Proceedings of the 20th International Congress on Acoustics, ICA 2010

20-27 August 2010, Sydney, Australia

A fully consistent constrained generalized coordinate formulation of the interaction of an acoustic cavity and surrounding structures

Jerry H. Ginsberg
G. W. Woodruff School of Mechanical Engineering,
Georgia Institute of Technology, Atlanta, Georgia 30332, USA

PACS: 43.20.Tb

ABSTRACT

This paper describes the derivation of a set of time domain equations descriptive of the interaction of an arbitrary acoustic domain and a structure that forms all or a portion of the boundary, with the remainder of the bounding surface allowed to be subregions on which the pressure or normal velocity are specified temporal inputs. The primary objective is development of a general semi-analytical method, but it is equally suitable as the foundation for a unified finite element formulation. The present formulation overcomes difficulties in prior formulations based on Hamilton's principle by considering the pressure at that interface to be a constraining surface traction that enforces velocity continuity conditions. The requirement that the acoustical response be irrotational is addressed by using a Ritz series to describe the velocity potential, whereas the structural displacement is described by a conventional Ritz series. The surface traction also is represented by a Ritz-like series with a set of basis functions that span the regions of the surface where velocity continuity must be enforced explicitly. The various series are used to describe the mechanical energies and virtual work in Hamilton's principle, from which equations governing the generalized coordinates of both media are derived by application of the calculus of variations. The motion equations are augmented by a set of algebraic kinematical constraint, which are obtained by requiring that the error in satisfying interface velocity continuity be orthogonal to the surface basis functions. The assembled set of equations are shown to be symmetric, and therefore consistent with the fundamental principle of reciprocity.

INTRODUCTION

Numerous formulations have been developed to describe the interaction of an acoustic cavity bounded by an elastic structure, possibly accompanied by active and passive regions along which either the pressure or normal velocity is specified. In most cases the dependent variable for the structure is the displacement, and that for the fluid is the pressure perturbation. The field equations for each typically are formulated independently, either in a discretized manner or with global series ansatz. The individual domains are mated by imposing continuity of the normal velocity pressure at interfaces, as well as by loading the structure with the acoustic pressure. An earlier attempt by Gladwell (1966) to use Hamilton's principle to formulate a unified description of the structure and acoustic domains failed to lead to a general approach. The present work discloses that this failure was due to formulating the principle in a manner that implicitly assumed that the continuity conditions would be met.

Moussou (2005) used Hamilton's principle as the basis for an analysis of the cavity in which the structural displacement is considered to be an input. Significant difficulties would be encountered if one were to attempt to implement Moussou's formalism for multidimensional problems. The acoustic field is represented as a superposition of a blocked field corresponding to the structure being rigid, and a compatibility field in which the fluid displacement normal to the fluid-structure interface matches the structure's displacement, with homogeneous boundary conditions elsewhere. Determination of the blocked field should be easier than that of the interacting domains, but it might require considerable effort, depending on the configuration of the system. More significant is the fact that determination of the compatibility field will be quite difficult. (Moussou recommended that it be determined by solving the Laplace equation.) The other difficulty entailed in Moussou's method is that it uses individual Ritz series to represent each component of the fluid's particle velocity. However, the basis functions for each component cannot be selected arbitrarily because it is necessary that the resulting displacement/velocity field be irrotational.

The present formulation is suggested by a version of Hamilton's principle that is used as the foundation for analyses of nonholonomic systems. It describes the virtual work that would be done if continuity of normal velocity at the structure-fluid interface is not satisfied, with continuity imposed as auxiliary conditions. The corollary is that pressure at the interface is treated as a constraining force that imposes this continuity condition, with the further consequence that it contributes to the virtual work, even though it is internal to the system. The constrained version of Hamilton's principle is satisfied by using globally defined Ritz series to represent the structural and fluid domains. The series for the acoustic response begins with a description of the velocity potential, thereby assuring satisfaction of the irrotationality condition is independently of the choice of basis functions.

VARIATIONAL EQUATIONS

The surface bounding the cavity is decomposed into three subdomains: S_e is the fluid-structure interface, the normal velocity is specified on S_v, and the pressure is specified on S_p. For the elastic structure the displacement is $\bar{u}_e$, and the forces doing work are any external agents, as well as the surface traction associated with the pressure applied by the fluid, which is designated $-\tau_e(\bar{x}_s,t)\,\bar{\gamma}(\bar{x}_s)$, where $\bar{x}_s$ designates a point on the bounding surface of the cavity and $\bar{\gamma}(\bar{x}_s)$ is the normal to the surface at that location, pointing into the fluid. For the fluid domain, the displacement is $\bar{u}_f$, and virtual work is done by the traction exerted by the boundary on the fluid. On S_e, this traction is $+\tau_e(\bar{x}_s,t)\,\bar{\gamma}(\bar{x}_s)$, while the traction that imposes the motion on S_v is $+\tau_v(\bar{x}_s,t)\,\bar{\gamma}(\bar{x}_s)$. The pressure specified on S_p is designated p_s, which may be null, corresponding to a pressure-release condition, or it may be related to the surface normal velocity by a time domain local impedance. An important aspect of these definitions is usage of the symbol τ to designate the surface pressures acting on S_e and S_v. Doing so permits these surface pressures to be treated as unknown constraint forces whose role is to enforce continuity of normal velocity. Of course, these quantities must match the acoustic pressure evaluated at the respective surfaces, but such matching is a condition sought in the eventual solution, rather than an *a priori* requirement. A similar perspective is standard in structural mechanics, where a force exerted by a support, such as the transverse force exerted on a beam by a pin connection, is treated like an external force, even though it can be directly related to the internal stress distribution.

In the linearized approximation the bounding surfaces are situated at their original location, so the surface normal $\gamma(\bar{x}_s)$ is assigned its value in the reference state of zero displacement. Hamilton's principle for the system is the sum of the principles associated with the structural and fluid domains. It will be noted in the below statement that if the displacements of the two domains along S_e are implicitly taken to satisfy the continuity condition, then $\bar{\gamma}\cdot\bar{u}_f=\bar{\gamma}\cdot\bar{u}_e$ along this region. The net virtual work done by τ_e in such a perspective, which is the fundamental source of the difficulty Gladwell (1966) encountered. In contrast, continuity will be imposed as an auxiliary condition, here so the surface traction's role is explicit in the principle's statement here,

$$\begin{aligned}\int_{t_0}^{t_2}\Big\{\ &\delta T_e+\delta T_f-\delta V_e-\delta V_f+\delta W+\\ &+\iint_{\mathcal{S}_e}\tau_e(\bar{x}_s,t)\,\bar{\gamma}(\bar{x}_s)\cdot[\delta\bar{u}_f(\bar{x}_s)-\delta\bar{u}_e(\bar{x}_s,t)]\,d\mathcal{S}\\ &+\iint_{\mathcal{S}_v}\tau_v(\bar{x}_s,t)\,\bar{\gamma}(\bar{x}_s)\cdot\delta\bar{u}_f(\bar{x}_s,t)\,d\mathcal{S}\\ &+\iint_{\mathcal{S}_p}p_s(\bar{x}_s,t)\,\bar{\gamma}(\bar{x}_s)\cdot\delta\bar{u}_f(\bar{x}_s,t)\,d\mathcal{S}\Big\}\,dt=0\end{aligned} \tag{1}$$

The structural displacement is represented as a conventional Ritz series using vectorial basis functions,

$$\bar{u}_e=\sum_{j=1}^{J}\bar{\psi}_{e,j}(\bar{x})\,q_j(t) \tag{2}$$

Correspondingly the structure's contribution to the kinetic and potential energy are quadratic sums whose coefficients are the elements of the inertia and stiffness matrices,

$$\begin{aligned}T_e&=\frac{1}{2}\sum_{j=1}^{J}\sum_{m=1}^{J}M_{e,nm}\,\dot{q}_j\dot{q}_m,\quad M_{mj}^{(e)}=M_{jm}^{(e)}\\ V_e&=\frac{1}{2}\sum_{j=1}^{J}\sum_{m=1}^{J}K_{e,nm}\,q_jq_m,\quad K_{mj}^{(e)}=K_{jm}^{(e)}\end{aligned} \tag{3}$$

It is necessary that the fluid particle velocity field be irrotational, which is assured by using a Ritz series to describe the scalar velocity potential,

$$\phi=L\sum_{n=1}^{N}\psi_{f,n}\,\dot{z}_n \tag{4}$$

where L is a length scale appropriate to the system. The representation of particle velocity is derived from the definition of ϕ, after which the displacement is found by a time integration,

$$\begin{aligned}\bar{v}_f&=\nabla\phi=\sum_{n=1}^{N}\hat{\nabla}\psi_{f,n}\,\dot{z}_n\\ \bar{u}_f&=\sum_{n=1}^{N}\hat{\nabla}\psi_{f,n}\,z_n\end{aligned} \tag{5}$$

where $\hat{\nabla}$ is a nondimensional gradient, such that $\nabla=\hat{\nabla}/L$. The variables z_n, whose dimension is length, are generalized coordinates for the fluid. The corresponding representation of the acoustic pressure is

$$p=-\rho c^2\nabla\cdot\bar{u}_f=-\frac{\rho c^2}{L}\sum_{n=1}^{N}\hat{\nabla}^2\psi_{f,n}\,z_n \tag{6}$$

An important aspect is that the pressure relation invoked in Eq. (6) is derived from the linearized equation of continuity, whereas the commonly used relation $p = -\rho\dot{\phi}$ is derived from the linearized momentum series, which already is incorporated into Hamilton's principle. The necessity to use the form in Eq. (6) is manifested by the fact that using $p = -\rho\dot{\phi}$ to form the potential energy density $p^2/2\rho c$ would lead to a description of the potential energy that depends on generalized accelerations $\ddot{z}_n$, in contradiction to the fundamental requirement that potential energy be a function only of position. The energy expressions for the fluid corresponding to the preceding are

$$\begin{aligned} T_f &= \frac{1}{2}\sum_{m=1}^{N}\sum_{n=1}^{N} M_{f,mn}\,\dot{z}_m\dot{z}_n \\ V_f &= \frac{1}{2}\sum_{m=1}^{N}\sum_{n=1}^{N} K_{f,mn}\,z_m z_n \end{aligned} \tag{7}$$

where the fluid inertia and stiffness coefficients are

$$\begin{aligned} M_{f,mn} = M_{f,nm} &= \rho L^2 \iiint\limits_{\mathcal{V}} \nabla\psi_{f,n}\cdot\nabla\psi_{f,m}\ d\mathcal{V} \\ K_{f,mn} = K_{f,nm} &= \rho c^2 L^2 \iiint\limits_{\mathcal{V}} \nabla^2\psi_{f,n}\nabla^2\psi_{f,m}\ d\mathcal{V} \end{aligned} \tag{8}$$

The surface traction on each subregion is described by a Ritz-like series using basis functions defined solely on each surface,

$$\tau_\alpha = \sum_{k=1}^{K_\alpha}\chi_{\alpha,k}\left(\bar{x}_s\right)\tau_{\alpha,k}\left(t\right);\quad \bar{x}_s\in\mathcal{S}_\alpha,\quad \alpha = e \text{ or } v \tag{9}$$

The basis functions $\chi_{\alpha,k}\left(\bar{x}_s\right)$ are a linearly independent set that may be extracted as subsets of the functions used to describe $\bar{u}_e$ and ϕ. For example, a convenient choice for $\chi_{e,k}$ is the normal component of the structural basis functions evaluated on the surface, $\bar{\gamma}\left(\bar{x}_s\right)\cdot\bar{\psi}_e\left(\bar{x}_s\right)$.

The virtual work terms are evaluated by noting that because the basis functions have been selected, a virtual displacement can only be obtained from virtual increments of the generalized coordinates, so that

$$\begin{aligned} \delta\bar{u}_e &= \sum_{j=1}^{J}\bar{\psi}_{e,j}\delta q_j \\ \delta\bar{u}_f &= \sum_{n=1}^{N}\hat{\nabla}\psi_{f,n}\delta z_n \end{aligned} \tag{10}$$

Introduction of these representations and Eqs. (9) into the virtual work terms in Eqs. (1) converts the contributions from S_e and S_v to sums of products of a traction coefficient and a generalized coordinate increment, while the contribution from S_p is that associated with generalized forces, specifically,

$$\begin{aligned} &\iint\limits_{\mathcal{S}_\alpha}\tau_\alpha\left(\bar{x}_s,t\right)\bar{\gamma}\left(\bar{x}_s\right)\cdot\delta\bar{u}_f\left(\bar{x}_s,t\right)d\mathcal{S} \\ &\qquad = \sum_{k=1}^{K_\alpha}\sum_{n=1}^{N} D_{\alpha,kn}\,\tau_{\alpha,k}\,\delta z_n;\quad \alpha = e \text{ or } v \end{aligned} \tag{11}$$

$$\iint\limits_{\mathcal{S}_\alpha}\tau_e\left(\bar{x}_s,t\right)\bar{\gamma}\left(\bar{x}_s\right)\cdot\delta\bar{u}_e\left(\bar{x}_s,t\right)d\mathcal{S} = \sum_{k=1}^{K_e}\sum_{j=1}^{J} E_{kj}\,\tau_{e,k}\,\delta q_j \tag{12}$$

$$\iint\limits_{\mathcal{S}_p} p_s\left(\bar{x}_s,t\right)\bar{\gamma}\left(\bar{x}_s\right)\cdot\delta\bar{u}_f\left(\bar{x}_s,t\right)d\mathcal{S} = \sum_{n=1}^{N} P_n\,\delta z_n \tag{13}$$

where the coefficients are

$$\begin{aligned} D_{\alpha,kn} &= \iint\limits_{\mathcal{S}_\alpha}\chi_{\alpha,k}\,\bar{\gamma}\cdot\hat{\nabla}\psi_{f,n}d\mathcal{S} \\ E_{kj} &= \iint\limits_{\mathcal{S}_e}\chi_{e,k}\,\bar{\gamma}\cdot\bar{\psi}_{e,\ j}d\mathcal{S} \\ P_n &= \iint\limits_{\mathcal{S}_v} p_s\left(\bar{x}_s,t\right)\bar{\gamma}\left(\bar{x}_s,t\right)\cdot\hat{\nabla}\psi_{f,n}\,d\mathcal{S} \end{aligned} \tag{14}$$

All terms appearing in Hamilton's principle for the system have been characterized. Equations of motion are obtained by applying the calculus of variations, which requires that the stationary property be satisfied when arbitrary increments are imparted to the generalized coordinates q_j and z_n. Ultimately, what emerges are two sets of equations of motion. Those for the structure are the coefficients of δq_j in the integrand,

$$\begin{aligned} &\sum_{m=1}^{J}\left(M_{e,jm}\,\ddot{q}_m + K_{e,jm}\,q_m\right) \\ &\qquad + \sum_{k=1}^{K_e} E_{kj}\tau_{e,k} = Q_j;\quad j = 1,...,J \end{aligned} \tag{15}$$

where Q_j are generalized forces associated with external forces applied to the structure,

$$\delta W = \sum_{j=1}^{J} Q_j\delta q_j \tag{16}$$

The equations of motion for the fluid domain are obtained from the coefficients of δz_n,

$$\begin{aligned} &\sum_{n=1}^{N}\left(M_{f,mn}\,\ddot{z}_n + K_{f,mn}\,z_n\right) \\ &\qquad - \sum_{\alpha=e,v}\sum_{k=1}^{K_\alpha} D_{\alpha,km}\tau_{\alpha,k} = P_m;\qquad m = 1,...,N \end{aligned} \tag{17}$$

The combination of Eqs. (15) and (17) is not sufficient in number because the traction coefficients $\tau_{e,k}$ and $\tau_{v,k}$ are unknown.

CONTINUITY CONDITIONS

Continuity of normal stress at the fluid-structure interface has been addressed by applying the traction on S_e to both the structure and the fluid. The other continuity conditions require that the components of $\bar{v}_f$ and $\bar{v}_e$ normal to the fluid-structure interface be equal, and that the same component of $\bar{v}_f$ match the normal velocity imposed on S_v, which is denoted as $v_s(\bar{x}_s, t)$. In an arbitrary situation x_s would be the current location of a point on the surface, and $\bar{\gamma}$ would be the normal to deformed surface. Linearization simplifies this by allowing $\bar{x}_s$ to be set to the original reference location and $\bar{\gamma}$ to be considered independent of time. Furthermore, linearization converts the total time derivative to a partial derivative. The consequence is that the velocity continuity conditions can be integrated in time to obtain displacement conditions,

$$\begin{gathered} \bar{\gamma}\cdot\bar{u}_f - \bar{\gamma}\cdot\bar{u}_e = 0; \quad \bar{x}_s \in \mathcal{S}_e \\ \bar{\gamma}\cdot\bar{u}_f - u_s = 0; \quad \bar{x}_s \in \mathcal{S}_v; \quad \dot{u}_s = v_s \end{gathered} \tag{18}$$

The number of generalized coordinates is finite, so it is not possible to satisfy these continuity equations everywhere. An approximate procedure is invoked. If one were to substitute the Ritz series with a specified set of values for the generalized coordinates, the result would be two error functions, consisting of the left side of each equation. These errors are required to be orthogonal to a set of basis functions. The functions $\chi_{e,k}$ and $\chi_{v,k}$ have been used to describe the spatial distribution of the surface traction, so it is logical that they also be used to orthogonalize the error in the continuity equations, Thus it is required that

$$\begin{aligned} &\iint\limits_{\mathcal{S}_e} \chi_{e,k}\left(\bar{\gamma}\cdot\bar{u}_f - \bar{\gamma}\cdot\bar{u}_e\right) dS = 0; \quad k = 1,...,K_e \\ &\iint\limits_{\mathcal{S}_v} \chi_{v,k}\left(\bar{\gamma}\cdot\bar{u}_f - u_s\right) dS = 0; \quad ; \quad k = 1,...,K_v \end{aligned} \tag{19}$$

Substitution of Eqs. (2) and (5) converts the preceding to a set of linear algebraic equations relating the generalized coordinates.

$$\begin{gathered} \sum_{n=1}^{N} D_{e,kn}\, z_n - \sum_{j=1}^{J} E_{kj} q_j = 0, \quad k = 1,..,K_e \\ \sum_{n=1}^{N} D_{v,kn}\, z_n = U_k(t), \quad k = 1,..,K_v \end{gathered} \tag{20}$$

where U_k are time functions representing the displacement input on $\mathcal{S}_v$,

$$U_k = \iint\limits_{\mathcal{S}_v} u_s\, \chi_{v,k} d\mathcal{S} \tag{21}$$

The number of equations described by Eqs. (15), (17), and (20) matches the number of unknowns contained in q_j, z_n, $\tau_{e,k}$, and $\tau_{v,k}$, so the set is solvable.

SOLUTION METHODS

It is convenient to assemble the governing equations in a stacked matrix form. The unknowns are

$$\{\zeta\}^{\mathrm{T}} = \left[\{q\}^{\mathrm{T}} \quad \{z\}^{\mathrm{T}} \quad \{\tau_e\}^{\mathrm{T}} \quad \{\tau_v\}^{\mathrm{T}}\right] \tag{22}$$

The coupling coefficients form the $K_\alpha \times N$ arrays $[D_\alpha]$, and the $K_e \times J$ array $[E]$. Then the equations of motion and associated constraint equations are

$$[M]\left\{\ddot{\zeta}\right\} + [K]\{\zeta\} = \{F\} \tag{23}$$

where

$$\begin{gathered} [M] = \begin{bmatrix} [M_e] & [0] & [0] & [0] \\ [0] & [M_f] & [0] & [0] \\ [0] & [0] & [0] & [0] \\ [0] & [0] & [0] & [0] \end{bmatrix} \\ [K] = \begin{bmatrix} [K_e] & [0] & [E]^{\mathrm{T}} & [0] \\ [0] & [K_f] & -[D_e]^{\mathrm{T}} & -[D_v]^{\mathrm{T}} \\ [E] & -[D_e] & [0] & [0] \\ [0] & -[D_v] & [0] & [0] \end{bmatrix}\{\zeta\} \\ \{F\} = \left[\{Q\} \quad \{P\}^{\mathrm{T}} \quad \{0\}^{\mathrm{T}} \quad -\{U\}^{\mathrm{T}}\right]^{\mathrm{T}} \end{gathered} \tag{24}$$

It is evident that the coefficient matrices are symmetric, from which it follows that the formulation is consistent with the principle of reciprocity, which Lyamshev (1959) proved from first principles applies for coupled acoustic cavity-structure systems. Secondly, the coupled system inertia $[M]$ is singular. Equations having this characteristic are said to be differential-algebraic (DAE), because some variables (the $\tau_{e,k}$ and $\tau_{v,k}$) only occur algebraically in them. There are a number of ways in which these equations may be solved.

Frequency domain analysis

The differential-algebraic nature of the equations has no significance if one seeks transfer functions representing the system response to a set of harmonic inputs. A caret shall denote the complex amplitude of a quantity, so the harmonic response is described as

$$\{\zeta\} = \mathrm{Re}\left(\left\{\hat{\zeta}\right\}\exp(i\omega t)\right) \tag{25}$$

The algebraic equations governing $\left\{\hat{\zeta}\right\}$ are not singular, except when $\omega = 0$, so they may be solved directly,

$$\left[[K] - \omega^2[M]\right]\left\{\hat{\zeta}\right\} = \left\{\hat{F}\right\} \tag{26}$$

Modal analysis

Eigensolutions are the nontrivial solutions of Eq. (26) when the excitation on the right side is not present. The rank deficient nature $[M]$ has the consequence of several eigenvalues being singular. The rank of $[M]$ is the number generalized coordinates, $J+N$, so it might appear that the number of singular values would be the number of constraint equations $K_e + K_v$, but the reduction is greater than that. The constraint equations represent $K_e + K_v$ linear algebraic equations relating $J + N$ generalized coordinates, so the number of generalized coordinates that can be arbitrarily selected without violating the constraint equations is $J+N-K_e-K_v$. Such a set of generalized coordinates are said to be unconstrained, and their number is the system's number-of-degrees-of-freedom. It follows that only $J + N - K_e - K_v$ eigenvalues extracted from Eq. (26) will be finite. Most numerical algorithms have difficulty handling singular eigenvalues, but this can be circumvented by taking $\lambda = 1/\omega^2$ to be the eigenvalue. This converts the eigenvalue problem to

$$[\lambda [K] - [M]] \{\Phi\} = \{0\} \tag{27}$$

An eigensolution of this equation should yield $2 (K_e + K_v)$ zero roots for λ. Some eigensolvers will fail to identify this number of null eigenvalues, but such a failure is not an issue. The eigenvectors corresponding to $\lambda = 0$ will have the property that their only nonzero elements are those that represent traction coefficients. Thus, these modes may be discarded, while the remaining eigensolutions may be used to form modal equations for system response.

Numerical time domain solutions

In some situations it might be desirable to use numerical methods to solve the system of equations described by Eq. (23). One approach for doing so is to use a DAE algorithm (Brenan, Campbell, and Petzold, 1995), but the equations have the characteristic of a stiff system, which requires greater numerical effort than that required to solve the same number of ordinary differential equations for a conservative mechanical system. An alternative approach is hinted at in the discussion of modal analysis. The method is a simplified version of the elimination method that has been developed to address nonholonomic systems. It entails selecting a subset of $K_e + K_v$ generalized coordinates as a constrained set. The constraint equations, whose number matches the number of constrained variables, are solved for those variables in terms of the remaining $J + N - K_e - K_v$, which constitute the constrained set. The expressions for the constrained set in terms of the unconstrained set are used to obtain energy expressions and virtual work that only feature the unconstrained set. The outcome is a set of equations like that of any other linear time-invariant system, with equivalent inertia and stiffness matrices and an associated set of generalized forces. Solution of these equation yields the response in terms of the unconstrained variables, from which the full set of generalized coordinates and the displacement field may be found from linear transformations. Unfortunately, space limitations prevent further elucidation of the method in the present venue; a detailed description of the elimination method may be found in the text by Ginsberg (2008).

CLOSURE

The present development enhances the variety of tools that one can employ to analyze the response of a structure that forms a portion of the boundary of a cavity containing a compressible inviscid fluid. The only restriction imposed is that the responses of both domains are sufficiently small to justify a linearized analysis. The focus here was on the development of analytical models featuring relatively few generalized coordinates, which generally results when one uses globally defined basis functions to define the Ritz series. However, introduction of local basis functions would allow the formulation to be used as the foundation for finite element methods. The fact that the coefficient matrices in the equations of motion are symmetric confirms that the formulation satisfies the principle of reciprocity, and the property also means that any computational efforts will be relatively efficient.

User discretion is required only in the selection of the basis functions, whose sole mandatory requirement is that they be linearly independent. The present development assumed that these functions did not satisfy any boundary conditions. However, the condition at a rigid surface can be satisfied identically by selecting fluid basis functions whose normal derivative vanish at that surface. Satisfaction of a pressure-release condition is not mandatory in the context of Hamilton's principle. Nevertheless, selection of functions that vanish identically at pressure-release surfaces may be expected to fit the response better, and therefore lead to more rapid convergence of the Ritz series. This freedom to select how boundary conditions are satisfied is an attractive feature, for it increases the options one has. It also can be used in a more general sense. Suppose the cavity's shape is an amalgam of basic shapes that fit the standard curvilinear coordinate systems. Individual Ritz series can be used to describe each subdomain. Correspondingly, Hamilton's principle would be modified by exposing the interfaces between domains, and adding virtual work terms for the traction forces on these exposed surfaces. This would be accompanied by additional constraint equations enforcing velocity continuity along these interfaces.

At this juncture, the development is abstract, in the sense that it has not be applied to a specific system. Substantial testing is required to assess the merits of various types of basis functions in a variety of systems. Furthermore, although the methodology has several attractive features, it is not obvious that this will translate to more efficient or more accurate solutions of specific problems. These issues require further examination.

REFERENCES

Brenan, K. E., Campbell, S. L., and Petzold, L. R. (1989). *Numerical Solutions of Initial-Value Problems in Differential-Algebraic Equations* (Elsevier, New York).

Ginsberg, J. H. (2008). *Engineering Dynamics* (Cambridge University Press, New York).

Gladwell, G. M. L. (1966) "A variational formula tion of damped acousto-structural vibration problems," *J. Sound Vib.* **4**, 172–186.

Lyamshev, M. (1959). "A question in connection with the principle of reciprocity in acoustics," *Sov. Phys. Dokl.* **84,** 405-409.

Moussou, P. (2005). "A kinematic method for the computation of fluid-structure interaction systems," *J. Fluids and Structures* **20**, 643-658.