

A Directional Loudspeaker Array for Surround Sound in Reverberant Rooms

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ABSTRACT

We present a method to enhance the performance of the sound field reproduction approach to 2-D surround sound applicable for reverberant rooms, under the constraint that only a small number of loudspeakers is permissible. The method is based on the idea of using steerable directional loudspeakers to exploit the room reverberation. In home theater applications, exact sound field reproduction is currently handicapped by the unreasonably large numbers of loudspeakers required for operation over audible frequencies. However by exploiting reverberant wall reflections, mirror-sources may be used as additional loudspeakers to help perform the reproduction. Utilizing mirror-sources, the number of loudspeaker locations required throughout the room may be reduced. A large array of omnidirectional loudspeakers can then be replaced by a small number of compact configurable directional loudspeakers. Simulating in a reverberant room with each directional loudspeaker modeled as an array of monopoles, we show that the performance is comparable to a circular array with a much larger number of elements. We quantify the accuracy of the sound field reproduction and the robustness to calibration error, comparing the proposed scheme with the more standard circular array geometry.

INTRODUCTION

In this paper we show that, using a small number of directionally-controlled loudspeakers, a sound field may be accurately reproduced in a reverberant room. The goal of surround sound is to reproduce a sound field within a control region. Using constructive and destructive interference from the waves emitted from a set of directional loudspeakers, sound field reproduction can be used to create an arbitrary sound field in the control region.

A common objective in surround sound is to place one or more phantom sources around the listener. To place a phantom source at any intended orientation, one would ideally distribute loudspeakers evenly around the listener, with sufficient numbers to avoid spatial aliasing. One such geometry is the uniform circular array (UCA). To meet aliasing requirements in 2-D, at least $2kR + 1$ loudspeakers are required [Ward and Abhayapala 2001]. However, neither this loudspeaker geometry nor the large numbers of loudspeakers are practical, as both aspects demand a large amount of physical space in the room which carries a low spouse-acceptance-factor. We suggest an approach to reduce the heavy requirements on numbers and arrangement of loudspeakers by using a loudspeaker configuration which exploits room reverberation.

Most approaches to surround sound operate on the basis of either ignoring the presence of reverberation or managing the levels produced. Earlier works [Ward and Abhayapala 2001; Poletti 2000] ignored reverberation in the reproduction problem. Recently, researchers have seen the possibility of reducing the reverberant level created by the loudspeaker system. By virtue of the equation governing the wave field synthesis approach to surround sound, the Kirchoff-Helmholtz equation, by surrounding the control region with an array of both monopole and dipole loudspeakers, the field external to the loudspeaker array could be canceled entirely. More practically, directional loudspeakers can also be used to reduce the strength of the sound reaching the room walls [Boon and Ouweltjes 1997; Poletti,

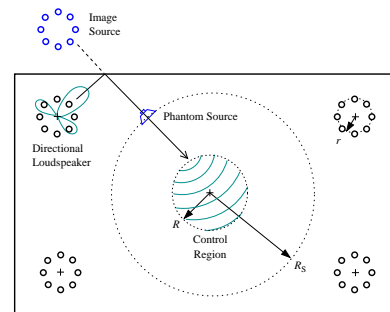


Figure 1: Creating a virtual sound source from a first order reflection. Shown are the array geometries of the corner array and the sound field control region.

Fazi, and Nelson 2010].

An alternative is to compensate for the reverberation in the surround sound system. Using exact knowledge of the acoustic transfer function (ATF) between the loudspeaker and every point within the control region, one may equalize the reverberation at the loudspeaker filters [Betlehem and Abhayapala 2005; Poletti 2005; Fuster et al. 2005; Spors et al. 2007; Gauthier and Berry 2007]. The challenge in this approach is in devising a scheme which is robust to ATF measurement error. An approach was proposed and tested against white noise in the ATF measurements in [Betlehem and Abhayapala 2005]. The reverberant compensation was tested using a circular array of monopoles.

This paper addresses the question of whether reverberant reflections could be *exploited* to enhance the application of surround sound in home theater. Instead of surrounding the listening area with a UCA of a large number of elements, a sparse set of steerable directional loudspeakers located near the corners of a room could be used. The configuration would then work by

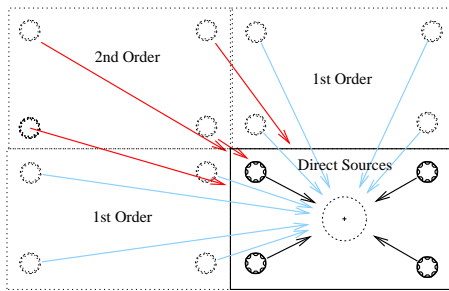


Figure 2: Virtual sound source directions available from utilizing direct source (black), the first order reflections (blue) and second order reflections (red).

exploiting wall reflections in a typical room which generate the reverberation to produce a large number of virtual loudspeaker locations for creating a phantom source.

One application of this concept is the digital sound projector [Yamaha 2010; Pioneer 2006] where first and second order reflections are exploited to create spatial sound. A large number of transducers mounted into a flat panel form five steerable beams used to create phantom sources in the positions required by Dolby surround sound systems. However this beamforming technology does not attempt the more exact sound field reproduction prescribed in [Vanderkoy and Lipshitz 1987; Nicol and Emerit 1999; Ward and Abhayapala 2001; Betlehem and Abhayapala 2005; Poletti 2005].

Exploiting reverberant reflection for exact sound field reproduction would only be feasible if the reproduction accuracy were insensitive to the loudspeaker gain mismatch and ATF measurement inaccuracies. Through exploring the performance of the corner array, we show that the reproduction accuracy and robustness can be comparable to that of the circular array. We use an array of four loudspeakers, each with a configurable directivity patterns. Performance is quantified with the mean square error in the reproduced sound field to indicate accuracy and measure to quantify robustness to perturbation of system parameters.

PROBLEM STATEMENT

We consider reproducing the sound field over a volume of space with a small number L of *steerable* directional loudspeakers. Each configurable directional loudspeaker is realized using an identical array of 2-D monopole elements, so that reverberation can be easily simulated using the image-source method [Allen and Berkley 1979]. We restrict attention to 2-D reproduction in a room using vertical line sources.

The purpose of the steerable loudspeaker approach is to generate additional phantom image directions by creating beams which bounce off reflective walls. Quantitative features of the reverberant sound field are accurately modeled by the image-source method for the case of specular reflection. By exploiting specular reflections, we can improve performance in reverberant environments.

We first overview the pressure matching approach of sound field reproduction. We then describe the approach to modeling the directional loudspeaker.

Pressure Matching

In the pressure matching approach, one reproduces a desired soundfield by matching the pressure at a finite number of points within the region of interest. We shall refer to these points as the *matching points* and the region as the *control region*. The

control region is a circular 2-D region of radius R . To reproduce the desired pressure field $P_d(\mathbf{x}; \omega)$ over the control region using the L directional loudspeakers of M 2-D monopole elements, one needs to satisfy the equation:

$$\sum_{\ell=1}^L \sum_{m=1}^M G_{\ell m}(\omega) H(\mathbf{x}_q, \mathbf{y}_{\ell m}; \omega) = P_d(\mathbf{x}_q; \omega), \quad q = 1, \dots, Q$$

where $H(\mathbf{x}_q, \mathbf{y}_{\ell m}; \omega)$ is the ATF between a monopole at $\mathbf{y}_{\ell m}$ and a point sensor at \mathbf{x}_q in a reverberant room environment. Pressure matching is performed at Q matching points $\{\mathbf{x}_1, \dots, \mathbf{x}_Q\}$. The set of equations required to be satisfied can be manipulated into the matrix-vector form

$$\mathbf{H}\mathbf{g} = \mathbf{p}_d, \quad (1)$$

where $[\mathbf{H}]_{q(M\ell+p)} = H(\mathbf{x}_q, \mathbf{y}_{\ell m}; \omega)$ is a matrix of acoustic transfer functions, $[\mathbf{g}]_{M\ell+m} = G_{\ell m}(\omega)$ is a vector of loudspeaker weights and $[\mathbf{p}_d]_q = P_d(\mathbf{x}_q; \omega)$ is a vector of desired pressures at the matching points. The loudspeaker weights \mathbf{g} required to achieve a small MSE robustly can be calculated through the regularized least squares solution:

$$\hat{\mathbf{g}} = [\mathbf{H}^H \mathbf{H} + \lambda \mathbf{I}]^{-1} \mathbf{H}^H \mathbf{p}_d. \quad (2)$$

where λ is the Tikhonov regularization parameter.

A class of desired pressure fields that shall be reproduced in this paper is the 2-D phantom monopole source:

$$P_d(\mathbf{x}; \omega) = P_0 H_0^{(2)}(k\|\mathbf{x} - R_s \phi_s\|),$$

where R_s is phantom source radius, $k = 2\pi f/c$ is the wavenumber, f is the frequency of interest, $\phi_s = [\cos \phi_s, \sin \phi_s]^T$, ϕ_s is the orientation angle of the phantom source, $H_0^{(2)}(\cdot)$ is the Hankel function of the second kind of order 0 and P_0 is a pressure amplitude constant.

For accurate sound field reproduction over a circular 2-D region of radius R , the number of monopoles required at wavenumber k [Poletti 2000] is:

$$L' = 2kR + 1. \quad (3)$$

This number corresponds to the number of spatial modes active within the control region.

Directional Loudspeaker Design

In general, a directional loudspeaker can be modeled with an N th order directivity pattern. The far-field directivity pattern $D_\ell(\phi; \omega)$ at angular frequency ω can be written as the phase mode expansion:

$$D_\ell(\phi; \omega) = \sum_{n=-N}^N \alpha_{n\ell}(\omega) e^{in\phi}, \quad (4)$$

whose $\alpha_{n\ell}(\omega)$ are the weighting coefficients for the n th order phase mode. Each directional loudspeaker is realized by arranging a number M of monopoles into an UCA of radius r . To ensure loudspeaker responses up to N th order are obtainable, one designs each monopole array choosing r and M as follows:

- Choose $r = N/k$ to excite a necessary number of spatial modes, up to order N [Ward and Abhayapala 2001].
- Choose $M \geq 2N + 1$ to ensure adequate number of degrees of freedom are available to create the loudspeaker responses.

This scheme ensures monopoles are spaced $\lambda/2$ or less apart to avoid spatial aliasing at frequency f . The array design could be realized in practice, by housing the M monopoles inside a cylindrical loudspeaker box. The monopole weights are then

chosen according to regularized least squares to suit the sound field reproduction problem.

We choose the loudspeaker parameters r and M to ensure the directional loudspeakers are designed to achieve second order directivity responses. The near-field directivity pattern D_ℓ of each configurable directional loudspeaker ℓ that results from the above pressure matching design is:

$$D_\ell(\phi, d; \omega) = \sum_{m=1}^M G_{\ell m}(\omega) H_0^{(2)}(k|r\phi_m - d\varphi|)$$

where d is the distance from the centre of the UCA of the loudspeakers, ϕ the angle made with the x -axis, $\varphi = [\cos \phi, \sin \phi]^T$, $\phi_m = [\cos \phi_m, \sin \phi_m]^T$ and ϕ_m is the orientation angle of each loudspeaker m .

Pressure Matching with an UCA

For comparison, we shall also reproduce the sound field with $L' = LM$ acoustic monopoles arranged into an UCA. Matching the pressure over Q points inside the sound control region, the loudspeaker weights are again obtained through the regularized least squares solution in (2) where instead $[\mathbf{H}]_{m\ell} = H(\mathbf{x}_m, \mathbf{y}_\ell; \omega)$ is now the ATF between a monopole at located at \mathbf{y}_ℓ in the UCA and a point sensor at \mathbf{x}_m .

ON ROBUST DESIGN

We briefly discuss aspects which contribute to the robustness of a surround sound system. The way the robustness is quantified is through the loudspeaker weight energy (LWE) $\|\mathbf{g}\|^2$. The white noise gain [Van Trees 2002, p. 69], quantifies the ability of a loudspeaker array to suppress spatially uncorrelated noise in the source signal. The major errors such as those in the amplitude and phase of the ATF estimates and loudspeaker position errors are nearly uncorrelated and affect the signal processing in a manner similar to spatially white noise [Cox, Zeskind, and Kooij 1986]. As the LWE is inversely proportional to the white noise gain, it provides a relative measure of the reaction to such errors.

We examine the factors affecting robustness with aid of the singular value decomposition (SVD). In the case $L' \leq M$, the SVD of the ATF matrix \mathbf{H} can be written:

$$\mathbf{H} = \sum_{n=1}^{L'} \sigma_n \mathbf{u}_n \mathbf{v}_n^H$$

where \mathbf{u}_n are the orthonormal output vectors of the sound fields reconstructible by \mathbf{H} , \mathbf{v}_n are the orthonormal input vectors of loudspeaker weights and σ_n are the singular values of matrix \mathbf{H} describing the strength of the sound field created by each loudspeaker weight \mathbf{v}_n . We shall assume singular values are ordered $\sigma_1 > \sigma_2 > \dots > \sigma_{L'}$. After substituting the SVD of \mathbf{H} into (2), the loudspeaker weights can be shown to be:

$$\mathbf{g} = \sum_{n=1}^{L'} \frac{\sigma_n}{\sigma_n^2 + \lambda} c_n \mathbf{v}_n,$$

where $c_n = \mathbf{u}_n^H \mathbf{p}_d$ is the projection of \mathbf{p}_d on the subspace of sound fields reconstructible by \mathbf{H} .

A straight-forward way of improving robustness is to increase the Tikhonov regularization parameter λ . The LWE can be shown to be:

$$\|\mathbf{g}\|^2 = \sum_{n=1}^{L'} \left(\frac{\sigma_n}{\sigma_n^2 + \lambda} \right)^2 |c_n|^2,$$

which is inversely related to λ . It is largest if we choose a vector as the sound field $\mathbf{g} = \mathbf{u}_{L'}$ with the smallest singular value,

where LWE is equal to $\sigma_{L'}^2 / (\sigma_{L'}^2 + \lambda)^2$. Increasing λ however reduces the size of the LWE at the expense of performance.

In contrast, manipulating the acoustic environment's geometry so that the desired sound field \mathbf{p}_d projects onto only the reconstructible sound fields \mathbf{u}_n having large singular values σ_n would also improve robustness. Robustness can be improved by:

- choosing a loudspeaker array geometry which couples strongly the principal components of the ATF matrix to the desired set of sound fields. One way to do this is to place a loudspeaker in-line with the desired phantom source;
- changing the acoustic sound environment to achieve the same ends. One way is to introduce reverberation to create an image-source in-line with the desired phantom source.

As illustrated by the blue and red arrows in Figure 2, first and second order reflections greatly increase the range of directions a phantom can be placed. There appears good scope for improving performance by exploiting these reflections.

In the case of the array of directional loudspeakers, the LWE includes a component attributable to the ease of realizing the directional patterns with the M monopoles. The measure hence relies on the directional loudspeaker being properly designed, which will be the case if the number and geometry of the monopoles are chosen correctly for the design frequencies.

RESULTS AND DISCUSSION

Simulation Setup

We compared performance of the corner array with a UCA in a 6.4×5 m room under different reverberant conditions:

1. anechoic chamber,
2. a single (north) wall only with reflection coefficient $\gamma = 0.9$,
3. all wall reflection coefficients set to $\bar{\gamma} = 0.9$ and
4. the same room with coefficients $\gamma = [0.4, 0.8, 0.2, 0.6]$,

simulating performance at 500Hz. The array geometries are summarized as:

Corner Array: Each directional loudspeaker consisted of $M = 8$ monopole sources arranged in an UCA at radius $r = 0.2$ m, which can robustly generate accurate second order loudspeaker responses. Each of the $L = 4$ loudspeakers was placed in a corner of the room at 1.5 m from both walls.

Uniform Circular Array: $L' = 32$ loudspeakers were used and located at $R_s = 2$ m from the center of the control region.

The control region was located at the center of the room with a radius of $R = 0.5$ m. We positioned the loudspeakers of the corner array away from the walls to increase the range of directions that can be attained from low order reverberant reflections.

Room reverberation was simulated using a 2-D implementation of the image-source method [Allen and Berkley 1979], with ATFs computed using:

$$H(\mathbf{x}_q, \mathbf{y}_\ell; \omega) = \sum_{i=1}^{\infty} \xi_i H_0^{(2)}(k\|\mathbf{x}_q - \mathbf{y}_\ell^{(i)}\|),$$

where ξ_i denote the accumulated reflection coefficient for the i th image-source and $\mathbf{y}_\ell^{(i)}$ the position of the i th image-source of monopole ℓ , truncating the impulse responses to the T30 reverberation time. The T30 reverberation times are 530 msec and 100 msec for reverberant rooms 3. and 4. respectively.

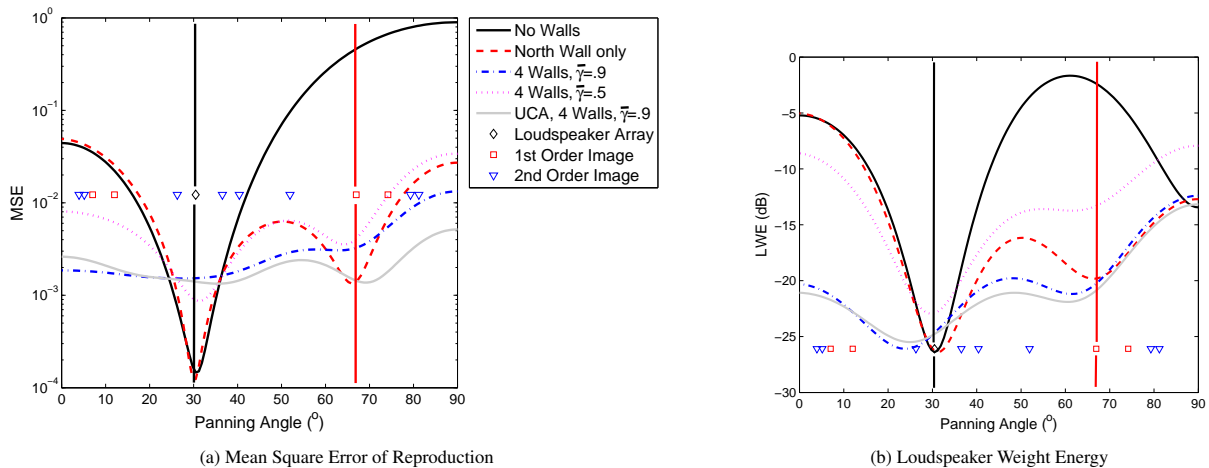


Figure 3: Performance comparison as a function of panning angle for a virtual source at 2m. Shown are (a) the MSE and (b) the LWE. Directions to the loudspeaker and first and second order image-sources are as marked. Plots clearly show that one or more wall reflections improves the reproduction performance of the corner array by up to two orders of magnitude above anechoic room conditions. Marked with vertical lines are the direct sound direction (black) and the most dominant reflection (red).

Sound field reproduction was carried out using the regularized pressure matching in (2) with Tikhonov parameter $\lambda = 0.1$ to create a 2-D monopole phantom source at 2 m from the center of the control region. Due to the symmetry in the room geometry, it was sufficient to pan the phantom source angle over a 90° angular range.

We compare the performance of the corner array with that of an UCA of 32 loudspeakers in reverberant room case 3. For a 0.5 m control region radius, only 11 monopoles are required by (3) at 500 Hz, so there are a number of additional degrees of freedom with which to perform the reproduction. These degrees of freedom are not wasted, as adding loudspeakers above the Nyquist sampling requirements improves the robustness.

Array Performance

The MSE reproduction performance of the corner array in several acoustic environments is shown in Figure 3, where we study the effect of adding one or more reflective walls to the room. In the anechoic environment, the corner array performs poorly when panning angles away from the directional loudspeakers (black curve in Figure 3). One or more strong reflections however improves on the sound field reproduction performance of the corner array configuration, by up to two orders of magnitude. The corner array compares favorably with the circular array. Both configurations perform with an error in the range 10^{-2} to 10^{-3} , except in the cases of sound propagating from either the north or east walls. Re-creating a phantom sound propagating from the north wall ($\phi_s = 90^\circ$) is the most difficult, as the loudspeaker image-sources are furthest away from this phantom source direction.

Marked on Figure 3 also are angles of the direct source, first order images and second order images. The MSE in the direction of the first order image at 67° is good; it almost matches the performance of placing the phantom source in-line with a directional loudspeaker at 30° . The loudspeaker array here is clearly exploiting the reverberant reflection to improve MSE. The first order image of the bottom-right directional loudspeaker beyond the bottom wall produces the most impact here, pulling down the MSE by two orders of magnitudes below the anechoic case at 67° .

Second order images also contribute to improving MSE performance. In Figure 3, the MSE is lower around the second

order images of the four wall cases than for the single wall and anechoic case. First order reflections are the easiest to exploit. Higher order images however, being further away from the control region, produce reflections that are diminished in amplitude. These reflections would be more difficult to exploit robustly than first order reflections, and neither is their impact on the MSE performance as dramatic.

The level of performance is dependent upon the strength of reverberant reflections. Reducing the strength of reverberant reflections decreases performance. The pink dotted curve in Figure 3, where average reflection coefficient is reduced from 0.9 to 0.5, shows a performance that is slightly degraded.

There appears to be an optimal choice of wall reflection coefficient. If wall reflection coefficients are too weak, then exciting a wall reflection becomes difficult. However if they are too strong, then exciting a first order reflection is not possible without also exciting much higher order reflections. Higher order reflections are more susceptible to perturbation.

Figure 4 shows how the level of the performance varies with direct-to-reverberant energy ratio as wall reflection coefficient varies from 0.1 to 0.9. These plots corroborate the hypothesis that there is an optimal reverberation level. Here we introduced -20 dB of noise into the ATF matrix \mathbf{H} to emulate imperfect ATF measurement. Both the circular array and the corner array perform very similarly at -6 dB reverberation. The raised curves for the circular array in Figure 4(a) at 0° and 90° are remnants of the degeneracy of the symmetrical room geometry.

Beampatterns

The directional loudspeaker array performance is best when the phantom source is in-line with either a loudspeaker or a low order reflection. Phantom sources are placed in directions of D and R illustrated in Figure 5 in room 3. The beampatterns for the four steerable loudspeakers are shown at the four corners of the same figure.

For both cases, the beampatterns exhibits a non-trivial structure but possess the properties: (i) a large main lobe in the phantom source direction for the loudspeaker whose image is in-line with the phantom source, and (ii) several other lobes used to cancel the reverberation created from other reflections. The main lobe may be obscured by the reverberation-cancelling lobes if the

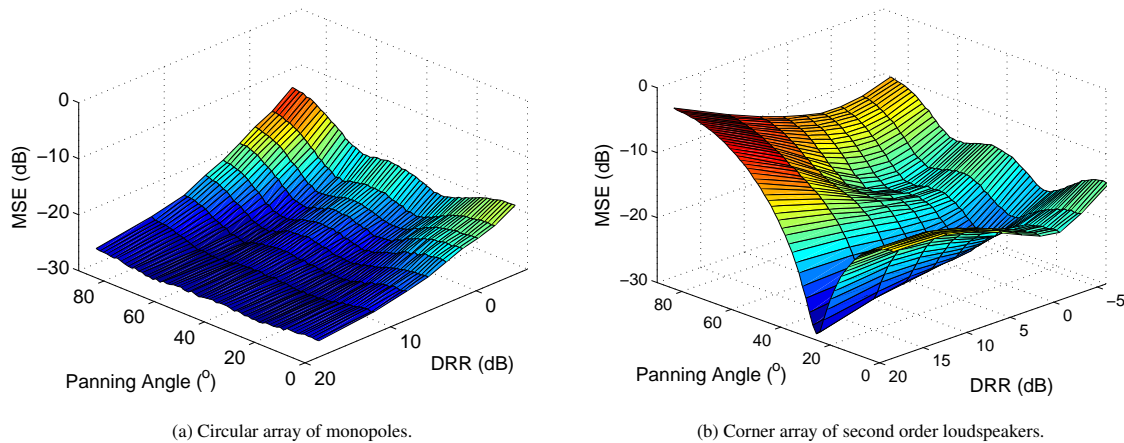


Figure 4: Mean square error (MSE) performance of (a) a 32 element circular array and (b) the four element corner array of directional loudspeakers in reproducing a phantom source at 500Hz. MSE is plotted against both phantom panning angle and direct-to-reverberant ratio (DRR). -20dB of white Gaussian noise has been added to each element of the matrix of acoustic transfer functions.

reproduction is not sufficiently regularized. Here we used a larger regularization parameter $\lambda = 0.5$ to ensure the main lobe is visible.

CONCLUSIONS AND FUTURE WORK

An approach to surround sound for exact sound field reproduction in a reverberant room was proposed utilizing steerable loudspeakers with configurable directional responses. An array of four configurable steerable loudspeakers with roughly second order directivity was shown to possess a reproduction performance comparable with a much larger circular array of loudspeakers, by exploiting the wall reflections in a reverberant room. The level of performance was seen to be dependent on the strength of specular reflections. For robust performance the room was seen to require strong wall reflections.

The notion of robustness of a surround sound system was investigated. As the reverberation is in general more difficult to control than direct sound, it is important that the robustness of the approach be explored. The pressure matching method, for example, in practise relies upon measurement of the acoustic transfer functions from each loudspeaker to a number of points. The approach must be robust to error in these measurements and can be made robust through regularization.

This preliminary study of performance opens up a number of research questions. The performance of a number of other geometries beyond the corner array in this paper were also assessed, including a diamond and pentagon. Although some geometries perform better than others, the corner array proposed here demonstrates the key features of using the steerable directional loudspeakers to exploit reverberation. The practical aspects of the design of a low order configurable directional loudspeakers is an open question. It may be more robust to use a configurable directional loudspeaker that is truly phase mode-limited, and it would be interesting to study how the performance depends upon the truncation order. We shall pursue these questions in future work.

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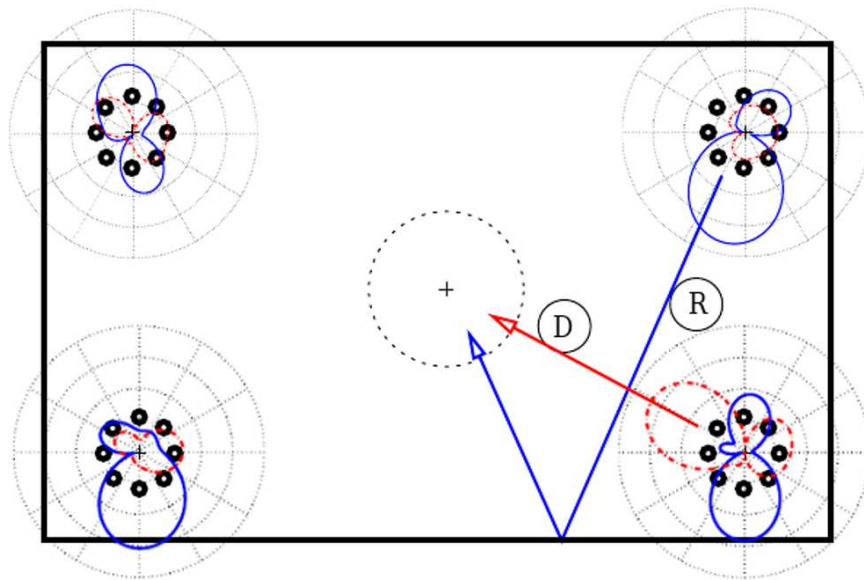


Figure 5: Beampatterns required of all four corner loudspeakers to place a phantom source in-line with direct ray D at $\theta_s(D) = -30.5^\circ$ (red) and in line with reflected ray R of the top-right loudspeaker $\theta_s(R) = -74.2^\circ$ (blue) at a radius of 2 m.