Acoustic streaming is a fundamental nonlinear phenomenon resulting from high frequency vibration in fluids. Known and investigated by Rayleigh and contemporary scientists, it attracts renewed interest due to the availability of powerful computational tools, advanced photography and precise laser velocimetry instrumentation, which can produce accurate experimental results. Its physical mechanism however is still not clearly understood. The analysis appears limited by the traditional premises of harmonic analysis, radiation force and wave propagation and reflection with the focus on nonlinear terms of the inertial frame formulations. Following our earlier analysis of nonlinear effects on rigid particles in a streaming medium using time domain (TD) finite element (FE) analysis with a moving mesh via Comsol™, we present the modelling of ultrasonic streaming alone. We use state of the art laser velocimetry instrumentation to measure the average velocity of 0.5μm latex tracer particles in a 0.3–4 mms⁻¹ streaming water insonified in the 1MHz frequency range. We use LabView™ virtual instrument to analyse light scattered by a swarm of particles in the moving fringes of crossed laser beams and find the ensemble particle motion from the frequency spectrum of the signal. In order to verify the FE modelling results with respect to the streaming velocity, the electric power is monitored at the transducer terminals. Our FE simulation, based on the Navier-Stokes (NS) equation for viscous incompressible fluids, does not involve wave propagation and radiation but is capable of representing the transient development of streaming, effects of boundaries and effects of the character of the ultrasonic source. Our investigation shows that streaming is neither implied by a time-varying topology nor associated with the asymmetry or even with the movement of the source or the fluid surface. Surprisingly, the streaming velocity is increased by making the enclosure fully symmetrical or by using a ‘bodiless’ distributed pressure source. Observations reveal not only evolving vortices but a more complex character of streaming than generally shown, including reversed streaming. Animations reveal transient development of a volume of streaming medium progressively ‘propagating’ away from the source and following a self contained volume of a pressure ‘valley’ travelling ahead of it and apparently attracting its motion.

INTRODUCTION

According to the well established view, acoustic streaming is a phenomenon of the medium movement away from the acoustic transducer which is attributed to wave propagation and radiation that generate a physical force acting in the medium: “This force comes from acoustic radiation pressure and is a nonlinear phenomenon derived from second-order terms” as summarised by Mitome (1998). Consequently, streaming or secondary motion appears and is derived for situations where a second order nonlinearity is identified. This is because of the time domain (TD) nonlinearity in the continuity equation or due to a spatial disturbance near boundaries (on a particle-fluid interface, over a boundary layer or in a standing wave). Practical applications such as manipulation of micro-objects, particle harvesting from industrial solutions or characterisation of medical transducers drive the intensified research in recent years and a matching volume of publications. Our focus is on the identification of the most elementary physical mechanism of the streaming rather than generating matching numerical and experimental results related to a chosen topology and conditions. In parallel with numerical simulations, we have conducted laboratory measurements of streaming using laser fringe velocimetry (LFV) (refer to the schematic setup in Fig. 1 and a photo in Fig. 2). Traditional analysis used harmonic formulations with approximate solutions employing originally 2nd order harmonic frequency components (Nyborg 1965, 1998) but relevant TD formulations are also available (Bradley, 1996). Those results led to the generally accepted conclusion that the streaming velocity is proportional to the medium viscosity, the square of the vibration frequency and amplitude of the fluid, although Mitome (1998) points out that the latter is only valid for weak sonication near the onset of streaming and further relation become linear. Attempts to improve these models, in search for a factor other the frictional forces alone that could generate the streaming, have attracted modelling as sophisticated as the full compressible Navier Stokes (NS) equations including both the wave propagation and fluid dynamics (Aktas and Farouk 2004).
The notion of the radiation force implying the condition of wave propagation in the medium however may be inappropriate since streaming has been commonly observed in incompressible liquids where classical wave propagation is usually not a part of the mathematical formulation and where the lack of compressional behaviour does not inhibit the streaming. More recent approaches have used numerical investigation of the nonlinear incompressible NS equation with various topologies and sources and even the accompanying heat transfer in the boundary layer to analyse conditions responsible for streaming (Loh et al 2002). Riley (2001), in a detailed review, defines ‘steady streaming’ in incompressible fluids as central to the phenomenon, stressing the importance of non-conservative forces as its source. Focusing on the NS formulations while retaining the harmonic approach commonly leads to the observation that streaming is supported under conditions of significant viscosity and high Reynolds numbers – conditions which are also responsible for the generation of vortices. Referring to the classical, 1960s analysis of the Rayleigh streaming, Riley questions the popular notion that streaming is generated in the bulk of the fluid because his derivations show that the stream function near a non-slip boundary develops a non-zero average velocity outside the Stokes layer under such conditions. This may be strict and convincing but it does little to reveal the physics of streaming. It appears from other research, that streaming is associated with vortices which move quite vigorously and thus change its direction (Cadwell, 1977). In his extensive experimental work, Cadwell used circular 800kHz transducer in a cylindrical glass cell (Ω=3.1cm, length 9.5cm) and advanced imaging. He found that a reverse streaming toroidal vortex appears first and this is followed and gradually replaced by a forward streaming vortex which appears at the transducer’s end and grows with increasing power of the ultrasonic excitation. Unfortunately, there are limitations of the experimental approach such as the difficulty in avoiding air bubbles (restriction of the allowed power), the limitation to a single frequency (as resonant transducers are used) and the inability to sample important variables throughout the cell at will. Therefore, increasingly often numerical techniques are used.

**METHODODOLOGY**

**Laser fringe velocimetry system**

The measurement of the streaming fluid velocity was monitored indirectly using tracer latex particles (Ω=0.5μm, density slightly above that of water) dispersed in filtered Milli-Q Milli-Ro, degassed water in glass vessels of various lengths and terminations, positioned horizontally.

The velocity of the particles was measured using a state-of-the-art laser fringe velocimetry (LFV) system (Thomas et al 2008). In our system, the modulated intensity light scattered from the ensemble of moving particles was fed via a single mode optical fibre collimator (Suparno et al 1994) to a photomultiplier tube (PMT). The output of the PMT was input to a 3rd order 30kHz cut-off low pass filter and then into a data acquisition system based on the LabView virtual instrument (VI). There, the time data representing the intensity signal was stored and Fast Fourier Transform (FFT) spectral analysis performed with up to 50kHz FFT bandwidth and 2Hz frequency resolution.

The velocity was obtained through spectral analysis of the intensity modulated light scattered by the particles in a millimetre size fringe created in crossed He-Ne laser light beams. This fringe-crossing technique or differential heterodyne method has been developed for the measurement of particle mobility. To remove the ambiguity of the velocity direction, the LFV system used frequency modulated light to produce a moving fringe (fringe spacing d=3.4μm, modulated at 2kHz). If the crossed beams have a small difference in frequency (ω), giving rise to moving fringes, the laser intensity is spatially and temporally sinusoidal over the interaction volume.

If scattering centres are stationary in the crossed beams then the time dependent scattered light signal varies in intensity at the same frequency as the shift frequency. If the particles are moving however, then this frequency will be shifted away from ω (upshifted for velocities counter to fringe velocity).

This shift in frequency of the intensity variation will be proportional to particle velocity, but not due to a Doppler effect per se.

Thus, use of the word Doppler (Drain, 1980) here is a misnomer and we prefer to call this technique, more correctly, Laser Fringe Velocimetry (LFV).
The light intensity modulation by a single particle (or a group of particles) in equation [1] contains the modulation frequency $\omega$ due to the moving fringe itself and its variation represented by time derivative of the angle $\phi$ due to the time-dependent particle position $x_0$.

$$\cos(\omega t + \phi)$$

where $\phi = 2\pi \frac{x(\tau)}{d}$

[1]

Note that the particle position $x_0$ is determined by the streaming velocity component $v_s$ assumed constant here and the high frequency (HF) ultrasonic vibration at frequency $\Omega$ and amplitude $A$ and is given by:

$$x_0 = A \sin(\Omega t) + v_s t$$

[2]

It can be shown that low pass filtering or short time averaging of the signal in equation [1] can remove the unwanted HF modulation without adversely affecting the resulting instantaneous short term averaged intensity if the ultrasonic phase variation remains within a few degrees. This is possible if the amplitude of ultrasonic particle vibration is much smaller than the fringe spacing ie if $A << d$.

The particle velocity was found from the frequency shift of the peak of the spectrum using equations [1] and [2] as $v_s = d \Delta \omega / (2 \pi)$. Whenever a spectrum did not have a clear maximum, several spectra were averaged.

The piezoelectric disc transducer generating the ultrasonic streaming ($\Omega = 2\text{cm}$ active piezoelectric area diam. $1.35\text{cm}$ type HM1630 by Honda El. Co. Ltd) was mounted in the bottom of the cylinder cell via a rubber flange. The top of the fluid was terminated by a lightly stretched thin latex membrane (thickness $\leq 0.05\ldots0.2\text{mm}$).

The transducer’s real power required to induce streaming at a few mm per second was in the order of $1\text{W}$, with the ultrasonic voltage between $V_p = 12...17V$ and current $I_p = 0.2...0.5A$ as measured at its electric terminals. The transducer was driven either by a self-tuned oscillator at $1.6 \text{MHz}$ (main resonance of the transducer) or at several lower frequencies using a sinewave generator with a power buffer. The additional frequencies corresponded with other resonances of the loaded transducer, where its current was in phase with the applied voltage. Both were monitored and measured on a digital oscilloscope (CRO).

The power could be varied only within a relatively small range. The lowest value was limited by the minimum needed for measurable streaming and the largest value limited by the onset of cavitation, which ruined the measurements. Inaccuracies arose mainly from two effects: the non-uniform shape of the peak of the spectrum of the captured signal scattered by millions of particles and by the fact that the transducer sinewave current was often harmonically distorted. A further source of uncertainty was instability of streaming ie its variation along and across the cylinder’s axis. Two main cylinder sizes were used: the smaller one ($\Omega = 19\text{mm}$, length $= 52\text{mm}$, the size used in the FE computer simulations) and the larger ones ($\Omega = 25\text{mm}$, lengths $8...10\text{cm}$ extended to $50\text{cm}$ by vinyl tubing.

Finite Element simulation conditions

We used a FE-TD analysis by Comsol™ (www.comsol.com) which allows for moving mesh simulations based on the arbitrary Lagrangian-Eulerian (ALE) scheme and optional user expressions. This is important since we have found that standard simulations using static mesh (equivalent to the inertial frame approach) do not lead to credible results whenever moving boundaries (fluid and transducer surface) are involved.

Ideally, the simulation platform should be entirely based on moving frame formulations (Lesniewski and Thomas 2008). While the moving frame implementation of the continuity and Euler equations is mathematically attractive (Lesniewski 2002), the tensor part of the NS equations requires further work.

However, with the traditional FE simulation approach we can approximate the moving frame conditions by defining suitable expressions for selected boundaries to make the mesh follow them while using the ALE algorithm everywhere else to maintain the convergence. Since ALE’s versatility is limited by its sensitivity to minor constraints (Masud, 2006) only when the stability or convergence could not be achieved, mesh movement along other boundaries was imposed and sometimes even internal dummy boundaries were included with expressions controlling the mesh movement over them.

We used the conventional incompressible single viscosity NS equation (in the inertial reference frame). At this stage we are not attempting to approach the true moving reference (MF) frame using the moving mesh as it is unlikely to let it move exactly in tune with the fluid without its entanglement (ie mesh inversion) over a longer time without crashing the simulation. We analyse streaming of the liquid and transient boundary forces while varying excitation frequency, amplitude of vibrations and their distribution over the transducer membrane in various cell geometries.

Most simulations have been carried in a micro-scale cylinder ($200\times200\mu\text{m}$) but some were done for a realistic size glass cylinder ($19\times52\text{mm}$).

The vibrating liquid (of viscosity $\eta = 1\text{ mPa.s}$ and density $\rho = 1000\text{ kg m}^{-3}$) is excited by nonuniform floor vibration in the $1\text{MHz}$ range.

Three different radial deflection profiles were used:
1. $\cos^2(\pi r/R_0)$ with only axial displacement and finite flux,
2. $\sin^2(\pi r/R_0)$ with only axial displacement and zero flux and
3. $\sin(\pi r/R_0)\times0.595$ with symmetric displacement and zero flux.

Here $r$ is the radial variable and $R_0$ is the cell radius. For non-axial symmetry geometries the profiles apply to one of the dimensions only.

Care was taken to arrange all transient signals and their spatial distribution to avoid any discontinuities – all time functions were continuous down to the $2^{nd}$ derivative (acceleration).

The number of mesh elements was within $10,000$ to $20,000$ for micro-scale cells but reached approximately $150,000$ for a realistically sized cylinder. The first indication of mesh size being too large and the onset of further undesirable effects leading eventually to the lack of convergence is the ‘noise’ on otherwise smooth plots of variables related to streaming. Note that these variables represent quantities smaller than the main vibrational variables by two or more orders of magnitude. Avoiding it was relatively easy for the micro-size vessel but for the regular scale we were close to reaching the limits of the simulation platform and the computer hardware. It is the number of mesh elements that increased the memory usage from the initial few GB to up to 30GB and slowed down the execution from a couple of days to a few weeks, for the worst cases.

Moving mesh simulations were carried out on a Dual Core, 2GHz Xeon CPU with 32GB RAM using WinXP-Pro 64bit OS, taking typically 30 hours per 0.5ms (for the micro size cell) and 120 hours per 1ms (for the large cell).

Note that inertial frame simulations are nearly twice as fast.

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RESULTS AND DISCUSSION

Laboratory measurements

Many results were taken and observations made. Typical results presented in Table 1 show tracer particle velocity measured as a function of position and transducer power. The main difficulty was to obtain repeatable, reproducible conditions either with respect to stable streaming or to stable resonant frequency of the transducer (unstable load and vortices had some affect on the resonant frequency which in turn would affect the fluid dynamics). Often, the spectrum did not have a clear maximum or its maximum was wide without a clearly defined single peak indicating a range of frequencies ie spread of particle velocities. In some cases two frequency peaks were observed, as if the streaming had two separate velocities (see comment at the end of the next section). All this is indicative of the complexity of this dynamic system with its tendency to create unstable vortices or even streaming in the reverse direction, towards the transducer. An example of two spectral profiles, one clearly defined and the other broad and less clear are shown in Fig. 3.

<table>
<thead>
<tr>
<th>Frequency [MHz]</th>
<th>Transducer Power [W]</th>
<th>Velocity [mms⁻¹]</th>
<th>Distance from the transducer, on axis [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.11</td>
<td>1.0</td>
<td>-2.9</td>
<td>~40</td>
</tr>
<tr>
<td>0.46</td>
<td>0.43</td>
<td>2.6</td>
<td>~40</td>
</tr>
<tr>
<td>1.75</td>
<td>1.1</td>
<td>2.5</td>
<td>~40</td>
</tr>
<tr>
<td>0.76</td>
<td>0.95</td>
<td>-0.28</td>
<td>~40</td>
</tr>
<tr>
<td>1.7</td>
<td>1.2</td>
<td>2.6-3.4</td>
<td>~40</td>
</tr>
<tr>
<td>1.6</td>
<td>0.74</td>
<td>2.1 (1.9-2.5)</td>
<td>40 (39-42)</td>
</tr>
<tr>
<td>1.6</td>
<td>0.81</td>
<td>2.7 (2.5-3.0)</td>
<td>40 (39-42)</td>
</tr>
<tr>
<td>1.6</td>
<td>0.83</td>
<td>2.8 (2.5-3.4)</td>
<td>40 (39-41)</td>
</tr>
<tr>
<td>1.6</td>
<td>0.9</td>
<td>3.5 (3.1-3.4)</td>
<td>40 (39-42)</td>
</tr>
<tr>
<td>1.6</td>
<td>0.98</td>
<td>3.9 (3.5-4.3)</td>
<td>40 (39-42)</td>
</tr>
<tr>
<td>1.6</td>
<td>1.1</td>
<td>4.0 (3.3-4.2)</td>
<td>40 (39-41)</td>
</tr>
</tbody>
</table>

An interesting phenomenon was observed when the dual peaks, one positive and one negative, appeared in the frequency spectrum. This occurred with measurements when the laser fringes were located near the cell walls. Measurements near the centre of the cell exhibited a single frequency peak.

Changing the surface (termination) from softer to a less elastic (using thicker latex or loading it with a layer of a jelly in the experimental investigation) sometimes led to the reversal of the direction of streaming but did not seem to inhibit or enhance it otherwise.

Reverse frequency shift (reverse streaming) has been observed in the following situations: along the edge of the cell (return flow), at lower transducer frequencies and, occasionally but consistently using the hard termination at lower frequencies or using a thin membrane with a mass loaded termination.

Finite Element simulations

Simulation gave us relative freedom in modifying most setup conditions and parameters which would not be possible experimentally.

The current objectives of the simulations were to identify the most critical factors causing streaming and to identify agreement with or departures from the known relations, such as dependence of the streaming velocity on the square of the excitation frequency and amplitude, and its proportionality to the bulk viscosity.

While we have already confirmed the latter, finding other specific effects proved difficult. This was not only because the FE simulations in a moving mesh take extremely long times (days to weeks) and require enormous computer RAM (4 to 30GB) as well as the storage space for data (files range from 2 to 200GB) but also because streaming is affected to some degree by most investigated parameters.

We have focused on investigating the effects of cell geometry including axial symmetry, vibration profile of the source, development of streaming with and without initial ultrasonic bursts and the role of the physical presence of the transducer. The latter was done by arranging separate simulation where the vibrations were created within a 2mm layer by a bodiless source generating spatially distributed volume source in place of the former vibrating membrane.
The results were quite surprising. We have eliminated a number of factors as not playing a significant role in the generation of streaming, these are (refer Fig. 4 and 5a,b.):

- The finite movement of the vibrating membrane,
- Increase of its amplitude while zeroing the flux,
- Presence of cell asymmetries, eg edges,
- Changing the surface response of the cell.

Figure 4. Comparison of centre point path (axial fluid displacement) in a 1.6MHz ultrasonic wave simulated using various conditions for the real size cell (also ref Note 1).

Traces:
(a) Red – single 3μs burst of 18μm centre membrane deflection followed by streaming.
(b) Blue - 3μs burst of 2μm centre membrane deflection, followed by continuous 1μm centre membrane amplitude vibration (radial profile 1).
(c) Black - 3μs burst of 2μm membrane deflection, followed by continuous 1μm centre membrane vibration (profile 2).
(d) Green – no initial burst, only continuous 1μm membrane amplitude vibration (slow start).

Note 1: For convenience, streaming has been presented using plots of transient axial displacement of a central point in the liquid (on axis, at half cell length). In most cases only average values were plotted (ie without showing short term vibrations following the ultrasonic movement of the membrane but of a fraction of its strength). Stronger initial bursts were used to speed up the onset of streaming.

In more detail, the lack of the membrane’s presence and replacing it by a vibrational volume force source as well as making the cell smooth by making its end spherical has increased rather than reduced the streaming (Fig 5b). Also using a cell with the above features and adding another symmetry (ie two coupled cells with a transducer in between) has only increased the streaming. Observation of the development of the streaming was done using animated cross section showing the spatial distribution of pressure and velocity (Fig 6).

This showed that streaming develops as a vortex rising from the ‘floor’ (source of vibrations) upwards and that there is a valley of the pressure rising just in front of the flow until it reaches the surface (considering the upper half of Fig 6.).

Figure 5a. Comparison of the centre point fluid axial displacement) in a 1MHz ultrasonic wave simulated for the micro-size cell for 4μm centre membrane deflection. The cell termination is ‘soft’ (pressure release) except for the green dashed trace.

Traces:
(a) Brown – Asymmetric waveform (push fast, retrace slower), radial profile 1
(b) Pink – radial profile 1 but ‘steeper’ (over half radius length)
(c) Red – radial profile 1
(d) Green – radial profile 2
(e) Green dashed – as traced above but with a mass loaded termination

Figure 5b. Comparison of the centre point fluid axial displacement) in a 1MHz ultrasonic wave simulated for the micro-size cells for a ‘bodiless’ force source equivalent to the velocity source as in fig. 5a. The cell termination is ‘hard’.

Traces:
(a) Orange – rectangular cell, no axial symmetry
(b) Dark Blue – half sphere terminated cylindrical cell
(c) Light Blue – as above, at 80% force amplitude
(d) Red – flat end cylinder
(e) Black Dashed – standard traces from fig.5a. (red and green).
This process was not significantly reduced by any of the investigated conditions. It was found that a more ‘spiky’ or narrower source radial profile (e.g., type 2 vs 1; but other forms were also simulated) visibly increased the streaming, especially where rotational flows were enhanced (type 2). Making the latter symmetrical however, reduced rather than increased the streaming. In the larger scale simulations a significant problem was posed by the slow progression of streaming – in a situation where computer memory was being exhausted by extending the simulation time. This was significantly reduced by initiating the vibrations with a short stronger burst of the same frequency and of a few periods duration only. Unexpectedly, this significantly affected the subsequent streaming by making it unsteady (Fig. 4).

It was interesting to find that some experimental spectra had the same previously puzzling features i.e., two frequency peaks, representing two superimposed streaming velocity components, large and small.

**CONCLUSIONS**

We have used experimental investigation with laser fringe velocimetry and simultaneous FE simulations with moving mesh of incompressible NS equations to investigate mechanisms of streaming. While earlier, commonly accepted nonlinear relations between streaming velocity and ultrasonic vibration amplitude and frequency and fluid viscosity could be reproduced on a micro-scale we have realised that streaming is affected by many more factors and it is unrealistic to expect any simple relation between them. This is the case even for the applied ultrasonic power. Furthermore, in both experiments and simulations, we have found that streaming tends to be progressively unsteady and its distribution also varies spatially as it is associated with vortex formation, yet it is progressively more complex when the cell size increases.

We have found that streaming is not increased i.e., not generated, by asymmetries of the cell containing the vibrating fluid, such as corners or lack of axial symmetry. It is also not caused by the finite displacement of the transducer membrane (i.e., a space-time asymmetry) interacting with the boundary layer there. In fact, increasing the symmetries increased the streaming. It was also found that streaming was significantly increased by short ultrasonic bursts with either a single burst or when a burst was used as a starting point for otherwise slow simulations.

We have found that laser velocimetry with moving fringes assisted by the LabView data processing is suitable for streaming measurements, especially as it is able to distinguish between forward and reverse streaming which has not been reported earlier by teams using common velocimetry.

We have also found that the FE TD simulations in a moving frame are a suitable tool for investigation of ultrasonic streaming in fluids giving results significantly different and more realistic than those with fixed mesh and boundaries. That simulation platform could be used in the future provided that multi-core PCs with 32GB RAM and clock speeds greater than 2GHz are utilized.

Future work should investigate the transient formation of streaming vortices and the energy distribution. The latter however would require finer spatial sampling and time stepping, significantly slowing down the computations. Therefore that part would need to be carried out as an extra section of the simulation after a period of initial, less stringent simulation.

**REFERENCES**


