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Probabilistic Prediction of Wheel Squeal under Field Humidity Variation

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ABSTRACT

This research examines the effect of changes in coefficient of friction due to humidity on the likelihood of wheel squeal events occurring in practice. Theoretical mechanics based modeling is developed and compared to a database of field measurements of wheel squeal occurrences at a field site in Australia. In particular, a relatively simplified model of wheel squeal is developed based on existing literature but notably incorporates probabilistic mechanics to account for field parameter variations and hence allows direct comparisons with field data. The model is then tuned to field site conditions at which over 2 million wheel passes have been monitored for a period of 3 years. The comparison indicates that field measured trends for the effect of relative humidity on coefficient of friction and hence the occurrences of wheel squeal have been able to be predicted using the very efficient model.

INTRODUCTION

“Wheel Squeal” describes the high pitched tonal noise a train’s wheels can make as they negotiate a curve (corner) of a railway line. It is somewhat similar in nature to brake squeal [1] however in contrast, it necessarily involves surfaces in rolling contact. Wheel squeal has plagued the railway industry for many years and is becoming more critical as railway usage increases and subjective human noise tolerance decreases. One of the perplexities of the occurrence of wheel squeal is that it appears to be sensitive to a wide range of parameters that naturally vary in the field.

Much modelling of wheel squeal has been performed particularly following the renowned works of Rudd[2] and review by Remington[3] and Thomson et al[4] in which the fundamental mechanism due to lateral creepage was more or less consolidated. Curve squeal is generally thought to originate from the unstable vibratory response of a railway wheel when subject to large creep forces whilst negotiating corners. Present consensus from the literature is that the unstable excitation of the squealing wheel originates from a lateral ‘stick-slip’ mechanism in the contact region analogous to the bowing of a violin string. Pure tone components of squeal, in the range 600 – 10,000 Hz, are generally related to wheel natural frequencies that correspond to the out-of-plane wheel bending (or axial) modes.

Various theories, although much simplified, have evolved over the years to describe the lateral traction ratio (lateral tangential force Q_y divided by the normal force N) and lateral creepage (≈ angle of attack) conditions during the excitation of a railway wheel in curves [14,5] as shown in Figure 1. Referring to Figure 1, under tractive rolling contact, traction first grows approximately proportially with creepage (see a)), as the area of adhesion (slip) within the contact region decreases (increases).

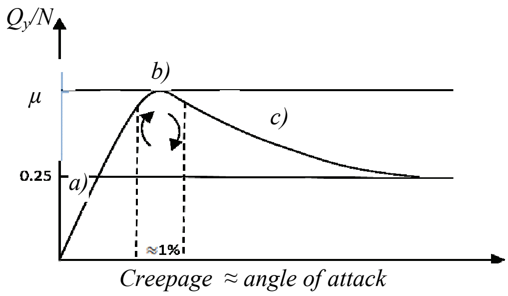

Figure 1. Traction/creepage characteristic of railway wheel-rail contact. a) region of slip/adhesion, b) point at which full sliding first occurs and c) negative slope region of increased sliding causing negative damping of creep oscillations.

For further increasing creepage, the traction reaches a maximum value equal to the static coefficient of friction, μ at b). Past this point there is insufficient friction within the contact region to prevent full sliding (no adhesion within the contact). The quasistatic creep force then shows a falling (negative slope) characteristic with the lateral force reducing with increasing creepage once the contact is in full sliding (see c)). The friction coefficient and shape and slope of the traction/creepage curve is effected by the so-called third body of the contact; an interfacial layer consisting of any lubricants, contaminants and material generated as a result of the contact interaction [6]. The negative slope in the creep curve at c) can be shown to be associated with negative damping of creep oscillations and hence squeal instability. Nearly all squeal models are based on this same mechanism: An angle of attack is imposed on the wheels by the bogie dynamics in the curve—this means the wheel slides laterally across the rail in addition to rolling—this is associated with a high lateral ‘‘traction-creep’’ force leading to self-excited “stick-slip” oscillations (about b) in Fig 1), which in turn excite wheel vibrations and radiated sound. The main differences between the models lie in details in the modelling of wheel/rail mechanical impedances (analytical [7,8,9,10,11], FEM [12,13,14]), vertical dynamics [13,14], contact forces and wheel sound radiation [12,13,14]. Some have also included wheel/rail roughness or wheel rotation effects [10,11]. Recently, a transient analysis of the lateral creepage of the wheel was performed to account for nonlinearities of friction forces and resultant excited wheel modes appeared to match field observations better[15].

Much recent research has also been focussed upon experimental validation of model predicted conditions under which squeal occurs and the effect of friction modifiers[16] on the phenomena. Recent predictive modelling includes that of [14,17] which include detailed representation of the dynamic behaviour of the wheel and rail and creepage in the saturated region. Twin disk and bogie testrigs have been utilised for validation under controlled environments [18]. Experimental results reported on the rolling contact force conditions during squeal include those by de Beer et al. [14], Monk-Steel et al. [17] and Koch et al [19]. In Monk-Steel et al. [17] the inclusion of longitudinal creep was shown to reduce the lateral creep force and thereby change the slope of the friction curve. This leads to a lower incidence of squeal in the presence of longitudinal creep, and an increase in the threshold of lateral creepage necessary for squeal. In Koch et al [19], measurements were carried out on a 1/4 scale test rig including a mono-block wheelset, and tests of anti-squealing solutions. A relation between noise level, rolling speed and angle of attack was confirmed experimentally and the average friction coefficient as a function of lateral creep was measured/inferred in dry conditions and with water. In [18] novel instrumentation directly on twin disc wheels close to the contact patch was used to obtain more direct measurements of lateral force to provide some validation of existing predictive models although the presence of a third body in the contact appears to affect the reliability of the testrig results. In [16] friction modifiers were shown to cause substantial (~12 dB) noise reductions associated with top-of-rail squeal and flanging noise at range of European mass transit sites. Despite these considerable efforts, there has been little success in correlating model predictions characterisitically "noisy" field measurements and observations of wheel squeal occurences. The effects of environmental factors have not directly been investigated in a quantified manner either.

The present research investigates the effect of changes in coefficient of friction due to humidity on the likelihood of wheel squeal events occurring in the field. The major contributions include:

- Field measured evidence of the effect of relative humidity on the occurrence of wheel squeal
- The development and validation of a very efficient probabilistic /mechanics based predictive model for prediction of the likelihood of wheel squeal occurrences in the field.

In the following, theoretical mechanics based modeling is developed and compared to a database of field measurements of wheel squeal occurrences at a troublesome squeal site in Australia. In particular, a relatively simplified model of wheel squeal is developed based on existing literature [2-14] but notably incorporates probabilistic mechanics to account for field parameter variations and hence allows direct comparisons with field data. The underlying mechanism of wheel squeal modelled is based on a flexible lateral mode of vibration of the wheel being self-excited due to unstable lateral stick-slip contact mechanics (providing negative damping i.e. energy input).

Modelling

The model developed in this paper is similar to the classic model derived in Rudd [2]. Although models of greater complexity exist (see review articles [3] and [4] for further references) this efficient model should be suitable for describing the underlying physics of the wheel squeal mechanism under changes in relative humidity. In essence, the model captures the wheel squeal mechanism of a flexible lateral mode of vibration of the wheel being self-excited due to unstable lateral stick-slip contact mechanics, providing negative damping (i.e. energy input).

Dynamics and Contact Mechanics Interaction

Consider the least stable lateral mode of wheelset/rail vibration for the situation depicted in **Figure 2**.

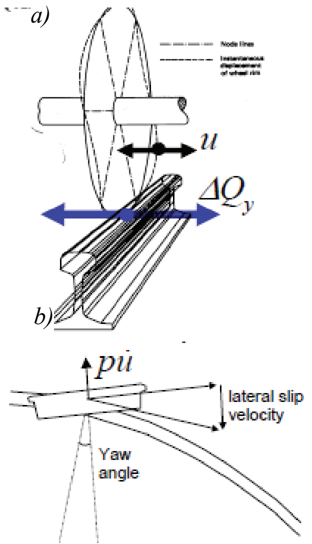

Figure 2. a) Wheel/rail interaction that leads to squeal, b) relationship between yaw angle (angle of attack≈ ξ_o) and lateral slip velocity $p\dot{u}$.

The equation of motion for the least stable mode is given by,

$$m\ddot{u} + c\dot{u} + ku = p\Delta Q_y, \tag{1}$$

where u is the modal displacement, m, c and k are modal mass, damping and stiffness, Q_y is lateral traction force and p is modal participation factor. For small oscillating deflections about a nominal condition (typical of vibrations) a Taylor series expansion can be used to linearise the traction – creep relation as

$$\Delta Q_y = \left.\frac{\partial Q_y}{\partial \xi}\right|_{\xi_0} \Delta\xi, \tag{2}$$

where ξ is the lateral creep. The nominal lateral creep ξ_o is dependent on the yaw angle according to **Figure 2** b). The lateral creep oscillations during squeal are associated with flexible lateral displacements of the wheel so that (2) may be expressed as

$$\Delta Q_y = \frac{-p\dot{u}}{V}\left.\frac{\partial Q_y}{\partial \xi}\right|_{\xi_0}. \tag{3}$$

The term $\partial Q_y/\partial\xi\big|_{\xi_o}$ is actually the slope of the creep law as shown in Figure 3. This figure highlights, the modeling focuses on small linear oscillations about a quasistatic nominal condition of nominal creep (ξ_o ≈angle of attack) for one wheel as shown.

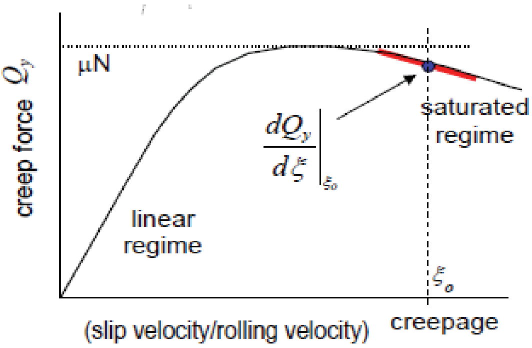

Figure 3. Creep/Force law showing saturated regime associated with wheel squeal.

and may be expressed as an explicit function of the friction coefficient,

$$\left.\frac{\partial Q_y}{\partial \xi}\right|_{\xi_0} = \mu N f'(\xi), \tag{4}$$

where N is the normal force and f is a function of the creep that depends on the physical properties of the contacting materials and local surface geometry. Combining equations (1), (3) and (4) to obtain the governing modal equation for squeal as

$$m\ddot{u} + S\dot{u} + ku = 0\ ,\ S = c + \mu N f'(\xi)\, p^2/V, \tag{5}$$

where S, represents the combined structural and contact damping of the system.

PROBABILISTIC CRITERIA FOR SQUEAL

System equation (5) is in the form of a simple spring-mass-damper (or 2nd order) system that has well-known vibration solutions. In particular, as the system damping S, approaches zero, transient vibration response to disturbances take longer to damp out and persist indefinitely at zero. If the system damping is negative, self-excited vibrations grow until a nonlinear amplitude is reached.

Hence it is believed squeal occurs when the system effective damping lowers beyond some critical minimum value (theoretically 0) due to a negative slope of the creep curve $f'(\xi)$ overcoming the positive structural damping in the wheel c. In particular, squeal occurs when the system damping S, becomes less than a critical value S_{crit}, i.e.

$$S < S_{crit}\ . \tag{6}$$

In order to determine the probability of squeal occurring given realistic variations in field parameters, we define the system parameter,

$$X = N f'(\xi)/V, \tag{7}$$

as a random variable, due to variations in the normal force, the equilibrium cornering creep and the passage speed, V, over successive passages at the same site. If we assume that X is normally distributed (due to the central limit theorem this may be reasonable due to the complicated relations between velocity and equilibrium cornering conditions), the average number of squeal events, $\langle n_{squeal}\rangle$, after n passes shall be approximately given by

$$\langle n_{squeal}\rangle = \frac{n}{2}\left(1 + erf\left(\frac{S_{crit} - \langle S\rangle}{\sigma_S\sqrt{2}}\right)\right) \tag{8}$$

where $\langle S\rangle$ and σ_S are the mean and the standard deviation of S and n is the number of wheelset passes.

Simplified Model: variations in system parameter *X* are independent of friction coefficient and speed

Now making the following assumptions

- S_{crit}, c and p are constants
- Changes in friction do not affect $\langle X\rangle$ or σ_X (mean and standard deviation of the system parameter X)

Note that these assumptions are not valid in general, but greatly simplify the analysis and may be approximately correct for small changes in friction coefficient. Under these assumptions it can be shown that, in terms of the system parameter random variable X, equation (8) becomes,

$$\langle n_{squeal}\rangle = \frac{n}{2}\left(1 + erf\left(\frac{S_{crit} - c}{\sigma_X\sqrt{2}\mu p^2} - \frac{\langle X\rangle}{\sigma_X\sqrt{2}}\right)\right). \tag{9}$$

Using equation (9) the effect of changes in the coefficient of friction on system damping with respect to the critical squeal value is qualitatively demonstrated in **Figure 4**.

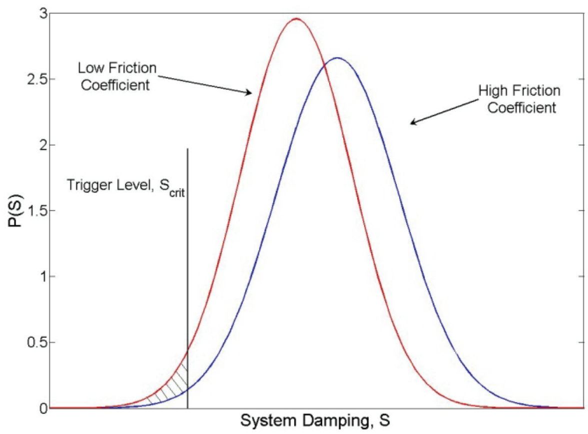

Figure 4. Effect of friction coefficient on distribution of the squeal triggering random variable.

In particular, the probabilistic distribution of the overall system damping is plotted for both low and high coefficients of friction. Squeal is predicted to occur where the system damping is less than the critical value S_{crit} as indicated on the left hand side of the distributions. The difference in area under the system damping distributions in this region (see hatching) indicates a higher probability for squeal to occur for the low friction case. The relatively large difference in area indicates that squeal is likely to be quite sensitive to changes in coefficient of friction. This effect is investigated quantitatively in the subsequent section under field measured conditions of variability due to changes in relative humidity.

Complex Model: variations in system parameter *X* are dependent on friction coefficient and speed

To more realistically model the effect of changes in coefficient of friction it is necessary to consider the influence of friction on equilibrium angle of attack [20]. In particular, it is noted that the variations in system parameter X will be predominantly driven by coefficient of friction and velocity variations. Thus, small changes in friction coefficient and velocity yields the approximate linear relationship,

$$X = X(\mu, V) \approx X_0 + \left.\frac{\partial X}{\partial V}\right|_0 (V - V_0) + \left.\frac{\partial X}{\partial \mu}\right|_0 (\mu - \mu_0), \tag{10}$$

where the subscript zero denotes the nominal equilibrium value. Substituting equation (10) into equation (5) yields

$$S = c + \mu p^2 \left(X_0 + \left.\frac{\partial X}{\partial V}\right|_0 (V - V_0) + \left.\frac{\partial X}{\partial \mu}\right|_0 (\mu - \mu_0) \right) \tag{11}$$

Therefore the following expressions for the mean and standard deviation of the system damping, S, can be derived as a function of the linear system parameter sensitivity coefficients in the form,

$$\langle S \rangle = c + \mu p^2 \left(A + \left.\frac{\partial X}{\partial V}\right|_0 \langle V \rangle + \left.\frac{\partial X}{\partial \mu}\right|_0 \mu \right),$$
$$A = X_0 - \left.\frac{\partial X}{\partial V}\right|_0 V_0 - \left.\frac{\partial X}{\partial \mu}\right|_0 \mu_0 \tag{12}$$

and

$$\sigma_S = \mu p^2 \left.\frac{\partial X}{\partial V}\right|_0 \sigma_V, \tag{13}$$

Equations (12) and (13) along with (8) can be used to determine the probability of squeal occurring where the effect of changes in the coefficient of friction and velocity on system damping have been taken into account explicitly.

In the following field case study it is also assumed that;

- S_{crit} is approximately zero ie squeal occurs when the system damping becomes negative.
- $\left.\frac{\partial X}{\partial \mu}\right|_0 > 0$, ie a decreasing coefficient of friction decreases the effective system damping due to the contact mechanics. The mechanism for this is via an increase in angle of attack with reduced friction [4].

FIELD INVESTIGATION

A field investigation of the effect of humidity on squeal was performed using a RailSQAD® system (Railway Squeal Acoustic Detection System) and climactic information from the Bureau of Meteorology in Australia. RailSQAD is a trackside acoustic microphone array and trackside sensor system that detects, follows, records and trends a range of train and wheel/rail noise sources. Rail cars are tagged with wagon identification AEI tags enabling the system to analyse and trend individual wheelsets. All data streams are connected to a PC-based data acquisition system that records and analyses the noise emitted by wheel/rail interaction as well as other parameters such as wheel speed and direction. Wheelset noise emissions are first classified as Squealing or Flanging, and then ranked according to noise level, against user agreed criteria. This processed data can then be statistically analysed against varying train operating parameters, rollingstock condition and environmental conditions.

In the present investigation, RailSQAD data has been compared with climactic information from the Bureau of Meteorology[1], along with noise complaint data from a local Council web survey on rail noise, to investigate vehicle, location and climactic factors. In particular, data was analysed for an average of 2100 axle passes/day over a 1 year period for a suburban railway site with a 300m corner radius on an inclined continuously welded track as shown in Figure 4.

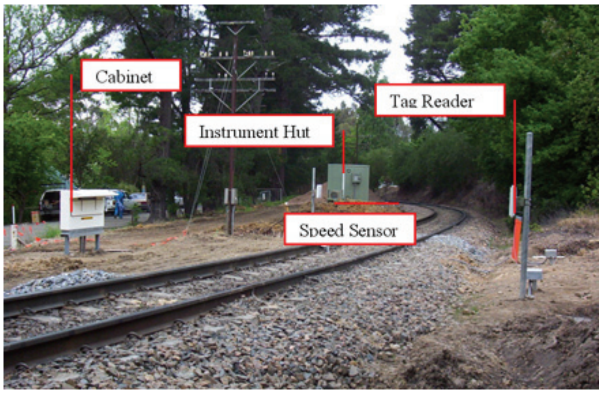

Figure 4. Field site showing RailSQAD microphone array and sensors (from www.railbam.com.au).

Figure 4 also indicates the main components of the RailSQAD measurement equipment including a cabinet of microphone arrays, a Tag reader to identify wheelsets and a vehicle speed sensor. The traffic was almost fully freight with only a few passenger trains per week. A statistical analysis (see [21]) of the data collected over one year showed a correlation between the average relative humidity and the number for squeal events recorded. Two levels of squeal are given; one where squeal sound power level exceeds 105dB and another when it exceeds 95dB. A sample of the relationship is shown in Figure 5.

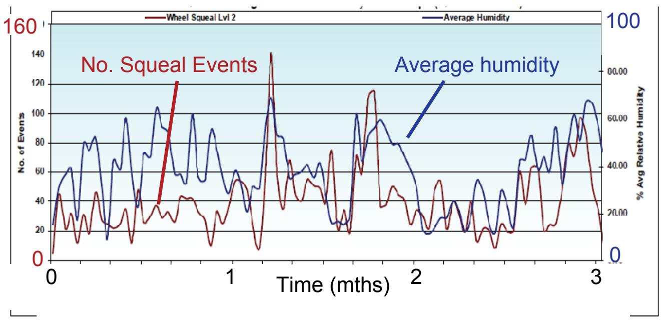

Figure 5. Relationship between average relative humidity (%) and squeal events (from [21]).

The correlation between squeal events and humidity is not directly discernable in Figure 5 but is investigated and shown in more detail for comparison with the modelling results subsequently (ie see Figure 7). In particular, it was of interest to examine if the field relationship between relative humidity and squeal (in Fig. 5) can be explained via the model derived as equation (8,12,13). The proposed mechanism is via a reduction in the coefficient of friction with an increase in relative humidity. In support of this, in [22], it is shown experimentally that humidity has an almost linear effect on coefficient of friction, as seen in Figure 6. This trend is most likely due to higher levels of relative humidity being associated with higher levels of local condensation in the contact patch.

[1] Climactic conditions were measured at a weather station, 2-3 kms away of similar, but slightly higher elevation.

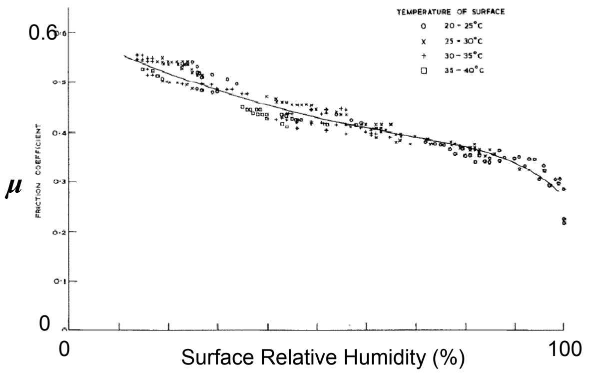

Figure 6. Effect of relative humidity on coefficient of friction (reproduced from [22]).

This in turn acts in the same manner as direct lubrication does on friction coefficient. A linear approximation of this experimental data yields the approximate relation that,

$$\mu=-0.0022H\%+0.6, \qquad (14)$$

where $H\%$ is the percentage relative humidity. The historical data for the average relative humidity and the number of squeal events and axle passes per day are required to fully examine the effect in mean trend of the data presented in Figure 5. This data has been provided by ARTC and the results of the measured data have been binned together in 10% increments of relative humidity to clearly show the mean trend of the data. In this case study, only wheel squeal 2 events (>90 dB) have been used as they are more frequent and hence are more statistically significant. At present, each industry body may have their own standard in this regard.

The theoretical trend expected requires the additional parameter values obtained from traffic data over the same period examined. In particular, the measured average and variation in vehicle speed was $\langle V\rangle$=37.2kph and σ_V=14.3kph from which,

$$\langle X\rangle/(\sigma_X \sqrt{2})\approx 1.84. \qquad (15)$$

Also required are the unknown values for the damping coefficient for the least stable mode of the system, c, the modal contribution factor, p, and the system parameters dependant on the equilibrium cornering conditions (see equation 13). For a comparison of the trend expected these parameters have been fit to the humidity data and are summarized as,

$$(-c-A-\mu\, \partial X/\partial\mu|_o))/(\sqrt{2}\, \sigma_V\, \partial X/\partial V|_o)=1.19-1.20\mu. \qquad (16)$$

RESULTS

The results of the measured data and model predictions are shown in Figure 7. In particular, the field measured mean effect of relative humidity on probability of squeal events is shown along with 95% confidence intervals for the mean trend. For comparison, the predictive model theoretical trend is also shown. Both trends indicate an increasing likelihood for squeal events as the relative humidity increases. This is consistent with the relationships with friction coefficient and system damping shown in figures 4 and 6. More specifically, it is likely that higher (lower) relative humidity increases (decreases) contact patch lubrication to decrease (increase) friction coefficient that in turn decreases (increases) the likelihood of full sliding which is associated with negative system damping ie vibrational instability.

Figure 7 shows that probabilistic theoretical model is reasonably successful in matching the trend of the effect of relative humidity on the probability of wheel squeal. In particular, the correlation coefficient squared (R2) between the theoretical and measured data is 0.87, indicating a good correlation between theoretical and field results. This result is obtained using the measured historical train data along with realistic parameters for the equilibrium cornering conditions for the train. This seems to confirm the mechanism of wheel squeal modelled in equations (1) to (6). Further confidence may be gained with accurate measurements/models of the cornering parameters used in this investigation (summarized in equation 16). Also the relative humidity of the rail and air are likely to be different due to heating of the rail, however the use of the relative humidity of the air alone should give an approximate representation of the trend until better estimation methods/measurements become available.

It is noted that the parameters of (16) are also able to be measured/estimated via more field data and performing theoretical cornering simulations to obtain the equilibrium cornering configuration as well as the sensitivity of this equilibrium to changes in speed and coefficient of friction. It is also interesting to note that the simplified model equation (9) is able to produce a similar trend to the measured data as shown in Figure 7, but only with an unrealistic critical value for system damping at which wheel squeal occurs (ie. positive rather than negative). This is counter to the accepted model of wheel squeal being associated with a negative slope of the creep force relation (as shown in Fig. 3), hence necessitating the need for the complex model.

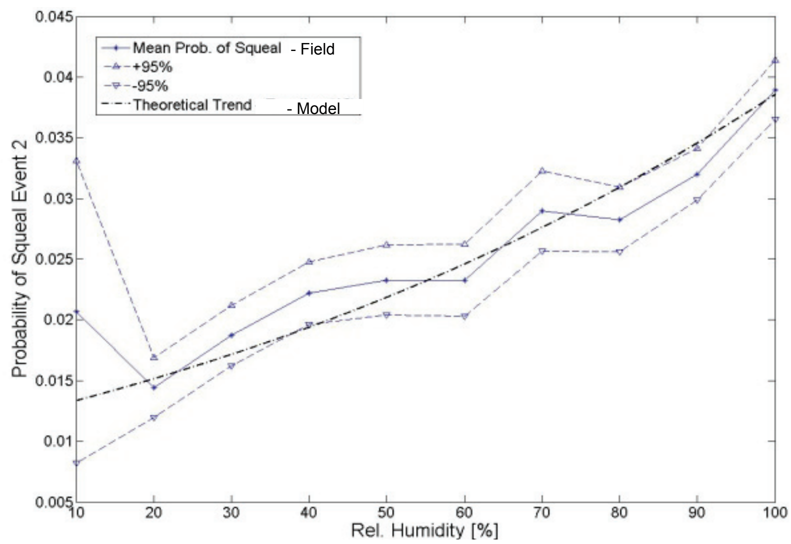

Figure 7. Field Measured and Model Predicted Effect of Relative Humidity on Probability of Squeal. Triangles indicate confidence intervals for field measurements in probability estimate.

CONCLUSIONS

Field measured trends for the effect of relative humidity on coefficient of friction have been able to be predicted using a relatively simplified wheel squeal model that is suitable for probabilistic (likelihood) measurements and interpretations. In particular, the model-based probability of a squeal event occurring over the full humidy range correlated with field measured results with 95% confidence. The model is based on a random variation of system damping based on measured variations in speed and humidity. The cornering parameters assumed for this investigation could be measured in the field.

Further research is underway, to provide further insight and extend the predictive modelling This includes: field measurements of the angle of attack, lateral and vertical forces of passing trains; the development of an estimation method for the relative humidity of the rail based on measurements of air temperature, rail temperature and relative humidity of the air; and a full investigation of the interrelationships between humidity, the creep curve, the equilibrium cornering behaviour and the probabilistic occurrence of wheel squeal.

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