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Sound radiation of orthotropic curved sandwich structures using a mixed boundary element/finite element approach

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ABSTRACT

Sandwich structures such as composite skins-Honeycomb core (NIDA) and Metal skins–Polymer core (MPM) panels are increasingly used in the aeronautics and automobile industries, respectively. It has been shown that this class of structures enables manufacturers to cut weight and cost while providing vibration and harshness performance. These materials induce however increased sound radiation which unfortunately leads, in some instances, to higher interior noise levels. In consequence, there is a need for accurate and reliable low cost numerical tools to efficiently estimate and optimize the vibroacoustic behaviours of such structures. This paper deals with the prediction of the vibroacoustic behaviour of curved orthotropic sandwich panels. A sandwich finite element is first presented and its ability to predict accurately and efficiently the structural response for such structures demonstrated by comparison with classical 3D solid modeling. Next, the element is used within a mixed boundary element/finite element approach (BEM/FEM) to illustrate the effect of curvature and orthotropy on the airborne sound transmission performance of these panels. Examples are presented which consist of both sandwich honeycombs and MPM panels.

INTRODUCTION

The reduction of noise and vibrations of sandwich panels is a major requirement for performance, sound quality and customer satisfaction in the automotive industry, construction of aircraft, spacecraft, and ships because of their high stiffness-to-weight ratios. Sandwich structures such as composite skins-Honeycomb core (NIDA) and Metal skins–Polymer core (MPM) panels and cylinders are classically used to improve the vibro-acoustic behaviour for structure airborne-borne excitations. These materials are widely used to reduce noise and vibration and to improve interior sound quality. It has been shown that this class of materials enables manufacturers to cut weight and cost while providing noise, vibration and harshness performance [1, 2]. Detailed states of the art have been presented by Nashif et al. [3], Sun and Lu [4], Beranek [5], Allen [6] and Vincent et al. [7]. Although initially confined to the aerospace field, sandwich structures are now applied in almost all industrial fields. This motivated the development of prediction methods for their vibration and acoustic performance.

Vibro-acoustic studies of these structures have been carried out both using classical analytical formulation [3-9] and numerical approaches [10-14] such as finite element analysis based on first- and higher-order shear deformation theories. Analytical solutions are only appropriate for simple structures such as beams or plates with simple boundary conditions. In practice it is often necessary to design damped structures with complicated geometry, complex loadings and non-uniform features such as material discontinuities. Consequently, it is natural to consider the finite element method (FEM) to represent correctly the physics of such problems. However, classical finite elements based methods rely on the use of full solids and /or plate solid- plate models which are both computationally expensive [13,14]. Many authors criticize theses approaches as being too complex and costly to use. Alternative models based upon sandwich beam/plate theory [15-20] have been developed consequently. The authors cited above consider only the structure-borne excitation; however it is often necessary to consider the acoustic excitation to study sound transmission through such panels and its control that are of interest in several practical problems. Particularly, acoustic engineers need analysis methods and tools allowing accurate predictions and optimization of dynamic and acoustic responses of sandwich panels.

Several authors have investigated analytical methods to compute the sound transmission loss of sandwich structures [21-25]. However, they limited their studies to the case of infinite plate and a plane wave excitation. Recently, Ghinet et al. extended their work [25] to predict the diffuse field sound Transmission Loss (TL) of infinite sandwich composite and laminate composite cylinders [26]. Tang et al. [27,28] studied the sound TL through one and two infinite concentric cylindrical sandwich shells with a honeycomb core subjected to an incident oblique plane wave. They derived a closed-form expression of TL based on a modal analysis. The later are well suitable for the high frequency domain of large structures. However, for the low-frequency domain, where modal behaviour is predominant, prediction methods are still limited to rectangular simply supported plates and far-field acoustic assumption. Assaf et al. [17] uses a numerical model based on a finite element formulation for a sandwich structure coupled to a variational boundary element method to account for fluid loading. Zhou et al. [29] studied the sound transmission characteristics of an aluminum panel and two composite sandwich panels by using two boundary element analyses. Both boundary element analyses were used to study the sound transmission loss of orthotropic symmetric sandwich panels excited by a random incidence acoustic field. Both authors neglected the curvature effect. This study deals with the prediction of the vibroacoustic behaviour of curved orthotropic sandwich panels. A sandwich finite element is first presented [30-32] and its ability to predict accurately and efficiently the structural response for such structures is demonstrated by comparison with classical 3D solid modeling. The finite element model is based on a discrete displacement approach and account for the unsymmetrical and the curvature effect. The rotational influence of the transverse shearing in the core on the skins behaviors, ensure a displacements consistency over the interfaces between the viscoelastic core and the elastic skins; thus resulting in an accurate representations of the physics. Next, the element is used within a mixed boundary element/finite element approach (BEM/FEM) [33] to illustrate the effect of curvature and orthotropy on the airborne sound transmission performance of these panels. Numerical examples dealing with both sandwich honeycombs and MPM panels are presented.

FINTE ELEMENT FORMULATION

Figure 1 describes the geometry of the developed element [30]. The displacement field of the skins is built using the Love-Kirchhoff's assumptions but is corrected to account for the rotational influence of the transversal shearing in the core. The Mindlin model is used to describe the displacement field of the core. The rotation effects of the transversal shearing in the core as well as the bending of the panel are described by the rotations γ_x and γ_y angles and the transversal displacement W.

The displacements fields of each of the three layers are written as follows (Figure 2):

$$\begin{cases} U_1(x,y,z,t) = U_{20}(x,y,t) - z\dfrac{\partial W(x,y,z,t)}{\partial x} + z_2\psi_x(x,y) \\ V_1(x,y,z,t) = V_{20}(x,y,t) - z\dfrac{\partial W(x,y,z,t)}{\partial y} + z_2\psi_y(x,y) \\ W_1(x,y,z,t) = W(x,y,t) \end{cases}$$

$$\begin{cases} U_2(x,y,z,t) = U_{20}(x,y,t) - z\dfrac{\partial W(x,y,z,t)}{\partial x} + z\psi_x(x,y) \\ V_2(x,y,z,t) = V_{20}(x,y,t) - z\dfrac{\partial W(x,y,z,t)}{\partial y} + z\psi_y(x,y) \\ W_2(x,y,z,t) = W(x,y,t) \end{cases}$$

$$\begin{cases} U_3(x,y,z,t) = U_{20}(x,y,t) - z\dfrac{\partial W(x,y,z,t)}{\partial x} + z_3\psi_x(x,y) \\ V_3(x,y,z,t) = V_{20}(x,y,t) - z\dfrac{\partial W(x,y,z,t)}{\partial y} + z_3\psi_y(x,y) \\ W_3(x,y,z,t) = W(x,y,t) \end{cases}$$

(1)

Where the following notations are used: $\psi_x = \dfrac{\partial W}{\partial x} + \gamma_x$ and $\psi_y = \dfrac{\partial W}{\partial y} + \gamma_y$.

z_2 and z_3 and are the core's top and bottom layers z-coordinates calculated from the reference axis as shown in Figure 1 ($Z_2 = -Z_3 = h_2/2$ in the case of symmetrical sandwich).

W: transverse displacement,

$W_{,x}$ and $W_{,y}$: two rotations of the face sheets related to the transversal shearing in the core,

U_{20} and V_{20} : in-plane displacements of the middle planes of the face sheets.

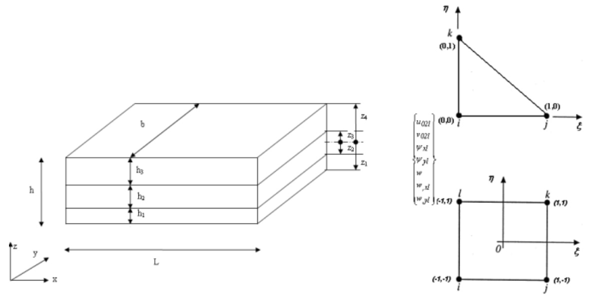

Figure 1 : Geometry of the sandwich plate

Using expressions (1), the linear displacements-strains relations of each layer are written as follows:

$$\varepsilon^{(i)} = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}_i ; \ \{\gamma\} = \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \quad \text{(i=1,2,3)} \qquad (2)$$

$$\begin{aligned} \varepsilon^{(1)} &= \varepsilon_m + z_2\psi - z\chi \\ \varepsilon^{(2)} &= \varepsilon_m + z\psi - z\chi \\ \varepsilon^{(3)} &= \varepsilon_m + z_3\psi - z\chi \end{aligned} \qquad (3)$$

With:

ε_m : Strain due to the membrane effect.

ψ : Strain due to the rotational effect.

χ : Strain due to the bending effect.

γ : Strain due to the shearing effect in the core.

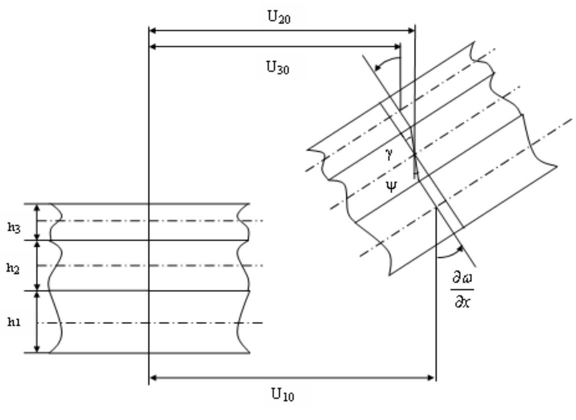

Figure 2 : Displacement field of the sandwich plate

Within the framework of linear elasticity, the internal strain energy of the plate reads:

$$U = \frac{1}{2}\int_v \langle\sigma\rangle\{\varepsilon\}\, dv \qquad (4)$$

$$U = \frac{1}{2}\sum_i \int_V \left(\varepsilon^{(i)T} C^{(i)} \varepsilon^{(i)} + \gamma^T C^{(c)} \gamma\right) dV \quad \text{(i=1:3)} \qquad (5)$$

Where C(i) is the behaviour matrix of the elastic material of i-th layer. For orthotropic material with respect to x, y and z axes, the matrices are given by:

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{yz}\end{bmatrix}=\begin{bmatrix}\overline{C}_{11} & \overline{C}_{12} & \overline{C}_{16}\\ \overline{C}_{12} & \overline{C}_{22} & \overline{C}_{26}\\ \overline{C}_{16} & \overline{C}_{26} & \overline{C}_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{yz}\end{bmatrix} \qquad (6)$$

$$\begin{bmatrix}\sigma_{xy}\\ \sigma_{xz}\end{bmatrix}=\begin{bmatrix}\overline{C}_{44} & \overline{C}_{45}\\ \overline{C}_{45} & \overline{C}_{55}\end{bmatrix}\begin{bmatrix}\varepsilon_{xy}\\ \varepsilon_{xz}\end{bmatrix} \qquad (7)$$

They are related to the the constitutive matrix [Cij] at the lamina level (defined, with respect to the principal axis making an angle θ THEta with the x-y axis), by:

$$\left[\overline{C}_{ij}\right]=\left[T_1\right]^{-1}\left[C_{ij}\right]\left[T_1\right]^{-T} \quad (\mathrm{i,j}=1,2,6) \qquad (8)$$

$$\left[\overline{C}_{ij}\right]=\left[T_2\right]^{-1}\left[C_{ij}\right]\left[T_2\right]^{-T} \quad (\mathrm{i,j}=4,5) \qquad (9)$$

[T1] and [T2] are transformation matrices.

$$[T_1]=\begin{bmatrix}\cos^2\theta & \sin^2\theta & 2\cos\theta\sin\theta\\ \sin^2\theta & \cos^2\theta & -2\cos\theta\sin\theta\\ -\cos\theta\sin\theta & \cos\theta\sin\theta & \cos^2\theta-\sin^2\theta\end{bmatrix} \qquad (10)$$

$$[T_2]=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix} \qquad (11)$$

$$\left[C_{ij}\right]=\begin{bmatrix}C_{11} & C_{12} & \\ C_{12} & C_{22} & \\ & & C_{66}\end{bmatrix} \quad (\mathrm{i,j}=1,2,6) \quad ; \qquad (12)$$

$$\left[C_{ij}\right]=\begin{bmatrix}C_{44} & \\ & C_{55}\end{bmatrix} \quad (i,\, j=4,5)$$

With: $C_{11}=\dfrac{E_x}{1-\nu_{xy}\nu_{yx}}$;

$$C_{22}=\frac{E_y}{1-\nu_{xy}\nu_{yx}} \;;$$

$$C_{12}=\frac{\nu_{xy}E_x}{1-\nu_{xy}\nu_{yx}}=\frac{\nu_{yx}E_y}{1-\nu_{xy}\nu_{yx}}$$

$$C_{44}=G_{xy}\ , C_{55}=G_{xz}\ , C_{66}=G_{yz}$$

$$\nu_{ji}=\frac{E_j}{E_i}\nu_{ij} \quad (\text{pour } i, j = 1, 2, 3)$$

θ is the lamination angle, E_x and E_y are the Young's Moduli in the two directions;
G_{xy}, G_{xz} and G_{yz} are the shear moduli;
ν_{xy} and ν_{yx} are the Poisson ratios.
For details see Ref. [7].

Using the displacement fields described above, rectangular and triangular elements for unsymmetrical three layer sandwich plate are developed and described in Figure 1 [32]. Each node of both elements contains seven degrees of freedom (dofs). These are the transverse displacement W, the in-plane displacements U, V and four rotations ψ_x, ψ_y, θ_x, θ_y.

The generalized displacements field of the four node element, $u=\left[U_{02} \quad V_{02} \quad \psi_x \quad \psi_y \quad W\right]^T$, are discretized with Lagrange bi-linear shape functions for the in-plane displacement and rotations, U_{02}, V_{02}, ψ_x and ψ_y and Hermite cubic shape functions for the transverse displacement W. This lead to a 28 degrees of freedom vector $q_e=\left[q_e^i \quad q_e^j \quad q_e^k \quad q_e^l\right]^T$ where $q_e^n=\left[u_{02n} \quad v_{02n} \quad \psi_{xn} \quad \psi_{yn} \quad w \quad w_{,xn} \quad w_{,yn}\right]^T$; $n=i,j,k,l$.

The formulation of the bending terms for the 3-noded triangular element is based on the same concept as the Discrete Kirchhooff triangular element (DKT) developed by Batoz and al. [34]. The DKT element is a thin plate element based on Kirchhoff assumptions introduced in discrete form. To obtain the strain energy due to the membrane and shear effects, Lagrange bi-linear functions are used to interpolate the displacement field $\left\lfloor U_{02} \quad V_{02} \quad \psi_x \quad \psi_y \right\rfloor$. On the other hand, the transverse displacement W is interpolated using the shape functions for the nine dofs triangle discussed in Zienkiewicz [35]. This leads to a 21 degree of freedom element $q_e=\left[q_e^i \quad q_e^j \quad q_e^k\right]^T$ vector where $q_e^n=\left[u_{02n} \quad v_{02n} \quad \psi_{xn} \quad \psi_{yn} \quad w \quad w_{,xn} \quad w_{,yn}\right]^T$; $n=i,j,k$ as shown in Figure 1.

To take into account the curvature effect, a sandwich plate is approximated as a faceted surface, formed by connecting together flat rectangular or triangular sandwich elements at vertex nodes. This element uses 7 degrees of freedom of the flat plate element plus 2 degrees of freedom in rotation $(\psi_z, \frac{\partial w}{\partial z})$ around the normal with the plane of the plate. Each element thus uses three translational (U, V, W) and six rotational $(\psi_x, \psi_y, \psi_z, \frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}, \frac{\partial w}{\partial z})$ d.o.f. per node. The rectangular and triangular elements have 36 and 27 d.o.f. respectively.

BOUNDARY ELEMENT FORMULATION

The geometry of interest is depicted in Figure 3. The curved sandwich structure is fixed in a rigid baffle separating two semi-infinite fluids. The formulation negelecting Fluid-structure coupling is given first. The structure is in consequence supposed to vibrate with a known displacement field us(y) and radiate acoustically in an infinite fluid (V) with density ρ_f and sound speed c_f. The radiated field can be expressed as the solution of the direct Helmholtz Integral Equation.

$$C^+(x)P(\underline{x})=\int_{S^*}\left[G(\underline{x},\underline{y})\frac{\partial P(\underline{y})}{\partial n_y}-P(\underline{y})\frac{\partial G(\underline{x},\underline{y})}{\partial n_y}\right]dS_y \qquad (15)$$

Where S is the coupling surface between the structure and the fluid, G*(x,y)* is the free-field Green's function, which can be expressed as:

$$G(\underline{x},\underline{y})=e^{-jkr}\big/4\pi r \qquad (13)$$

k is the wavenumber in the fluid (ω/c_f)
For a baffled structure:

$$G(\underline{x},\underline{y})=e^{-jkr}\big/4\pi r+e^{-jkr'}\big/4\pi r' \qquad (14)$$

and its normal gradient:

$$\frac{\partial G(\underline{x},\underline{y})}{\partial n_y}=\frac{1}{4\pi}\left[\left(-\frac{1}{r^2}-\frac{ik}{r}\right)e^{-ikr}\frac{\partial r}{\partial n_y}+\left(-\frac{1}{r'^2}-\frac{ik}{r'}\right)e^{-ikr'}\frac{\partial r'}{\partial n_y}\right] \tag{15}$$

Where r denotes the distance from x to y and r' is the distance from x to y' (the image of y with respect to baffle plane S_H) as shown in figure 3.

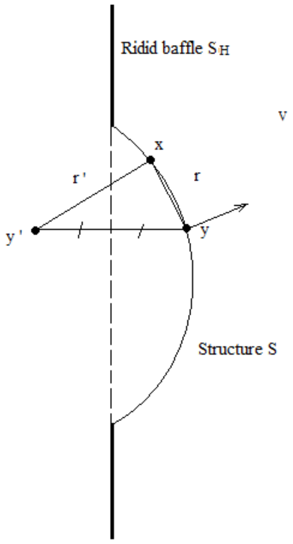

Figure 3. Half space limited by an infinite rigid plane S_H and boundary S

C^+ is the Green's constant given by [33]:

$$C^+(x)=\begin{cases}1+\int_{S^*}\frac{\partial G_0(\underline{x},\underline{y})}{\partial n_y}dS_y & \forall \underline{x}\in S\\ 1 & \forall \underline{x}\notin S \text{ et exterior}\end{cases} \tag{16}$$

$\int_{S^*}(.)dS_y$ represent Cauchy's principal value of the integral.

For a regular surface: $C^+(x)=1/2$.

To compute the surface pressure, the normal derivative of eq. (15) should be carried out.

To eliminate the resulting Hadamard's finite part, a variational form is preferred. It is given by [36]:

$$\frac{1}{2}\int_S\left(\underline{u}(\underline{x}).\underline{n}_x\right)\delta p(\underline{x})dS_x=\frac{1}{\rho_0\omega^2}\int_S\int_S P(\underline{y})\frac{\partial^2 G(\underline{x},\underline{y})}{\partial n_x\partial n_y}\delta p(\underline{x})dS_x dS_y-$$
$$\int_S\int_{S^*}\left(\underline{u}(\underline{y}).\underline{n}_y\right)\frac{\partial G(\underline{x},\underline{y})}{\partial n_x}\delta p(\underline{x})dS_x dS_y \tag{17}$$

With:

$$\int_S\int_S P(\underline{y})\frac{\partial^2 G(\underline{x},\underline{y})}{\partial n_x\partial n_y}\delta p(\underline{x})dS_x dS_y=$$
$$\int_S\int_S k^2\underline{n}_x\underline{n}_y P(\underline{y})G(x,y)\delta p(\underline{x})dS_x dS_y-$$
$$\int_S\int_S\left[\underline{n}_x\Lambda\nabla_x\delta p(\underline{x})\right]\left[\underline{n}_y\Lambda\nabla_y\delta p(\underline{y})\right]G(\underline{x},\underline{y})dS_x dS_y \tag{18}$$

FLUID-STRUCTURE COUPLING

If one is interested in solving the coupled problem of the sandwich structure radiating in an infinite fluid, an additional variational form of the equation for the structure has to be written down [33]:

$$\int_{\Omega_S}\underline{\underline{\sigma}}(\underline{u}):\underline{\underline{\varepsilon}}(\delta\underline{u})d\Omega-\int_{\Omega_S}\rho_s\omega^2\underline{u}.\delta\underline{u}d\Omega+\frac{1}{2}\int_S P(\underline{x})(\underline{n}_x\delta\underline{u}(\underline{x}))dS_x+$$
$$\int_S\int_{S^*}P(\underline{y}).\frac{\partial G(x,y)}{\partial n_x}(\delta\underline{u}(\underline{x}).\underline{n}_x)dS_x dS_y-$$
$$\rho_0\omega^2\int_S\int_{S^*}(u(\underline{y}).\underline{n}_y).\frac{\partial G(\underline{x},\underline{y})}{\partial n_x}(\delta\underline{u}(\underline{x}).\underline{n}_x)dS_x dS_y$$
$$=\int_{\partial\Omega_S/S}\delta\underline{u}(\underline{x})\underline{F}(\underline{x})dS_x \tag{19}$$

Then, the governing vibroacoustic algebraic system of the coupled structural-acoustic problem is then given by:

$$\begin{bmatrix}K-\omega^2\left(M+\rho_0 M_1(k)\right) & C_1+C_2(k)\\ C_1^t+C_2^t(k) & -\frac{1}{\rho_0\omega^2}D(k)\end{bmatrix}\begin{Bmatrix}u\\ p\end{Bmatrix}=\begin{Bmatrix}F\\ 0\end{Bmatrix} \tag{20}$$

With:

K: Stiffness matrix of the structure

M: Mass matrix of the structure

$$\begin{aligned}&\frac{1}{2}\int_S p(\underline{x})\left(\delta\underline{u}(\underline{x}).\underline{n}_x\right)dS_x && \Rightarrow \delta u^T C_1(k)p\\ &\int_S\int_{S^*}P(\underline{y})\frac{\partial G(\underline{x},\underline{y})}{\partial n_y}\left(\delta\underline{u}(\underline{x}).\underline{n}_x\right)dS_y dS_x && \Rightarrow \delta u^T C_2(k)p\\ &\int_S\int_S\left(u(\underline{y}).\underline{n}_y\right)G(\underline{x},\underline{y})\left(\delta\underline{u}(\underline{x}).\underline{n}_x\right)dS_y dS_x && \Rightarrow \delta u^T M_1(k)u\\ &\int_S\int_S P(\underline{y})\frac{\partial^2 G(\underline{x},\underline{y})}{\partial n_x\partial n_y}\delta p(\underline{x})dS_x dS_y && \Rightarrow \delta p^T D(k)p\end{aligned} \tag{21}$$

By condensing the pressure variable *P*, one obtains:

$$\left[-\omega^2 M+j\omega Z(k)+K\right]u=F \tag{22}$$

With:

$$Z(k)=j\omega\rho_0(C_1+C_2(k))D^{-1}(k)(C_1+C_2(k))^T+j\omega\rho_0 M_1(k) \tag{23}$$

The solution of (20) leads to the nodal displacements field in the sandwich plate and to the radiated parietal acoustic pressure:

$$p=-\rho_0\omega^2 D^{-1}(k)(C_1+C_2(k))^T u \tag{24}$$

The radiated power is given by:

$$\Pi_r=\frac{1}{2}R\left(\int_S p(j\omega\underline{u}.\underline{n})^* dS\right)=\omega.\mathrm{Im}\left(u^{*T}C_1 P\right) \tag{25}.$$

NUMERICAL VALIDATIONS

Free vibration analysis of a sandwich plate with honeycomb core

This example deals with a three layer rectangular simply supported sandwich plate with honeycomb core. The plate has elastic isotropic face sheets and a thick soft core made from orthotropic polymer material. This sandwich plate has been analysed by a number of authors using various methods

[37-41]. Its dimensions and material properties are summarized in Table1.

Table 1: Plate configurations and materials properties used for the numerical validation.

Lx=1.83mm; Ly=1.22mm; h1=0.406 mm; h2=6.4mm; h3=0.406 mm;			
E1 (Pa) = E2 = 68.98 x 109	v1 = 0.3	ρ1(kg/m3) = 2768	Face layer
G13 (Pa) = 0.134 x 109	G23 (Pa) = 0.052 x 109	ρ2(kg/m3) = 122	Core layer

The eigen frequencies of the structure calculated using the present model are presented in Table 2 for the first nine modes. They are compared to another numerical model proposed by Reddy and based on higher-order shear deformation plate theory (HSDT) [38], analytical results [37] and experimental data [37]. The plate was divided into 6 by 6 elements for both numerical models.Taking the analytical results as the reference, it can be seen that the presented finite element is more accurate than HSDT model for the same number of elements.

Table 2. Plate's configurations and the material's properties used for the numerical validation.

Modes	Analytical	Test	HSDT (6x6 elements)	FES (6x6 elements)
1	23	-	23	23
2	45	45	46	44
3	71	69	74	69
4	80	78	86	78
5	91	92	96	86
6	126	129	136	116
7	129	133	149	126
8	146	152	164	142
9	165	169	186	155
10	174	177	197	160

Dynamic analysis of honeycomb sandwich panel

A square honeycomb sandwich panel of area 1.365 m^2 is now considered. A 19.05 mm thick Nomex core is sandwiched between two aluminum face sheets, similar to honeycomb panels found in aircraft applications. Selected material properties of the aluminum face sheets and the Nomex core are listed in Table 1.

Table 1. Material properties for the aluminum honeycomb face sheet and the Nomex core.

Aluminum face sheet		Nomex honeycomb core	
h (mm)	0.508	h (mm)	19.05
E (N/m2)	7.1e10	Gxy (N/m2)	4.482e7
		Gxy (N/m2)	2.344e7
ρ (kg/m3)	2700	ρ (kg/m3)	48.06

For the numerical modeling, two approaches are compared. The first is usual and uses a plate-solid-plate model with offsets (MSC/NASTRAN is used with Cquad4 elements for the skins and hex-8 solid elements for the core). The second used the proposed Finite Element Sandwich (referred to as FES in the figures).
The present model uses a mesh of 10x10 of quad4 elements. Nastran's model uses 50 x 50 CQUAD4 elements for each face sheet and 50 x 50 HEXA8 elements for the core.

Figure 4 shows the comparison of the honeycomb panel mean square normal velocity calculated using both approaches. Excellent agreement is seen between MSC/Nastran and developed sandwich element. The resonance frequencies, the resonance amplitudes and width (damping) of the first four modes are accurately estimated.

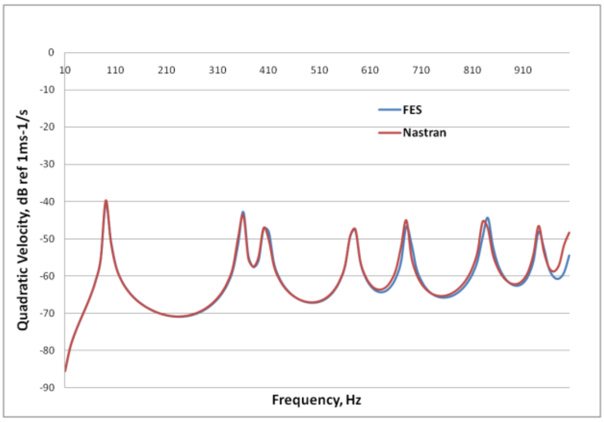

Figure 4: Quadratic velocity of honeycomb panel: FES vs Nastran

Sound transmission loss computation

The sound transmission loss of orthotropic honeycomb sandwich plate is investigated in this section using boundary element method described above. It is evaluated using the following formula:

$$TL = 10Log\frac{\Pi_i}{\Pi_t} \quad (26)$$

where Π_i and Π_t are the incident and the transmitted acoustic power respectively. For an incident progressive plane wave with angles of incidence (θ,ϕ) (see figure 5) ,

$$P_i = |P_i|\exp(i\omega t - ik(x\sin\theta\cos\phi + y\sin\theta\sin\phi + z\cos\theta)) \quad (27)$$

and the incident sound power is given by:

$$\Pi_i = \frac{1}{2}\mathrm{Re}\int_S P_i v_{i,n}^* dS \quad (28)$$

in which P_i and $v_{i,n}$ represent the incident sound pressure and the normal particle velocity respectively (*denotes the complex conjugate).

The transmitted sound power is given by:

$$\Pi_t = \frac{1}{2}\mathrm{Re}\int_S P_i v_n^* dS = \frac{\omega}{2}\mathrm{Im}\int_S P w_n^* dS \quad (29)$$

Where P is the sound pressure applied as an external loading to the finite element model, given by (24).
For a flat plate embedded in an infinite rigid plane baffle and radiating in a semi infinite fluid, P is given by (15).

$$P = \rho\omega^2 \int_S w_n G_b(x,y) dS \quad (30)$$

Where $G_b = e^{-ikr}/2\pi r$ is the half-space free field Green's function satisfying $\partial G_b / \partial n = 0$ on the baffle. For this case, the impedance matrix given by (23) reduces to:

$$Z(k) = 2j\omega\rho_0 M_1(k) \quad (31)$$

The governing vibroacoustic algebraic system of the coupled structural-acoustic problem is given by (22), where F is the source vector resulting from the acoustic excitation:

$$F = \int_s P_b N^T dS \qquad (32)$$

P_b is the blocked pressure which is the total pressure on the incident side when the plate is considered as acoustically rigid (on the plate $P_b=2P_i$). N is the vector of the shape function associated with the transverse displacement W.

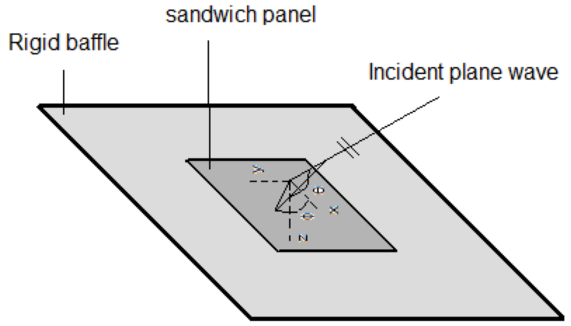

Figure 5: baffled sandwich plate excited by plane wave

Using the above development, the predicted sound TL of the honeycomb panel considered previously (see table 1) for diffuse sound field excitation is given in Fig. 6 and compared to experimental and existing numerical results [42], Reasonable agreement is found between the three results.

An illustration of the effect of curvature and orthotropy on the airborne sound transmission performance of the studied panels will be presented during the oral presentation.

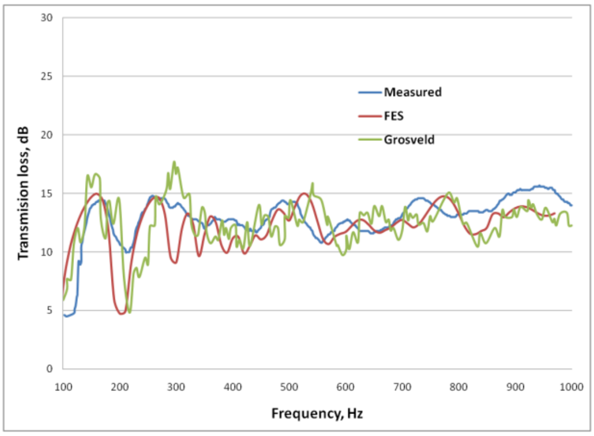

Figure 6: Transmission Loss of the honeycomb sandwich panel

Conclusion

A sandwich finite element was presented and its ability to predict accurately and efficiently the structural response of curved orthotropic sandwich panels, such as composite skins-Honeycomb core (NIDA). The element is coupled to a boundary element method to account for fluid loading. Comparisons of the presented element versus both tests and classical 3D solid modeling prove its accuracy for the modeling of the vibro-acoustic behaviour of the studied laminated steels. The element can be used to illustrate the effect of curvature and orthotropy on the airborne sound transmission performance of these panels.

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