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Left-handed shear and longitudinal elastic waves in 2D phononic crystals made of a solid matrix

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ABSTRACT

Waves propagating in left-handed materials have unusual properties such as phase and group velocities of opposite signs and negative refraction index. Periodic lattices have been shown to exhibit such properties both for electromagnetic (photonic crystals) and in-fluid acoustic (phononic crystals) waves. This work addresses the question of the existence of left handed elastic waves in phononic crystals. Two-dimensional phononic crystals made of square lattices of cylindrical cavities or inclusions in a solid matrix are considered. Dispersion curves are computed using plane wave expansion method for real wave vectors in the Brillouin zone and finite element method for complex wavenumbers along a specific propagation direction. From these results, the existence and symmetry of the left-handed propagation mode in the phononic crystal is discussed and its relationship with lattice geometry and constitutive materials is analyzed.

INTRODUCTION

Electromagnetic, acoustic and elastic left-handed materials

Ideal Left-Handed (LH) materials were first theoretically studied by Veselago [1] in the frame of electromagnetism. LH materials have real speed of light c together with negative dielectric permittivity e and negative magnetic m. For a monochromatic plane electromagnetic wave in which all quantities are proportional to $e^{j(kz-wt)}$, where w is the circular frequency, t the time, $k = w/c$ the wave number and z the space coordinate, Maxwell's equations are written

$$\vec{k} \times \vec{E} = (wm/c)\vec{H} \ , \tag{1}$$

$$\vec{k} \times \vec{H} = -(we/c)\vec{E} \ , \tag{2}$$

where $\vec{k}$ is the wave vector, $\vec{E}$ is the electric field and $\vec{H}$ the magnetic field. From equations (1) and (2), it can be verified that $\vec{k}$, $\vec{E}$ and $\vec{H}$ form a left-handed triplet when e and m are negative and a right handed triplet when e and m are positive. Electromagnetic Poynting vector $\vec{P}$, which is associated to energy flux, is defined as

$$\vec{P} = \vec{E} \times \vec{H} \ . \tag{3}$$

and therefore $\vec{P}$, $\vec{E}$ and $\vec{H}$ always form a right-handed triplet. It appears then that $\vec{P}$ and $\vec{k}$ are colinear and of opposite signs for an harmonic monochromatic plane wave propagating in a LH material.

For monochromatic plane acoustic waves propagating in a linear ideal homogeneous isotropic fluid, Euler's equation is written as

$$p\vec{k} = (r_f w)\vec{v} \ , \tag{4}$$

where p is the acoustic pressure, $\vec{v}$ the particle velocity and r_f the fluid density at rest. The dot product of each term of equation (4) and $\vec{v}$ gives

$$p\vec{v}.\vec{k} = (r_f w)|\vec{v}|^2 \ . \tag{5}$$

From equation (5), the wave vector $\vec{k}$ and the intensity vector $\vec{I} = p\vec{v}$ are colinear and of opposite signs when r_f is negative. In that case, the adiabatic compressibility c_S is also negative in order to have a real speed of sound $c_f = 1/\sqrt{r_f c_S}$ [2].

The case of a monochromatic shear or longitudinal plane elastic wave propagating in a linear elastic isotropic medium is finally considered. Particle motion is governed by Newton's law

$$\overline{\overline{T}}.\vec{k} = -(r_s w)\vec{v} \ , \tag{6}$$

where $\overline{\overline{T}}$ is the stress tensor and r_s the solid density. Equation (6) is transformed by performing the dot product of each term and $\vec{v}$ and by using the symmetry of $\overline{\overline{T}}$ to give

$$\vec{k}.\left(-\overline{\overline{T}}.\vec{v}\right)=\left(r_s w\right)\left|\vec{v}\right|^2 \ . \tag{7}$$

In that case, the wave vector $\vec{k}$ and the elastodynamic Poynting vector $\vec{P}=-\overline{\overline{T}}.\vec{v}$ are colinear and of opposite signs when r_s is negative. In addition, longitudinal (resp. shear) wave velocity must be real in order to have propagating plane longitudinal (resp. shear) waves. In an isotropic medium, this implies to have negative elastic moduli for the corresponding waves i.e.

$$M=\frac{E(1-n)}{(1+n)(1-2n)}<0 \ , \tag{8}$$

for longitudinal waves and

$$G=\frac{E}{2(1+n)}<0 \ , \tag{9}$$

for shear waves, E being Young's modulus and n Poisson's ratio. Thus, different zones can be defined in the (E,n) space of a virtual elastic material in order to describe the existence of propagating longitudinal and shear waves together or separately (Fig. 1).

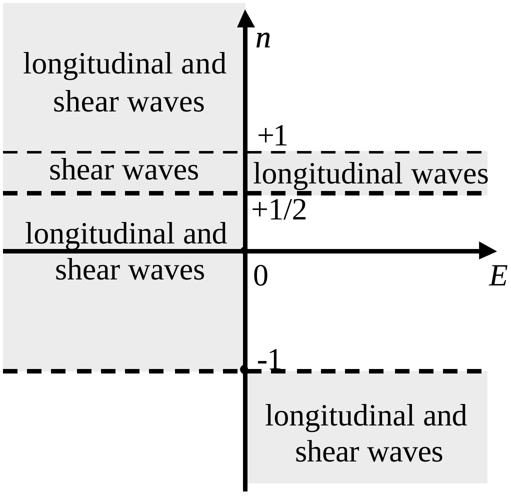

Figure 1. Domains of existence of longitudinal and shear propagating waves in a left-handed elastic material, considering a negative density r_s

Acoustic meta-materials versus phononic crystals

LH materials are usually artificial materials which are designed and fabricated in order to exhibit specific properties. Two main concepts are considered: meta-materials and phononic crystals.

It is known that resonators such as a simple spring-mass system can be seen, just above resonance, as a spring having an effective negative stiffness. Thus, effective negative properties could be obtained in a reduced frequency range by including adequate local resonators in a bulk material [2]. Properties of these locally resonant materials also called meta-materials result from steady-state of a resonant system with two important consequences: 1) a local effect is used in order to define effective materials properties. Consequently, the size of the resonators and the distance between resonators have to be small compared to wavelength to enable material homogeneization; 2) the material needs a certain amount of time, related to quality factor of the resonators, to build up the resonance and display the effective properties of a left-handed material.

LH properties can also be obtained by using multiple diffractions and reflections of wave propagating in a periodic material [3]. This is the well-known concepts of Phononic Crystals (PC) for acoustic waves. The fact that effective properties are obtained in steady-state after multiple reflections and diffractions in a crystal of high symmetry has two main consequences. 1) the medium cannot be represented by local properties and LH character must be described by wave properties (phase propagation, flux of energy) ; 2) a certain amount of time is also needed in that case to display required wave properties.

Content of the paper

The work presented in this paper focuses on the evaluation of LH properties of elastic phononic crystals. LH character is therefore associated to propagation mode and not to local effective properties of a medium. Identification of LH properties is performed from wave propagation dispersion curves in the PC which are computed by using Plane Wave Expansion (PWE) method [4] or Finite Element Method (FEM) [5]. Results are presented for two different 2D elastic phononic crystal constituted by a solid matrix with cylindrical cavities or hard cylindrical inclusions. Main question addressed by the paper concern the symmetry of the LH propagation mode in the PC and its relation with lattice geometry and materials.

LEFT-HANDED ELASTIC WAVES IN A SOLID PHONONIC CRYSTAL WITH CAVITIES

Geometry of the phononic crystal and dispersion curves

The 2D phononic crystal considered in this section is a square lattice of air-filled cylindrical cavities in aluminum matrix [4, 6]. The density of aluminum is r_a = 2808 kgm^{-3} and the longitudinal and shear velocities are respectively V_{La} = 6337 m.s^{-1} and V_{Ta} = 3130 m.s^{-1}. Distance between axes of two adjacent holes is a = 3.9 mm and hole diameter is d = 3.2 mm.

Dispersion curves of elastic waves are presented in the first Brillouin zone, on the ΓXM path in terms of frequency versus wavevector (Fig. 2a). The band structure displays an isolated branch corresponding to a LH propagation mode with phase velocity opposite to group velocity in the [470kHz, 530kHz] frequency range. A predominantly shear behavior is identified for this propagation mode by studying the transmission of incident longitudinal and shear waves through a CP of finite thickness at normal incidence [4]. No LH mode with predominantly longitudinal behavior is observed on the dispersion curve contrary to the usual result obtained for acoustic waves propagating in lattice of rigid cavities [3, 7].

For further analysis, dispersion curves are drawn (only along ΓX) in complex wave number space in order to describe evanescent wave solutions (Fig. 2b). Displacement fields associated to different branchpoints at Γ or X are displayed in Figures 3 and 4. Two families of solutions can be distinguished according to the symmetry or antisymmetry of the displacement field with respect to the wavenumber axis. At CP boundary and at normal incidence, symmetrical modes will naturally couple to longitudinal waves and antisymmetrical modes to shear modes. Branches corresponding to each type of symmetry are analyzed in more details hereafter.

Antisymmetrical propagation modes

Antisymmetrical propagation modes (the lowest branch and its multiple foldings) are constituted by branches with alternatively Right-Handed (RH) and LH properties connected at points Γ and X by small loops in the complex plane (i.e. corresponding to small imaginary parts for complex wavenumber). Γ1 corresponds to a rigid motion of the whole lattice perpendicular to propagation direction. It can be seen as the limiting case of a pure shear wave when wavenumber tends to zero. Modes denoted by Γ3 and Γ4 correspond to antisymmetrical resonances of the matrix around the cavity of order 4 and 3 (resonance of order n is associated to a displacement close to cos(nq) at cavity surface). These resonances result from the coupling of isolated resonances via the lattice. They correspond to cases for which all cavities (in directions parallel and perpendicular to wave propagation) vibrate in phase. Finally, Γ6 is an harmonic resonance which corresponds to order 2 cavity resonance mixed with wavelength vibration of the unit cell perpendicularly to propagation direction. It exhibits a cut-off frequency (connection to a purely imaginary branch starting at zero frequency). Similarly, at X, modes denoted by X1, X2 , and X5 correspond to antisymmetrical resonances of the matrix around the cavity of order 2, 3 and 5. In that case, vibrations are in phase for cavities belonging to the same row (perpendicular to progagation direction) and out of phase for cavities of successive rows. RH mode branches on real axis are Γ1X1 and Γ4X5. Several branches (X1X2, Γ3Γ4 and X4X5) take place in the complex plane and connect resonance modes having the same phase distribution (successive rows both in phase or out of phase) along propagation direction. Finally, X2Γ3 is a branch corresponding to a LH propagation mode. Along this branch, displacement field progressively transforms from third order into fourth order cavity resonance.

Symmetrical propagation modes

Symmetrical propagation modes display only RH properties for this CP. Two branchpoints are found at Γ: Γ2 corresponds to rigid motion of the whole lattice parallel to propagation direction which can be seen as the limiting case of a pure longitudinal wave when wavenumber tends to zero ; Γ5 is associated to a third order cavity resonance. Its displacement field is identical to Γ4 displacement field if a rotation of -π/2 is applied to propagation direction. Both modes have therefore same resonance frequency at zero wavenumber. At X, symmetrical modes at X3 and X4 are associated respectively to second and fourth order cavity resonances. Identification of resonance order associated to X6 is difficult: strong intercellular coupling generates complicated cavity surface displacement field which cannot be reduced to cos(nq) . RH mode branches on real axis are Γ2X3 and Γ5X4. No LH mode branch is found. Finally, two different types of branches are observed in the complex plane: X4X6 loop connects at constant real part two branchpoint in X as observed for antisymmetrical modes; X3Γ5 is a more complicated complex branch that relates branchpoints in Γ and X through a path having very large imaginary part. This last branch which could be due to the fact that Γ5 mode has a lower resonance frequency than X4 mode, leading to the configuration [complex branch X3Γ5 followed by RH real branch Γ5X4] instead of [complex branch X3X4 followed by LH real branch X4Γ5]

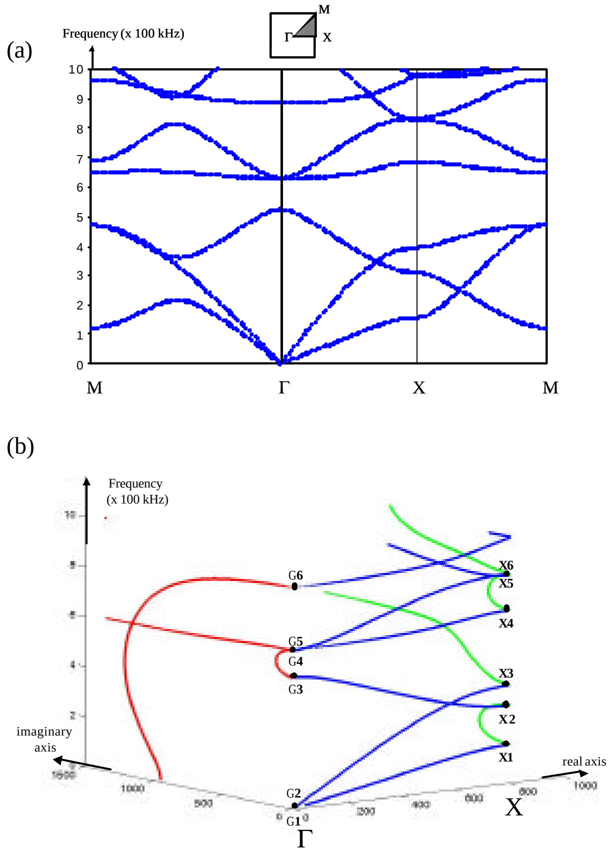

Figure 2. 2D phononic crystal constituted by a square lattice of air-filled cylindrical cavities in an aluminum matrix. (a) dispersion curves along ΓXM path for real wavevectors ; (b) dispersion curves along ΓX path for complex wavenumbers

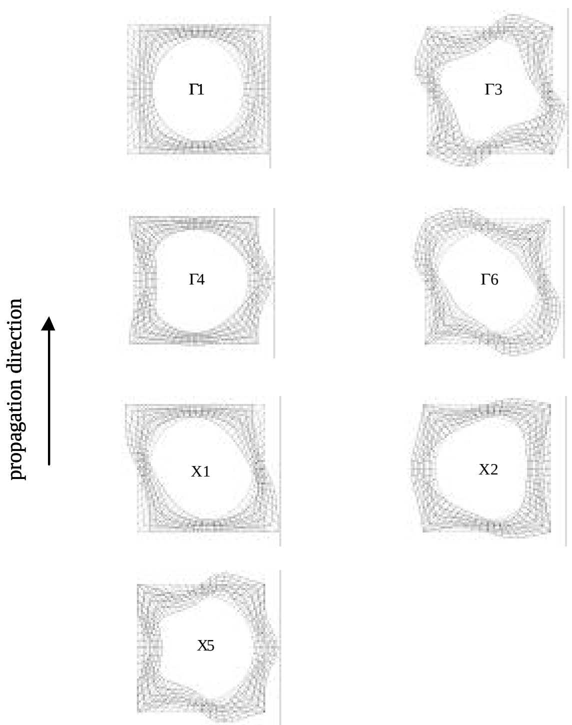

Figure 3. 2D phononic crystal constituted by a square lattice of air-filled cylindrical cavities in an aluminum matrix. Displacement fields of antisymmetrical modes at different branchpoints of dispersion curves

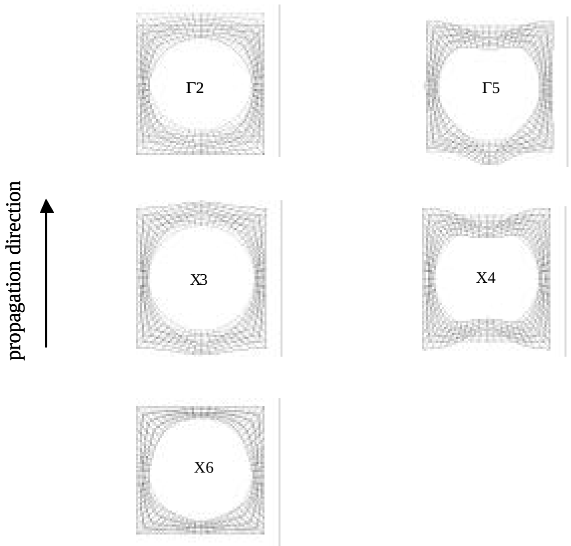

Figure 4. 2D phononic crystal constituted by a square lattice of air-filled cylindrical cavities in an aluminum matrix. Displacement fields of symmetrical modes at different branchpoints of dispersion curves

LEFT-HANDED ELASTIC WAVES IN PHONONIC CRYSTALS WITH HARD INCLUSIONS

Geometry of the phononic crystal and dispersion curves

The 2D phononic crystal considered in this section is a square lattice of steel cylinders in a nylon matrix. Physical properties are $r_s = 7800$ kg.m^{-3}, longitudinal velocity $V_{Ls} = 6181$ m.s^{-1} and shear velocity $V_{Ts} = 3244$ m.s^{-1} for steel, $r_n = 1152$ kg.m^{-3}, longitudinal velocity $V_{Ln} = 2454$ m.s^{-1} and shear velocity $V_{Tn} = 1085$ m.s^{-1} for nylon. Distance between axes of two adjacent steel cylinders is $a = 2.5$ mm and cylinder diameter is $d = 1.545$ mm.

Fig. 5 displays the dispersion curves of elastic waves on the ΓXM path in terms of frequency versus real wave vector (Fig. 5a) and on ΓX path in terms of frequency versus complex wave number (Fig. 5b). The band structure displays a branch corresponding to a LH propagation mode in the [650kHz, 810kHz] frequency range. Displacement fields associated to different branchpoints at Γ or X are displayed in Figures 6 and 7 for antisymmetrical and symmetrical propagation modes respectively.

Antisymmetrical propagation modes

Antisymmetrical propagation modes have a more complicated band structure than in the previous case. The lowest branches on real axis Γ1X1 and X2Γ3 display usual RH and LH properties respectively. Successive rows (along propagation direction) vibrate in phase at Γ and out of phase at X. Γ1 and X1 are associated to lateral motion of the steel cylinder while Γ3 and X2 correspond to rotational motion of the steel cylinder. Higher branches Γ4X4 and X7Γ6 are almost flat. Related displacement fields at Γ4, X4, X7 and Γ6 involve mainly shear strain of the nylon matrix with limited motion of the steel cylinder. Two branches (X1X2 and X4X7) take place in the complex plane and connect resonance modes having out of phase vibration of successive rows along propagation direction. Γ3 mode has unusual cut-off frequency behaviour as point of maximum frequency, denoted M, is not located on real axis. Imaginary branch connected to Γ4 moves toward high imaginary part and high frequency and is difficult to interpret.

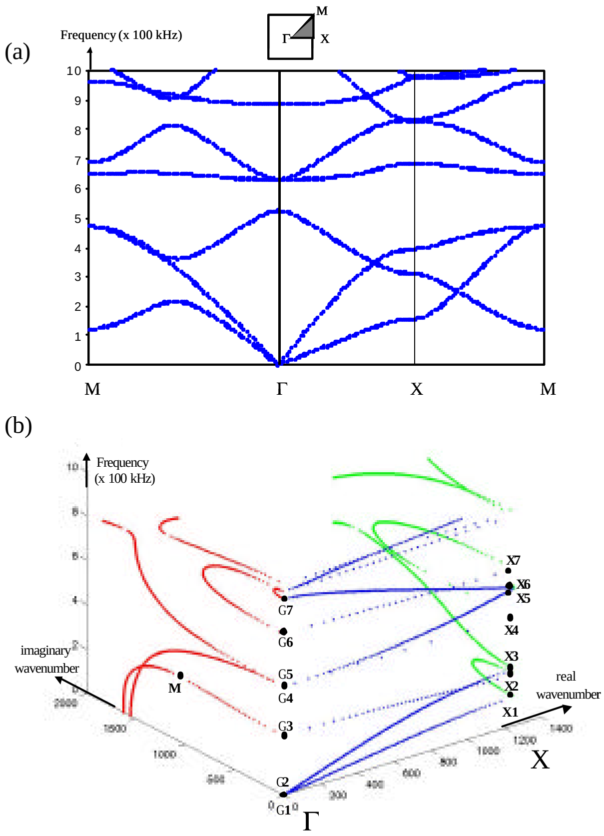

Figure 5. 2D phononic crystal constituted by a square lattice of steel cylinders in a nylon matrix. (a) dispersion curves along ΓXM path for real wavenumbers ; (b) dispersion curves along ΓX path for complex wavenumbers

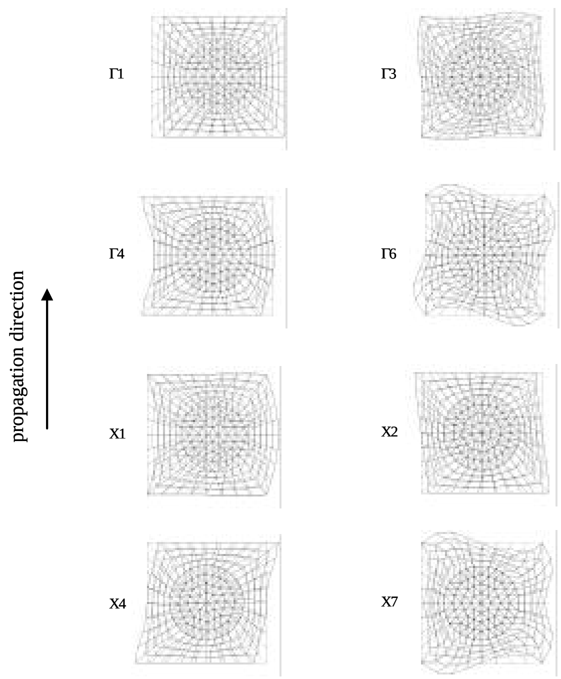

Figure 6. 2D phononic crystal constituted by a square lattice of steel cylinders in a nylon matrix. Displacement fields of antisymmetrical modes at different branchpoints of dispersion curves

Symmetrical propagation modes

Three different branches are found for symmetrical propagation modes. The first two branches, Γ2X3 and Γ5X5, display usual RH properties. Γ2 and X3 are associated to longitudinal motion of the steel cylinder while Γ5 and X5 involve shear strain (sy mmetrical with respect to propagation direction) of nylon matrix with limited motion of the steel cylinder. The third branch X6Γ7 corresponds to a LH propagation mode. The associated displacement at Γ7 involves a shear strain of the nylon matrix (symmetrical with respect to propagation direction) similar to Γ5 mode, mixed with compressional strain along propagation direction. A very small branch between X5 and X6 in the complex plane connect resonance modes having out of phase vibration of successive rows along propagation direction. Γ5 mode has a cut-off frequency shown by the purely imaginary branch starting at zero frequency. Imaginary branch connected to X3 moves toward high imaginary and high frequency and is difficult to interpret.

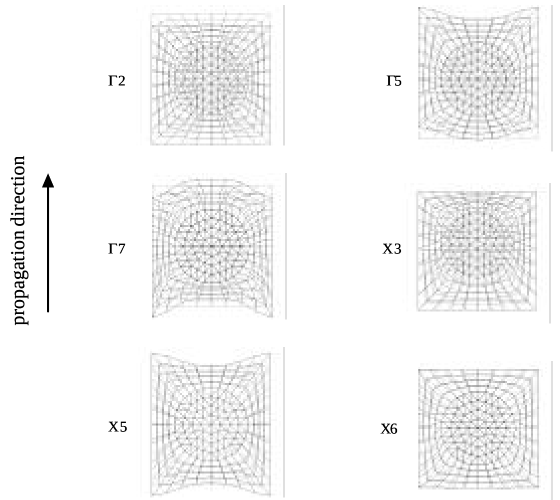

Figure 7. 2D phononic crystal constituted by a square lattice of steel cylinders in a nylon matrix. Displacement fields of symmetrical modes at different branchpoints of dispersion curves

CONCLUSION

No general rule can be identified to state the existence of symmetrical or antisymmetrical left-handed branches in the band structure from the analysis of the two phononic crystals presented in this work. The representation of dispersion curves in the complex wave number domain gives useful information on the connection between propagation mode branches. It opens up the possibility of a more progressive method based on unit cell resonance analysis (isolated and in interaction) which could improve our understanding of the relationship between existence of left-handed branches and phononic crystal geometry and constitutive materials.

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