations of the double and triple integrals match the expected results.

Source: (Author, 2010)

**Figure 2**. Example tutorial problem on vector calculus

**Question 2.1** By using Cauchy's residue theorem, evaluate

$$I = \oint_{C:|z|=1} z^n e^{\frac{1}{z}}\, dz,$$

for integer $n > 0$.

**Question 2.2** By using Cauchy's residue theorem, evaluate

$$(a) \quad I_1 = \oint_{C:|z|=\frac{1}{2}} \frac{z+2}{z(z+1)^2}\, dz, \qquad (a) \quad I_2 = \oint_{C:|z|=2} \frac{z+2}{z(z+1)^2}\, dz.$$

**Question 2.3** Suppose that rational function $f(z)$ has a simple pole at $z = z_0$, such that

$$f(z) = \frac{g(z)}{h(z)} \quad \text{where} \quad h(z_0) = 0, \quad g(z_0) \neq 0,$$

where $g$ and $h$ are polynomial functions. By writing

$$h(z) = (z - z_0)R(z) \quad \text{where} \quad R(z_0) \neq 0,$$

show that the residue of $f$ at $z = z_0$ is given by

$$\mathrm{Res}\,\{f(z)\}_{z=z_0} = \frac{g(z_0)}{h'(z_0)}.$$

Find the four simple poles $z = z_n$ for $n = 1, 2, 3, 4$ of the rational function

$$f(z) = \frac{z^2}{z^4+1}$$

and hence show that

$$\mathrm{Res}\,\{f(z)\}_{z=z_n} = \frac{1}{4z_n}$$

Source: (Author, 2010)

**Figure 3**. Example tutorial problems on complex variables

Q1. Calculate the modes for a rigid rectangular guide with cross-section

$$-a < z < a, \qquad -b < y < b,$$

the guide being aligned with the $x$-axis.

- Write down the Helmholtz equation in 3D Cartesian coordinates.
- Let $\Phi(x,y,z) = X(x)Y(y)Z(z)$.
- Substitute this into the Helmholtz equation and rearrange so as to introduce two separation constants.
- Apply the boundary conditions on the guide walls.
- Calculate the expression for a general mode in the guide.
- What are the cut-off frequencies?

Source: (Author, 2010)

**Figure 4**. Example tutorial problem on waveguides

Supplementary lectures about general mathematical methods and 'guest' lectures on more specialised topics were intended to give attendees an introduction to a wide range of applications of mathematics in acoustics and to the history of mathematical acoustics. Table 2 lists 'general' and 'specialist' lecture topics in 2003, 2005 and 2007. Lecturers and their institutions are listed in Table 3.

**Table 2**. 'Specialist' Lecture topics

| *2003* | *2005* | *2007* |
|---|---|---|
| Corrugated pipe acoustics | A short history of bad acoustics | The musical saw |
| Asymptotic methods | Porous material acoustics | Models for porous meshes |
| Inverse methods | Rayleigh's legacy | Predicting the future |
| Rigid-porous material acoustics | Acoustic scattering | Acoustic levitation |
| Outdoor sound propagation | Acoustics of rough surfaces | Parabolic Equation methods |
| Quantum orbit acoustics | Asymptotic methods | Approximations |

Source: (Author, 2010)

**Table 3**. Contributing Lecturers

| *Lecturer* | *Institution* |
|---|---|
| David Abrahams | Manchester |
| Keith Attenorough | Hull |
| Simon Chandler-Wilde | Reading |
| John Chapman | Keele |
| Bill Davies | Salford |
| Ian Drumm | Salford |
| Phil Duncan | Salford |
| Iain Dupere | Manchester |
| Trevor Cox | Salford |
| John Elliott | Hull |
| Chris Howls | Soton |
| Steve Langdon | Reading |
| Jane Lawrie | Loughborough |
| Chris Linton | Loughborough |
| Chris Jones | ISVR |
| Yui Wei Lam | Salford |
| Maureen McIver | Loughborough |
| Peter Monk | Delaware, US |
| Phil Marston | Washington State, US |
| Phil Nelson | ISVR |
| Nigel Peake | Cambridge |
| Allan Pierce | Boston, US |
| Rod Self | ISVR |
| Olga Umnova | Salford |
| Paul White | ISVR |
| Matthew Wright | ISVR |

Source: (Author, 2010)

An example abstract of a specialist lecture follows:-

> Data showing the influence of surface roughness on reflection of long-wavelength sound from a point source will be presented. The presence of roughness on an acoustically-hard surface gives it an effective impedance for wavelengths large compared to the roughness. The sound field above a rough surface in which the roughness elements are idealised by semi-cylinders can be calculated using a boundary element method, by a 'boss' theory and by a multiple scattering method. These methods of calculating the sound field and the resulting predictions are compared with each other and with data.

Attendees in 2005 and 2007 were fortunate to receive guest lectures from prominent US academics. As well as giving guest lectures, Allan Pierce and Peter Monk participated as tutors. In 2007 Phil Marston attended as an 'observer' and gave a guest lecture also.

The 2003 Summer School resulted in a text "Lecture Notes on mathematics for acoustics" edited by Matthew Wright and published by Imperial College Press [2]. The book was provided together with supplementary material to attendees at the 2005 and 2007 summer schools. A list of the chapters in this book is given in Table 4.

**Table 4**. Chapter titles in *Lecture Notes on Mathematics for Acoustics* [1]

| | | | |
|---|---|---|---|
| 1 | Vector Calculus | 8 | Acoustics of Rigid–Porous Materials |
| 2 | Functions of a Complex Variable | 9 | Generalised Functions in Aeroacoustics |
| 3 | Integral Transforms | 10 | Monopoles, Dipoles, and Quadrupoles |
| 4 | Asymptotic Expansion of Integrals | 11 | Corrugated Pipe Acoustics |
| 5 | The Wiener–Hopf Technique | 12 | Digital Filters |
| 6 | Waveguide Theory | 13 | Measurement of LTI systems |
| 7 | Wavefield decomposition | 14 | Numerical Optimisation |

Source: (Author, 2010)

Tutorial slots were planned to follow each mathematical presentation. Additional tutors were recruited to assist the lecturers for each of these sessions. Three computer laboratory tutorial sessions were held also supported by local postgraduate demonstrators.

Opportunity for arranging individual 'appointments' with tutors on topics chosen by the participants were given through the further particulars for registrants. Typically however they were arranged more or less *ad hoc* and subject to the availability of the tutors (including evenings) during the week.

## RECRUITMENT

The numbers of registrants and their institutions are listed in Table 5. Attendances at the 2003 and 2005 schools were bolstered by the large cohorts of graduate students and researchers in Southampton. Relatively few researchers from Southampton attended the 2007 school in Salford. On the other hand there were eleven attendees from Salford. The institutions described as 'other' in Table 5 include the Universities of York, Hull, Bradford, Sheffield, Reading, Brighton, Surrey and London and research organisations such as the National Physical Laboratory.

**Table 5**. Recruitment

| Institution | *Soton* | *Salford* | *Cantab* | *Other* | *Total* |
|---|---|---|---|---|---|
| 2003 | 8 | 4 | 5 | 26 | 43 |
| 2005 | 14 | 6 | 8 | 19 | 47 |
| 2007 | 4 | 11 | 6 | 8 | 29 |

Source: (Author, 2010)

## FEEDBACK

Feedback questionnaires were distributed and collected among the attendees in 2003 and 2005. The questions related to the course outcomes, the level of material, course content and lecture quality. Table 5 lists some responses to a question concerning the appropriateness of the Summer School.

**Table 5**. Example responses to "*How well did the summer school meet your needs*?"

| | |
|---|---|
| *SMART-1* | '..now aware of maths in acoustics in a general sense', 'to a great extent', 'helped quite a lot', 'needs met to a highly satisfactory level', 'I have lots of new ideas on my project now' 'has given a broad idea of what kind of maths might be required in the future', |
| *SMART-2* | "The SMART school was very helpful to me, especially because it occurs during my problem formulation for PhD. I have gained a good background for the mathematical methods I was planning to use and some good ideas for alternative methods." "….even though some of the material was way too hard to grasp over the course of five days, I know that if I ever come across any of the material in the future I will be able to have a better understanding of how to tackle the problems posed" "..the material (book and handouts) is very useful as it gives a deep insight of the maths for acoustics research, being also well presented and easy to follow" |

Source: (Author, 2010)

The 2003 school included a focus group discussion as part of a PhD research project into mathematical communications [3]. Responses to the question '*What one thing would you pick out as the best thing at this event?*' are summarized under the various components as follows:

Surgeries and tutorials

- The tutorials and coffee time talks with staff and other students, give good insights
- The surgeries
- Good tutorial staff who go over the problems in small groups
- The surgeries and tutorials are an opportunity to go over points that you didn't understand with other staff.

Materials

- Matlab functions and scripts that can be taken away and used
- Information and material to take away.
- The course material is most valuable resource
- References given on the handouts/ slides

Lectures and lecturers

- Lectures motivated you to investigate more difficult areas that you would not have been able to teach yourself
- Lecturers coming from the different views and approaches

Networking and sharing

- Sharing problems with so many individuals from the same field
- Opportunity to do networking with 40+ people in the same field.

Inspiration

- Inspiration to take back to your research work.
- Finding out about stuff you didn't know existed, at a level beyond undergraduate knowledge especially.

## CONCLUDING REMARKS

The three Summer Schools in mathematics for acoustics outlined in this paper have filled an important niche in engineering and acoustics education in the UK.

The primary aim of the Summer Schools was to convey awareness of the essential mathematical tools and concepts that could enable the participants, typically first year graduate researchers in physical and engineering acoustics, make a rapid impact in their research. However the Schools have also fulfilled a secondary aim which was to encourage a sense of community and offer opportunities for networking. This has proved particularly important for research students working in acoustically-related topics who were relatively isolated or in small groups.

One of the attendees in 2003 (Chris Powles, ISVR) subsequently acted as a tutor in 2005 and 2007.

No doubt there is a continuing need for such Summer Schools so it is to be hoped that some similar provision will happen in the future.

## ACKNOWLEDGEMENTS

The SMART Summer Schools would not have been possible without funding from the Engineering and Physical Sciences Research Council (UK) through grant numbers GR/S47021/01, EP/C547349/1 and EP/D507820/1. The support and help of many people is gratefully acknowledged, particularly from Matthew Wright at ISVR who worked on many of the initial ideas, was the source of many contacts and helped to make both ideas and contacts work. The grants were awarded while the author was Head of Department and Research Professor in Engineering at the University of Hull.

## REFERENCES

1 http://www.lboro.ac.uk/research/helm/
2 M. C. M. Wright ed., *Lecture Notes on the Mathematics of Acoustics* (Imperial College Press, London, 2005)
3 L. Leventhall, EPSRC Maths for Acoustics Summer School Focus Group Evaluation, unpublished communication (2003)