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Study of elastic modulus for viscoelastic absorption materials

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ABSTRACT

Based on wave propagating theory of multi-layered medium and the optimizing algorithm, the complex elastic modulus of viscoelastic materials are optimized with different physical conditions to improve material absorption performance. Isoclines of absorption coefficient on complex elastic modulus of absorption materials are presented with certain boundary conditions. Assuming the absorption coefficient is larger than 0.8, the scope of elastic modulus and loss factor of the viscoelastic materials with different boundary conditions are given and discussed. The results show that the sound absorption performance can be improved effectively by adjusting complex elastic modulus of viscoelastic materials. The scope of elastic modulus is found to be very sensitive to the boundary conditions while the absorption coefficient is larger than 0.8. The difficulty of adjusting complex elastic modulus can be reduced with certain steel backing, but the absorption performance of viscoelastic materials become worse with water backing.

I. INTRODUCTION

Rubber is a kind of viscoelastic materials, which are applied to the fields of noise control and underwater anechoic coating. Absorption performance of viscoelastic materials is very sensitive to complex elastic modulus. In order to design effective absorption viscoelastic materials, it's necessary to ensure the scope of complex elastic modulus. About each kind of parameter optimized design of the noise elimination level, some researches have been done for a long time. Scharnhorst optimized density and longitudinal wave modulus of the heterogeneous texture using the Lagrangian multiplication law [1], the minimal thickness of viscoelastic materials are given for plane waves normally incident on it. Stefan optimized material parameter of layered homogeneous medium, the optimal sound velocity of every layer material are obtained with different incident angle [2]. The resonance model of a sphere cavity in an infinite elastic solid is described in Gaunaurd's classical paper, in which effective complex elastic modulus are obtained in acoustical structure [3]. The relation between complex elastic modulus and absorption performance is discussed in most of papers, but the scope of complex elastic modulus of absorption materials are not given[1-3]. In fact it is important to improve material absorption performance by adjusting complex elastic modulus of viscoelastic materials.

The complex elastic modulus of viscoelastic materials are optimized based on wave propagating theory of multi-layered medium and the optimizing algorithm in this paper. The scope of complex elastic modulus of viscoelastic materials are presented with different backing in effective absorption frequency. The results indicate that material absorption performance are greatly improved by optimizing complex elastic modulus, but the span of elastic modulus is from one magnitude to three magnitudes in wider frequency. The scope of complex elastic modulus can be increased with certain steel backing. On the contrary, it is very difficult to adjust elastic modulus when the absorption materials are immersed in water.

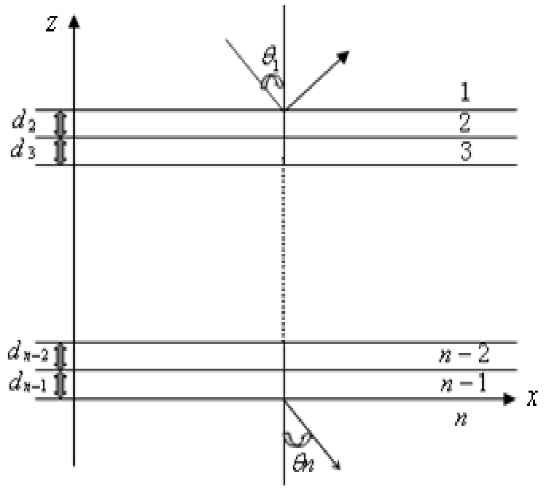
II COMPUTATION OF TRANSITION MATRICES OF MULTI-LAYERED MEDIUM FOR ASORPTION MATERIALS

The sound absorption performance of homogeneous materials can be computed by wave propagation theory and absorption coefficient is given by the transition matrix. Absorption coefficient is found to be very sensitive to complex elastic modulus, thickness, density and the boundary conditions. In this paper complex elastic modulus is equivalent to complex modulus of longitudinal waves [4]

$$\tilde{S}_e = S(1 + \mathrm{j} \cdot \eta) \tag{1}$$

where elastic modulus S and loss factor η are main optimized parameters, in the same way shear, Young's, and volume moduli can be optimized.

The absorption materials and backing are divided into many layers by physical model from figure 1, where $d_i\,(2 \le i \le n-1)$ is thickness of every layer, the first and last layer are fluid, and assuming half-infinite space. Sound waves are incident on absorption materials from direction z in the first layer with angle θ_1, while the nth layer is medium of sound waves transmission with angle θ_n.



Source: (BAI Guofeng, 2010)
Figure 1. Sketch map of layered structure

Assuming particle velocity and stress of every interface are continuous, transition matrices of multi-layered medium are presented by continuity. So transition matrices of all layers can be computed by from the 2nd to (n-1)th transition matrices multiplying in succession, as indicated, such that[5]

$$\begin{bmatrix} v_x^{(1)} \\ v_z^{(1)} \\ \sigma_x^{(1)} \\ \sigma_z^{(1)} \end{bmatrix} = \begin{bmatrix} A_{11}A_{12}A_{13}A_{14} \\ A_{21}A_{22}A_{23}A_{24} \\ A_{31}A_{32}A_{33}A_{34} \\ A_{41}A_{42}A_{43}A_{44} \end{bmatrix} \begin{bmatrix} v_x^{(n)} \\ v_z^{(n)} \\ \sigma_x^{(n)} \\ \sigma_z^{(n)} \end{bmatrix} \tag{2}$$

where v_x and v_z are particle velocity components, σ_x and σ_z are stress components, the upper index is interface number. When the first and nth layer are fluid, shear stress $\sigma_x^{(1)}$ and $\sigma_x^{(n)}$ vanish. The reflection coefficient can be written as

$$R = \frac{M_{32} + Z_1 M_{33} - (M_{22} + Z_1 M_{23}) Z_{n+1}}{M_{32} + Z_1 M_{33} + (M_{22} + Z_1 M_{23}) Z_{n+1}} \tag{3}$$

where M_{22}, M_{23}, M_{33} and M_{32} are shown in equation (4).

$$\begin{aligned} M_{22} &= A_{22} - A_{21}A_{42} / A_{41}, M_{23} = A_{23} - A_{21}A_{43} / A_{41} \\ M_{33} &= A_{33} - A_{31}A_{44} / A_{41}, M_{32} = A_{32} - A_{31}A_{42} / A_{41} \end{aligned} \tag{4}$$

where $Z_1 = \rho_1 \omega / \theta_1$, ρ_1, ω are medium density of incident waves and angular frequency, and Z_{n+1} is defined as Z_1.

The reflection coefficient of free boundary condition can be written as

$$T = \frac{2Z_1}{M_{32} + Z_1 M_{33} + (M_{22} + Z_1 M_{23}) Z_{n+1}} \tag{5}$$

The nth layer medium is air, which can be regarded as boundary condition, $\sigma_x^{(n)}$ and $\sigma_z^{(n)}$ vanish at the same time. The reflection and transmission coefficient of free boundary condition can be easily deducted from free backing condition,

When plane waves are normally incident, longitudinal waves only propagate in homogeneous materials. Pressure and particle velocity of every interface are continuous, and transition matrices of every layer medium are presented as[6]

$$\begin{Bmatrix} p_i \\ u_i \end{Bmatrix} = \begin{bmatrix} \cos(\tilde{k}d_i) & \mathrm{j}\rho\tilde{c}\cdot\sin(\tilde{k}d_i) \\ \mathrm{j}\sin(\tilde{k}d_i)/(\rho\tilde{c}) & \cos(\tilde{k}d_i) \end{bmatrix} \begin{Bmatrix} p_{i+1} \\ u_{i+1} \end{Bmatrix} \tag{6}$$

where p and u are pressure and particle velocity, the lower index is interface number, $\tilde{c}$ is complex velocity of dilatation waves, $\tilde{k}$ is complex wave number, and ρ is density.

Absorption coefficient

$$\alpha = 1 - |R|^2 - |T|^2 \tag{7}$$

III. OTIMIZATION ALGORITHM OF COMPLEX ELASTIC MODULUS

The minimal reflection coefficient is regarded as objective function to optimization problem of viscoelastic absorption materials. The complex elastic modulus is optimization variable and the restricted circumstance involves material thickness, backing boundary and other structure parameters. Optimization algorithm can be used as designing complex elastic modulus in order to achieve satisfactory absorption performance. Quasi-Newton method is a effective optimization algorithm, which form curvature information in the course of iterative vector $\mathbf{X}$, and the quadratic model can be written as

$$\min_X \left[\frac{1}{2} \mathbf{X}^T \mathbf{H} \mathbf{X} + \mathbf{C}^T \mathbf{X} + b \right] \tag{8}$$

where $\mathbf{H}$ is a symmetrical positive Hessian matrix, $\mathbf{C}$ is a constant vector, b is a constant, T denotes matrix transpose.

The optimal solution x can be obtained by the partial derivative of objection function

$$\nabla f(x) = \mathbf{H}x + \mathbf{C} = 0 \tag{9}$$

where $f(x)$ is objective function and ∇ denotes gradient of function.

The sequence quadratic programming is used to the quadratic approximate Lagrangian function, and Hessian matrix can be expressed by the equation

$$H_{k+1} = H_k + \frac{q_k q_k^T}{q_k^T S_k} - \frac{H_k^T H_k}{S_k^T H_k S_k} \tag{10}$$

where

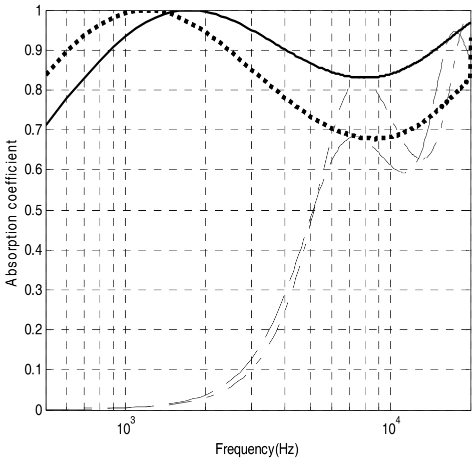
$$S_k = x_{k+1} - x_k ,$$

$$q_k = \nabla f(x_{k+1}) + \sum_{i=1}^{n} \lambda_i . \nabla g_i(x_{k+1}) - (\nabla f(x_k) + \sum_{i=1}^{n} \lambda_i - \nabla g_i(x_k))$$

IV. COMPUTATION RESULTS AND DISCUSSION

A. Optimization curves of complex elastic modulus for viscoelastic materials

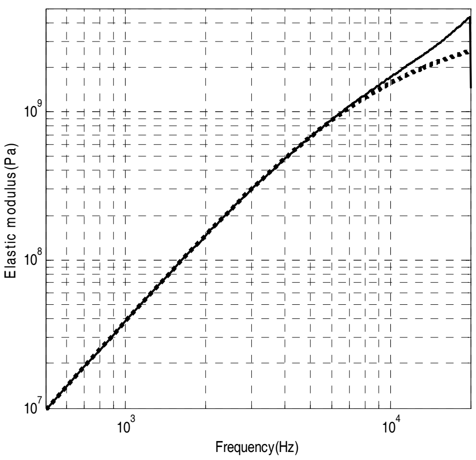
In this paper complex elastic modulus is optimized and absorption coefficient is computed when plane waves are incident from water and the backing is air. The density of absorption materials 1000kg/m^3, and thickness is 50 millimeters. The optimization results are presented by Quasi-Newton method, where the incident angle θ_1 is 45°, elastic modulus is 1.2×10^9Pa , loss factor of elastic modulus is 0.2, shear modulus is 0.4×10^7Pa and loss factor of shear modulus is 0.5.



Source: (BAI Guofeng, 2010)

Figure 2. Absorption coefficient curve of elastic modulus of homogeneous materials

(dashed-line: $\theta_1 = 90^\circ$, dashed-dotted-line: $\theta_1 = 45^\circ$, optimized-dotted-line: θ_1 =90, optimized -solid-line: θ_1 = 45,)



Source: (BAI Guofeng, 2010)

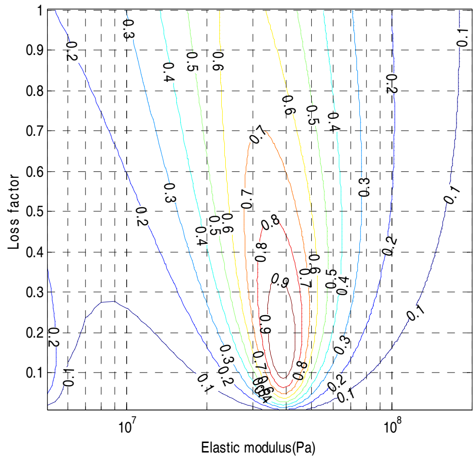
Figure 3. Optimization elastic modulus

(solid-line: $\theta_1 = 90^\circ$, dotted-line θ_1 =45°)

The absorption performance of viscoelastic materials is not very good in low frequency before optimization from dashed-line and dashed-dotted-line of figure 2. The incident angle is not sensitive to the curve of elastic modulus from figure 3. The results show that sound absorption performance can be improved effectively by adjusting complex elastic modulus of viscoelastic materials. In general, the higher is frequency, the more is elastic modulus. The elastic modulus increases from 10^7 to 5×10^9 Pa with frequency.

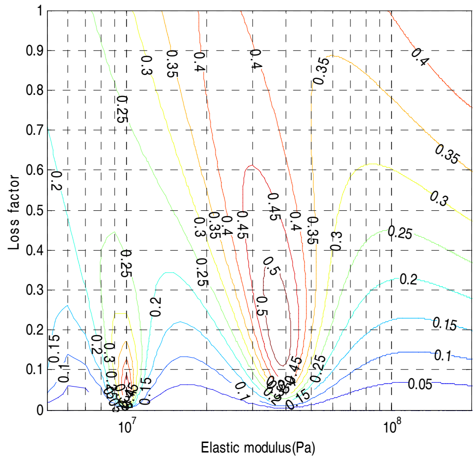
B. Absorption coefficient isoclines of complex elastic modulus

When plane waves are normally incident and the backing is air, the relation between complex elastic modulus and absorption performance can be analyzed by absorption coefficient isoclines. It is a requirement that the absorption coefficient is larger than 0.8 for optimizing complex elastic modulus.



Source: (BAI Guofeng, 2010)

Figure 4. Absorption coefficient isoclines with air backing in frequency 1kHz



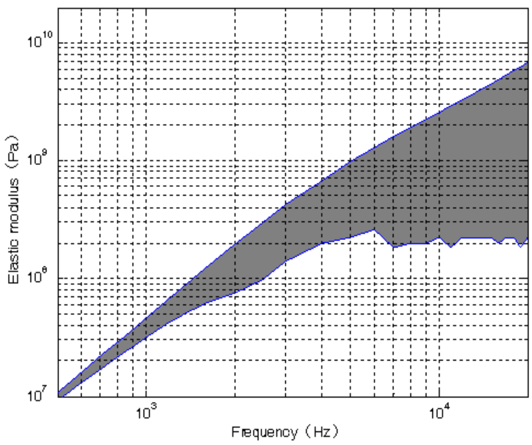
Source: (BAI Guofeng, 2010)

Figure 5. Absorption coefficient isoclines with water backing in frequency 2kHz

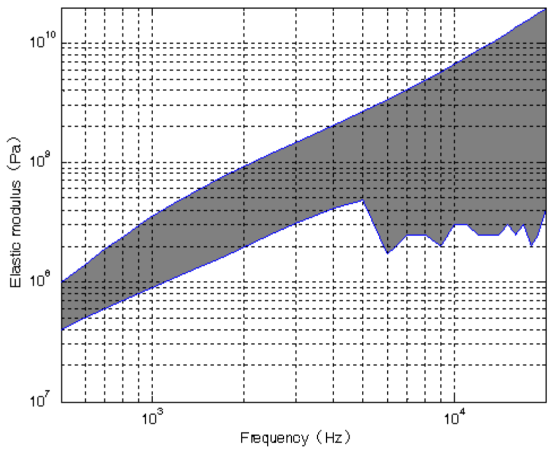
From figure 4 it can be seen that the variation extent of elastic modulus is around 10^7 Pa with air backing in 1kHz frequency when the absorption coefficient is larger than 0.8. Sound absorption coefficient isoclines in low frequency are regular ellipses at specific condition. There are not one region where the absorption coefficient is larger than 0.8 with water backing in 2kHz frequency from figure 5.

C. Scope of complex elastic modulus with different backing

Frequency, elastic modulus, loss factor and absorption coefficient are four dimensions ignoring other parameters. The elastic modulus and loss factor can be analyzed dividedly with frequency and absorption coefficient. In the largest area assuming absorption coefficient is larger than 0.8, the maximum and minimum elastic modulus are given. The scope of elastic modulus and loss factor can be obtained while the acoustic waves are normally incident with different backing.

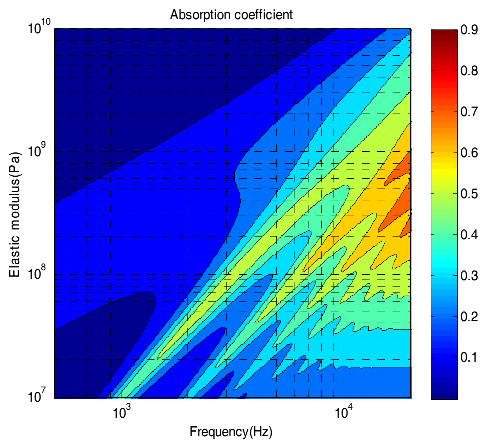


(a) air backing



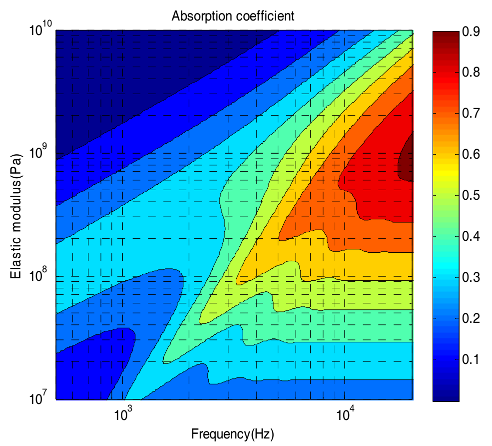
(b) 20mm steel backing
Source: (BAI Guofeng, 2010)
Figure 6. The scope of elastic modulus of homogeneous materials with air backing

Assuming absorption coefficient is larger than 0.8, the scope of elastic modulus of homogeneous materials with different backing can be seen by the shaded area of figure 6. In fact adjusting elastic modulus is more difficult than that of loss factors. The scope of elastic modulus is very narrow in low frequency with air backing. While the thickness of steel backing is 20 millimeters, the scope of shade area increases more greatly in low frequency than that of air backing. As showed in figure 7, the elastic modulus isoclines are obtained with water backing when the loss factor is invariable.



(a)

(a) Loss factor is 0.2



(b) Loss factor is 0.6
Source: (BAI Guofeng, 2010)
Figure 7. Elastic modulus isoclines of homogeneous materials with water backing

In comparison to air backing, the scope of great absorption coefficient is very narrow and even ignored from figure 7. It seems like favourable to absorption materials when the loss factor is larger.

V.SUBMISSION

The span of elastic modulus of viscoelastic materials is from one magnitude to three magnitudes in wider frequency. Assuming the absorption coefficient is larger than 0.8, the scope of elastic modulus increases with frequency. It is more difficult to adjust elastic modulus in low frequency for its narrow scope. The results indicate that the scope of elastic modulus becomes wider in low frequency with certain steel backing, which reduce the difficulty of adjusting complex elastic modulus. When the absorption materials are immersed in water, it is very difficult to adjust elastic modulus for its narrow scope.

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