On the Study of Longitudinal Wave traveling through an Elastic Plate Immersed in Water

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ABSTRACT

In this paper, an attempt is made for a problem of a longitudinal wave travelling through an elastic plate in water. The problem is reduced to that of an experimental and theoretical investigation of the reflected and transmitted waves. To solve the problem, a test sample of lateral dimension 202mm x 100mm x 10mm (Carbon steel) immersed in water was considered. An incident pulse was generated by a transducer on one side of the plate and received by an identical transducer on the other side of it. The generated pulse was also simulated using the 1D model in the time domain. Simulated and experimental ultrasonic traces have been used to analyze signals from the plate.

INTRODUCTION

Steel is essential in the building of bridges and civil structures. However, problems facing builders of such structures include cracking and corrosion. These factors reduce the strength in steel structures. Cracking is due to dynamic loading or fatigue while corrosion is due to the environmental chemicals on steel structures.

Here, we study the longitudinal wave reflection and transmission using a one-dimensional model in the time domain. Longitudinal waves which are launched into the plate are reflected from the plate, however, these waves can be perfectly transmitted without the reflections.

The study of longitudinal waves has important applications in nondestructive testing (NDT). The amount of reflection and transmission that occur when an ultrasound wave strikes an interface between water and the plate depends mainly on the physical state of water and the plate. Here, water is used as a couplant. Ultrasonic testing in water is widely used in nondestructive testing. Water is very often used for immersion coupling, because of its easy access and availability.

In this study ultrasonic techniques such as pulse echo and through transmission are considered. Longitudinal or normal transducers are generally used to study the reflection and transmission of ultrasound waves. Ultrasonic pulse echo and through transmission measurements are used for quality control of products in many industries. These techniques enable NDT technicians to monitor deviations of thickness or ultrasound velocity under industrial conditions. It is essential to estimate the appropriate thickness and velocity values based on the measurement of the time of flight of an ultrasonic signal in the plate. The estimation of delay time in ultrasonic through transmission measurements is also essential in this study.

When transmitting and receiving transducers are placed facing each other in a water tank, two signals are being studied. These signals are transmitted (a) between the two transducers only through water and (b) through the immersed steel plate placed between the two transducers. It is necessary to measure these signals. Generally, the waveform of the signal transmitted through the elastic plate is distorted due to the frequency dependent attenuation and phase velocity dispersion [1-3]. The investigation of several features of wave propagation in parallel fluid/solid layer systems was done by Plona et al. [4]. The reflection coefficients of sound waves incident on a liquid-cubic-crystal half-space has been calculated by Atalar [5].

The particular problem being discussed in this paper concerns the propagation of incident wave normal to the layer of the elastic plate placed between identical linear half-spaces, which in this case, is water.

The description of the wave propagation was made possible by considering reflections and transmissions due to the initial incident wave.

ULTRASOUND PULSE ESTIMATION

The transducer signal to be used in our experiment can be estimated using a formula given as [6-8]

\[ s(t) = A \exp(-(t-t_0)^2/(2\sigma^2)) \cos(2\pi ft + \phi) \]  

where \( A \) is the amplitude of the pulse, \( \sigma \) is a parameter to control the pulse width and determine the pulse delay time, \( t_0 \) is used to center the pulse, \( f \) is the center frequency of the pulse and \( \phi \) is the phase.

In order to simulate the signal that closely resembles the real signal generated by the transducer, we properly select the
The central frequency of the simulated spectrum corresponds to that of the real transducer which is 5MHz. The time-dependent simulated pulse matches well the experimentally measured echo pulse signal as seen in figure 2. The simulated pulse will be used to determine the theoretical total reflected and transmitted responses.

![Image](Image)

**Figure 1:** Estimated ultrasound pulse and its frequency spectrum

In the time domain, the finite duration incident pulse produced by the transducer in water is shown in figure 2.

![Image](Image)

**Figure 2:** Experimentally measured initial pulse produced by the immersion transducer of 5MHz. The distance between the two probes without the plate is 7mm. The vertical axis indicates the amplitude while the horizontal axis represents the elapsed time.

Figures 1 and 2 give the comparison between the initial pulse of a 5MHz transducer and the synthesised pulse from Eq. (1). It is essential to simulate the real transducer signal so that the theoretical total reflected and transmitted responses can resemble the real responses.

### THEORETICAL SIGNAL RESPONSES

A typical situation in the measurement of both the reflection and transmission properties of immersed plate is considered. Here, a longitudinal wave is incident from region 1 onto an imaginary plate of lateral dimension 202mm x 100mm x 10mm. When an ultrasonic longitudinal wave is normally incident onto the interface of two different media, part of its energy and sound pressure reflects back at the interface and the rest transmits through the interface as seen in figure 3 [9, 10].

The reflection and transmission ratios of energy can be determined by the acoustic impedances of the two media. The time of flight of the transmitted wave is given by

\[ t = \frac{d}{v} \]  

(2)

while the two-way travel time of a reflected wave is given by

\[ t = \frac{2d}{v} \]  

(3)

where d is the thickness of the plate. The initial conditions we considered in this study suppose that the plate is interacting with a wave pulse given by Eq. (1). The theoretical total reflection response z(t) of the plate in the time domain can be obtained by the convolution of the impulse response q(t) with the input signal s(t) appearing in Eq. (1). Mathematically, this can be written as

\[ z(t) = q(t) * s(t) \]  

(4)

where \( * \) represents a convolution operator and s(t) is the simulated pulse of Eq. (1). The modelling procedure consists in determining the total reflection coefficients, of a single layered plate at discrete frequencies belonging to the spectrum of the incident signal of Eq. (1). The overall material frequency response is obtained by multiplying the reflection coefficient by the corresponding frequency component of the excitation signal. Thus, the received output signal spectrum is given by

\[ Z(\omega) = Q_R(\omega)S(\omega) \]  

(5)

where the subscript R represents the reflection mode, \( Q_R(\omega) \) and \( S(\omega) \) are the respective Fourier transforms of q(t) and s(t). Here, the angular frequency is given by \( \omega = 2\pi f \). The Fourier transform of the incident signal of Eq. (1). The overall material frequency response is obtained by multiplying the reflection coefficient by the corresponding frequency component of the excitation signal. Thus, the received output signal spectrum is given by

\[ Z(\omega) = Q_R(\omega)S(\omega) \]  

(5)

with \( R_{12} \) being the reflection coefficient from the left of the plate, \( t_1 \) is the time delay for the pulse traveling the distance between the transducer and the plate in water and \( Q_{1} = (1 - (R_{12})^2)e^{j\omega t}1/(1 + R_{12}R_{23}e^{j\omega t}) \) with \( t_2 \) being the time delay for the pulse travelling the plate thickness d. The time of flight of the transmitted wave is given by Eq. (2) while the two-way travel time of a reflected wave is given by Eq. (3) where d is the thickness of the plate.

Similarly, for the through transmission technique, Eq. (5)
takes the form

\[ Z(\omega) = T_2 T_3 e^{-j\omega t_2} S(\omega) \]  

(7)

where \( S(\omega) \) is the input signal spectrum of the excitation signal from Eq. (1), \( t_2 \) is the time delay for the pulse traveling through the plate thickness \( d \). The output signal is then determined in the time domain by a numerical inverse Fourier transform as in figures 4 and 5. Figure 4 shows the calculated reflected response.

![Figure 4: The reflected pulse due to the simulated pulse in figure 1](image)

In figure 5 the calculated transmitted response is displayed.

![Figure 5: The transmitted pulse due to the simulated pulse in figure 1](image)

**MEASUREMENT SET-UP AND PROCEDURE**

**Experimental set-up**

Pulse echo and through transmission techniques in a water tank are considered. As the water tank is filled, the attenuation increases as a result of air dissolving in water [12]. This increase can be prevented or reduced to some extent, as long as the water is kept at slow pace or the air is outgassed before measurement. That is, the water in the immersion tank must be free of visible bubbles. The electricity connector for the immersion testing transducer must be waterproofed. The facing material of an immersion transducer is not designed to resist wear, therefore direct contact with the surface must be avoided. A 5MHz immersion testing transducer was used in this study. The equipment used is the Epoch 4B flaw detector.

The ultrasonic beam pattern is the relative sensitivity of a transducer as a function of spatial angle. The beam patterns of transducers are reciprocal, that is, the beam will be the same whether the transducer is used as a transmitter or a receiver. The larger the diameter of the transducer as compared to a wavelength of sound, the narrower the sound beam.

Here, the ultrasonic beam pattern is represented by the horizontal or vertical lines, see figures 6, 7 and 8, for the simplicity of representing the direction of propagation of the longitudinal pulse. The sound beam is directed through the water into the material using a straight beam technique for generating longitudinal waves in the steel plate. This sound beam spreads out to some extent. It is essential to note that the sound beam intensity decreases as the distance from the centre of the beam increases. When the sound beam is transmitted, it passes through the water which acts as the couplant, into and through the steel plate.

When the transducer vibrates it does that at a fixed frequency and energy moves through the water as ultrasonic vibration. The velocity is constant for a given medium but varies from one medium to another. For our purpose, a plate of lateral dimension 202mm x 100mm x 10mm, is considered. Three types of measurements were performed: one only through a water path, another with one transducer to allow reflection and the other through the plate placed between two transducers. In figure 6, the transmitting transducer and the receiving transducer were carefully aligned by maximizing the amplitude of the transmitted pulse.

![Figure 6: Principle of through transmission technique without a plate. T and R represent the transmitting and reflecting transducers respectively. The horizontal lines are used to indicate the direction of propagation of the longitudinal pulse.](image)

The transmitting transducer \( T \) sends pulses represented by \( s(t) \) through water without the plate and the receiving transducer \( R \) receives them. The measurement was taken without the plate placed between the transducers.

**Pulse echo technique**

When the ultrasound strikes the steel plate, the ultrasound enters the steel plate and within it the velocity is constant. Pulse echo immersion testing can use time of flight measurements to determine the sound speed. These measurements are made between the plate’s front surface and the back surface when there are no discontinuities. Pulse echo measurements provide very good resolution. Here, the time of flight measurements are made with the aid of a time gate.

Figure 7 shows the pulses sent by the transmitting transducer and reflected back by the front surface and back surface of the immersed steel plate to the same transducer.
The transducer was positioned such that the time the sound beam takes to travel from the transducer to the surface of the steel plate and back is longer than the time the sound beam takes to travel from the front surface of the steel plate to the back surface and back to its front surface.

### Through transmission technique

The through transmission technique is widely used because it tests the entire volume of a material within a single test. Through transmission ultrasonic testing is used for detection, verification, sizing and growth rate monitoring of cracks in materials. This technique uses two transducers in a pitch-catch arrangement. It is most widely known as the method of inspection in automated immersion testing for detection of laminas in steel or disbonding in composite materials.

For our purpose the through transmission being discussed is done manually from external plate surfaces. This technique is especially used for highly attenuating materials. However, through transmission technique requires both sides of the plate to be accessible. In this technique, homogeneous test objects without defects are appropriate to test because the reflection by defects is absent.

In the through transmission configuration, both transducers were mounted to produce a beam perpendicular to the surface. Because of the directivity pattern of the transducer, the two transducers must be exactly oriented so that the receiving transducer receives the maximum amount of sound energy. Here, the part of the steel plate immediately below the surface is not obscured by the initial pulse.

In figure 8, the transducer-plate separation distance is 8.8mm and the transducers are placed 27.6mm apart.

### Measurement of sound velocity

The ultrasonic wave speed can be obtained by measuring the travel time of pulses of ultrasonic longitudinal waves over a known path length. To measure the travel time of pulses, the transmitting transducer transmits a wave pulse into the plate in the case of pulse echo technique. In the case of the through transmission, the travel time is measured when the receiving transducer separated from the transmitting transducer, receives the pulse through the plate.

Here, the arrival time of the received signal, $\tau_1$ is recorded without placing the sample material between the transducers. Then, the plate is placed between the transducers and the arrival time of the received signal $\tau_2$ is recorded again. The sound velocity in the plate can now be determined by

$$\Delta t = \frac{d}{v_\text{water}} - \frac{d}{v}$$

where $v_\text{water}$ is the sound velocity in water determined from Eq. (2), $v$ is the sound speed in the plate, $d$ is the thickness of the plate and $\Delta t = \tau_1 - \tau_2$. Eq. (8) is then solved to give $v$ as

$$v = \frac{d}{(d/v_\text{water}) - \Delta t}$$

A change in sound speed in water is caused by the changes in temperature. Thus, the speed of sound is mainly a function of temperature. The sound speed in fresh water at 20°C with the density of 1000kg/m$^3$ has a velocity of 1480m/s and acoustic impedance of 1.48x10$^6$kg/m/s. The water used in the experiment had a temperature of 19°C. In order to determine the sound speed in the plate the thickness of the plate and the speed of sound in water must be known.

Figures 9 and 11 show signal traces recorded in an experiment with and without the plate respectively. If the plate is flawless, then the amplitude of the sound energy received by the receiving transducer is maximum.

### RESULTS AND DISCUSSION

The results are summarized in figures 9, 10 and 11 about which the following observations can be made. Figure 9 shows the indications made by the transmitting transducer and the receiving transducer placed 27.6mm apart. The highest indication is received by the receiving transducer indicating that the medium is flawless. The lower pulse will be received when the discontinuity is largest.
The time interval between the initial pulse and front surface indications represents the travel time of the ultrasonic signal through the water. The time interval between the front surface and back surface represents the time travel in the steel plate.

The first indication represents the initial pulse, the second indication represents the front surface of the plate. The third indication represents the back surface. The rest of the indications represent the multi-reflections of the second and third indications. That is, the pulse is trapped inside the plate. The first, second, third and fourth indications occur at 1.03mm, 11.49mm, 20.33mm and 30.09mm, respectively. Finding the differences of the above consecutive values and averaging them gives the thickness of the plate, that is, 10mm.

The change in time, \( \Delta t \), is assumed to be the time delay between two consecutive echoes. The speed of sound is assumed to be \( v \) in the plate. The average time, \( \Delta t \), for all the reflections is 3.27\( \mu \)s. Thus, the velocity of ultrasound in a plate is calculated using Eq. (3) to give 6116.21m/s.

Figure 11 shows the indications that occur as a result of the pulse passing through the carbon steel plate. The first strong indication represents the initial pulse while the following indication represents the pulse emerging the plate. The other indications are repetitions of the initial ones while the longer range chosen. From the flaw detector screen, the echoes occurred at 1.33mm, 10.38mm, 20.86mm, 28.29mm and 39.65mm respectively. Finding the differences of these values give the thickness of the specimen to be 9.58mm.

Table 1 gives the comparison between the theoretical and experimental velocities in water and steel.

**Table 1.** Comparison between velocities in water and steel found theoretically and experimentally. PE (Pulse echo technique) and TT (through transmission).

<table>
<thead>
<tr>
<th>Material</th>
<th>Theoretical value ( V ) (m/s)</th>
<th>PE value ( V ) (m/s)</th>
<th>TT value ( V ) (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>1480 20°C</td>
<td>-</td>
<td>1500 19°C</td>
</tr>
<tr>
<td>Steel</td>
<td>5900</td>
<td>6116.21</td>
<td>5299</td>
</tr>
</tbody>
</table>

The measured values of thickness of plate read from the flaw detector for the pulse echo and through transmission techniques were compared with those of vernier caliper as indicated in Table 2.

**Table 2.** Comparison of thickness measurements using vernier caliper, pulse echo (PE) and through transmission (TT)

<table>
<thead>
<tr>
<th>Sample</th>
<th>Vernier (mm)</th>
<th>PE (mm)</th>
<th>TT (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>10</td>
<td>10</td>
<td>9.6</td>
</tr>
</tbody>
</table>

**CONCLUSION**

Ultrasonic data has been collected and displayed as RF A-scan presentations as in figures 9, 10 and 11. These presentations display the amount of received ultrasonic energy as a function of time.

The scanning of the ultrasonic transducer has been performed in the region with dimensions 202mm along x axis, 100mm along y axis and 10mm along z axis. The numerical model of the plate under investigation was developed and simulation of the expected waveform, reflected by the plate and passed through the plate, was performed. The simulated and experimental A-scan signals obtained from the plate are presented in figures 4, 5, 10 and 11. The multiple reflections from the interfaces have been also taken into account.

The work describes a qualitative study of how acoustic longitudinal waves behave in water and a plate immersed in water. We have presented a mathematical technique to perform a simulated input pulse which closely resembles the initial pulse generated by an immersion transducer of 5MHz. The computation of the estimated pulse included the frequency spectrum with a center frequency of 5MHz. The formula used gives a good representation of the pulse in both time and frequency domains.

Experiments demonstrated reflection and transmission of the pulse passing through water with and without the plate.

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