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Acoustically induced micro-scale capillary wave turbulence

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ABSTRACT

The theory of non-linear wave interactions leading to so-called interfacial wave turbulence, where a broadband distribution of capillary wave phenomena may be induced by a monofrequency oscillator, is well known, but experimental results are rare. In particular, it is challenging to set up a physical system where both capillary wave amplitudes are easy to measure and capillary forces dominate gravitational forces. Though capillary forces dominate at small scales, the small oscillation amplitudes and generally high oscillation frequencies preclude measurement via cameras or other traditional means. Instead, we use a laser Doppler vibrometer, capable of measuring oscillations up to 40 MHz, and providing a minimum detectable deflection of picometres. Using ultrasonic surface acoustic wave excitation at 19.5 MHz, we generate wave turbulence on the free surface of a water drop. Energy at the driving frequency does not directly enter the cascade ; rather the driving frequency excites a low-frequency resonance. This resonance appears to, in turn, excite higher harmonics, forming the cascade of length scales seen in the frequency spectrum of wave heights. The initial low-frequency resonance, contrary to expectations via Faraday wave theory, is not at one-half the excitation frequency. Instead, we find the low-frequency resonance to be on the order of 100 Hz, which probably arises due to a balance of capillary and inertial forces; the Faraday wave is not observed due to the high frequency of the excitation. By condensing each spectrum to the value of its power exponent, we find that the turbulence decays as the electrical input power increases beyond 500 mW, a SAW amplitude of about 1 nm. At these powers the probability of very large waves deviates strongly from the Gaussian distribution, indicative of strong non-linearity. Wave turbulent theory is therefore invalid in this high-power regime as the highly non-linear nature of the waves violates the theory's fundamental assumption of weak non-linearity.

THEORY OF WAVE TURBULENCE

The scaling of kinetic energy E against wavenumber k in isotropic hydrodynamic turbulence,

$$E(k) \sim k^{-5/3}, \tag{1}$$

derived by Kolmogorov in 1941, is well known (see for example [5]). An analogous power law describing the non-linear interactions of interfacial capillary waves was derived by Zakharov, scaling the "density of the wavenumber in k-space" n_k against the wavenumber as

$$n_k \sim k^{-19/4} \text{ [13]}, \tag{2}$$

although this is more conveniently expressed in terms of surface deflection η and frequency ω as

$$\eta \sim \omega^{-17/12}. \tag{3}$$

By this analogy, such wave interactions acquire the name "wave turbulence". However where the Kolmogorov spectrum is obtainable through dimensional scaling alone, Zakharov's result requires an exact solution of the stationary kinetic equation. In the construction of the kinetic equation, the system is assumed to consist of waves with random phase, and be statistically homogeneous (so-called "weak turbulence" [13]). A frequency spectrum of surface height can be obtained by measuring the time series of surface height, then applying a Fourier transform; if we consider each point of the spectrum in the frequency domain to be a random variable, the expected values will conform to the power law where weak turbulent theory holds.

The range of frequencies where the weak turbulent theory applies is referred to as the "inertial range"; according to the theory it is bounded at low frequency by the exciting force, and at high frequency by viscous damping. The role of viscosity is simply to absorb energy at short length scales; it plays no role over the inertial range of frequencies.

The analogy between hydrodynamic and wave turbulence has been investigated [2], finding a discrepancy between the theoretical and measured energy flux in the frequency domain, as well as significant statistical inhomogeneities at short time scales. However, the $-17/12$ power law scaling remains associated with capillary wave turbulence.

Previous verification of wave turbulence

The power law for capillary wave turbulence has been numerically [8] and experimentally [1–3] observed, but it is acknowledged that such results are rare [2]. Typically, the fluid is contained in a well, excited by a mechanical oscillator at around 10 Hz [1, 3]. A limiting factor is that the prevalent time and length scales are often short enough to preclude traditional non-invasive measurement techniques such as high-speed photography. To achieve the necessary breadth and resolution in the frequency domain, single-point measurements using such techniques as capacitive wires [3] and reflection of low-angle laser beams [1] have been used; a successful measurement over an area of surface was made using a technique measuring the diffusion of white light through the surface [12].

EXPERIMENT

The study presented here differs somewhat from the earlier literature; we approach the problem from a microfluidics perspective. Our fluid (de-ionised water is predominantly used) lies as a sessile drop on a microfluidic substrate, and is excited by an ultrasonic surface acoustic wave (SAW) at 19.5 MHz. Such an arrangement can cause mixing [10] or concentration [6], bulk transport [11] and atomisation [9] of the fluid. This work is therefore relevant to the broader field of microfluidics.

The drops are pipetted on to the substrate; the volume of the drops decreases over time due to both evaporation and atomisation, so the drop must be regularly replaced. The drops used in this work ranged in volume from 1 µl to 2.5 µl.

The surface acoustic wave was generated on a substrate of single-crystal 128° Y–X cut Lithium Niobate using an interdigital transducer (IDT). The electrical power applied to the IDT was varied from 100 mW up to 1 W. At higher powers, the large surface deflections are difficult to measure; in addition, atomisation of the fluid can occur. The amplitude of the SAW itself is on the order of 1 nm.

Measurements were made at a single point using a Polytec M400 laser Doppler vibrometer (LDV) capable of measuring oscillations up to 40 MHz. This measurement technique was selected as it is able to measure the excitation wave frequency, and can also be used to measure the wave itself on the substrate. The LDV provides a minimum detectable deflection of picometres. As this measurement technique requires a perpendicular surface to reflect the laser, only the peak of the drop can be measured with any accuracy. The arrangement of experimental apparatus is shown diagrammatically in Fig 1.

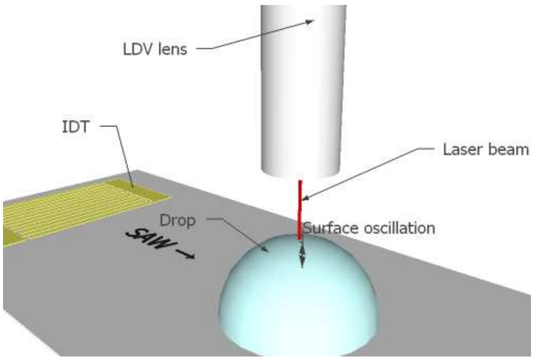

Figure 1: Diagrammatic representation of the experimental equipment. The SAW propagates along the Lithium Niobate substrate from the IDT towards the drop; the energy is absorbed by the fluid and is expressed as a surface vibration. The vibration is in turn measured by the laser Doppler vibrometer (LDV).

To interpret the vibrometer's full frequency range requires two separate interference signal decoders. To achieve full bandwidth and good resolution in the frequency domain, at least two individual scans must be performed; the data is assembled in post-processing.

RESULTS

Typically, the weak-turbulent cascade is visible when multiple recorded spectra are averaged. Two such averaged spectra exhibiting the turbulent cascade are shown in Fig 2, together with the theoretical $-17/12$ power gradient for reference. As excitation amplitude increases, more energy is absorbed by the fluid, and a corresponding increase in the observed spectrum amplitude reflects this increase in energy. Interestingly, for the low-amplitude result, the spectrum descends faster than the theoretical gradient over the region of 8–20 kHz, before closely approximating the theoretical gradient; in the higher-amplitude result, this effect is not apparent. Such steep gradients are often associated with finite-size effects, where the wavelength is of the same order of magnitude as the surface size. In the high-power case, the turbulent cascade is observed up to 1 MHz; to our knowledge this is the highest frequency at which interfacial wave turbulence has been observed.

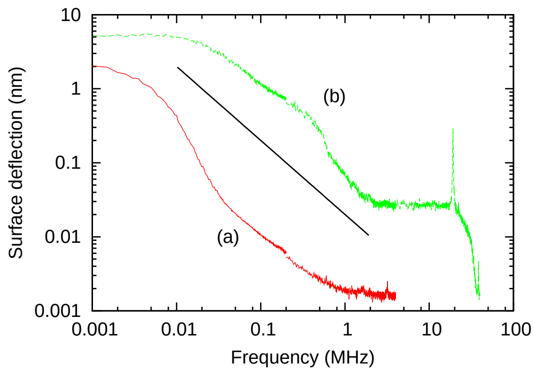

Figure 2: Comparison between typical spectra from drops exposed to (a) a low excitation amplitude of 0.26 nm (approximately 35 mW at the IDT) and (b) an intermediate amplitude of 0.64 nm (190 mW). The plot axes are logarithmic, so the theoretical power law becomes a straight line with a slope of $-17/12$, shown for reference between the two spectra.

The driving frequency at 19.5 MHz is clearly visible in spectrum (b) of Fig 2 as a single peak. The turbulent cascade is observed at frequencies below the driving frequency, contrary to Zakharov's original theory and other results found in the literature. If Zakharov's theory is correct, the high frequency excitation energy must shift to a lower frequency in order to enter the cascade.

The classical explanation for a downshift in frequency is the Faraday wave, whereby a fluid surface parametrically driven by a vibrating support will exhibit an oscillation at exactly half the driving frequency [4]. This phenomenon has been extensively studied [7] and is observed in other works dealing with capillary wave turbulence (for example [1]).

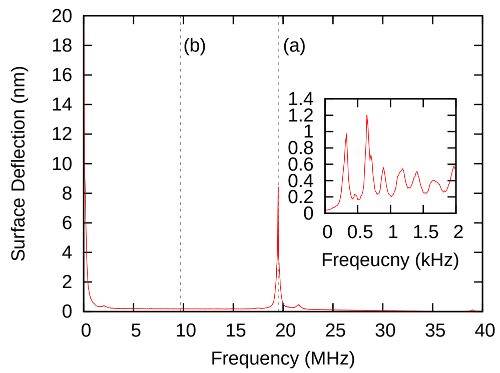

Figure 3: An illustration of the absence of the Faraday wave. The 19.5 MHz excitation frequency is marked (a); the Faraday wave would occur at 9.75 MHz, marked (b). Inset: finer-resolution plot showing the actual low-frequency response of the surface.

Figure 3 shows that Faraday wave theory is not applicable in this case; it can be clearly seen that for our system the 19.5 MHz excitation wave does not induce a resonance at half its frequency, 9.75 MHz. Instead, as has been previously observed for a similar experimental setup [9], there exists a resonance many orders of magnitude lower in frequency, in our case at $O(100\,\text{Hz})$. As illustrated by the inset to Fig 3, the low frequency resonance excites further harmonics; the interactions of these waves are then able to generate the apparent broadband excitation of the frequency cascade.

It is suggested in previous work [9] that the low-frequency resonance is due to a balance between capillary and inertial forces. This would require that the resonance frequency varies with some representative length scale L, fluid density ρ and surface tension γ as

$$\omega \sim \sqrt{\frac{\gamma}{\rho L^3}}. \tag{4}$$

While the ratio γ/ρ is difficult to vary without using surfactants, the relation between resonance frequency and length scale can be easily shown. Defining L as the cube root of drop volume, the relation (plotted in Fig 4) shows good linearity. We suggest the resonant frequency is related to the elastic vibration of the capillary shell, while the Faraday wave is suppressed due to the high frequency of the excitation. If these suggestions are correct, the resonance frequency will strongly depend on the drop shape; this is an avenue for further investigation.

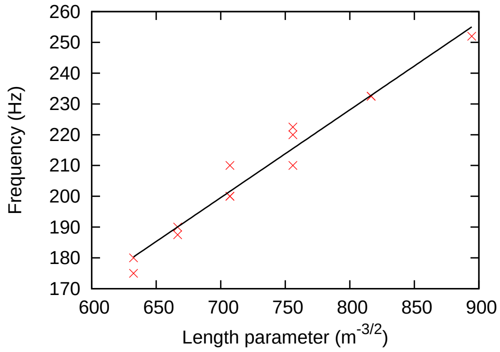

Figure 4: The relation between the low-frequency resonance ω and the drop length scale L (the cube root of volume is used here) is shown by plotting ω against $L^{-3/2}$ as per Eq 4. The solid line is a linear regressive fit to the data with zero intercept. The values of the length parameter represent drop volumes from 2.50 µL to 1.25 µL.

Returning to the effect of excitation amplitude on the frequency cascade, it would be desirable to quantify the agreement between experimental results and the theoretical power gradient. Many individual spectra were recorded, and by measuring the steepest power gradient for each spectrum over one octave of frequency, such a quantitative comparison is possible. Figure 5 shows gradient measurements plotted against their corresponding electrical input powers, thus illustrating the effect of changing excitation amplitude on the frequency cascade.

At low powers, up to 100 mW, this method of determining the power gradient is inaccurate, resulting in a great degree of scatter. Between 100 mW and about 500 mW, the agreement between theory and recorded gradients is quite good, although outliers are present where finite size effects discussed above dominate the spectrum. As power increases above a threshold of some 500 mW, a SAW amplitude of about 1 nm, the cascade becomes increasingly shallow. This is a consistent deviation from wave turbulent theory which we seek to explain.

By calculating the probability density function (PDF) of the time series of surface height, we gain another perspective on the apparent decay in gradient at high excitation levels. Figure 6 shows two functions; where the excitation power is 105 mW the PDF exhibits an approximately Gaussian profile, whereas the PDF arising from 590 mW excitation has significantly deformed tails. This shows that very large waves are disproportionally more likely at higher powers. This is a highly nonlinear phenomenon; it seems that the original “weakly nonlinear” assumptions of wave turbulence theory are violated, and hence the turbulent cascade is no longer apparent.

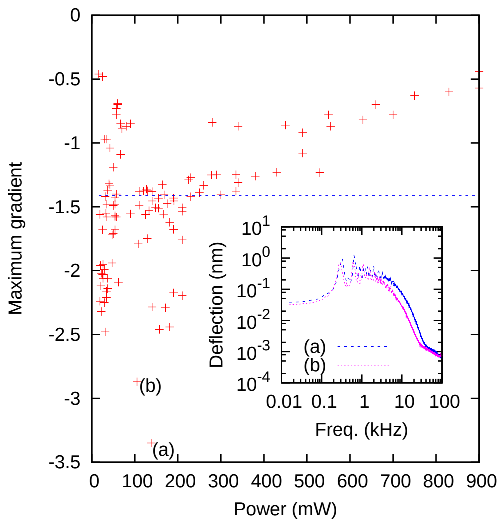

Figure 5: The effect of changing excitation amplitude on the turbulent cascade is shown by reducing many individual spectra to single points, representing their maximum power gradient. These points are plotted against the power applied to the IDT. The theoretical $-17/12$ result is shown as a line across the plot. Many outlying points in the region 100 mW to 300 mW exhibit steeper spectra associated with finite size effects, which dominate the shallower capillary-wave turbulent power law. Two such outliers, marked (a) and (b), are plotted in the inset.

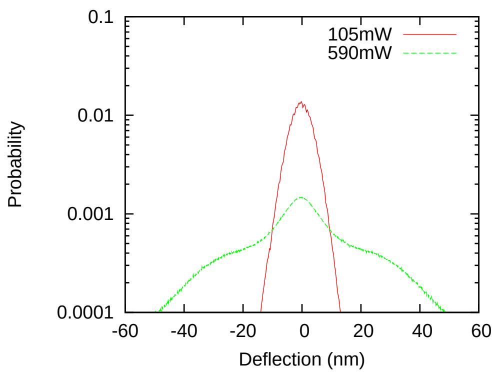

Figure 6: Probability density functions of timeseries for a weak-turbulent surface (excitation power 105 mW) and a non-weak-turbulent surface (excitation power 590 mW). The turbulent function is almost exactly Gaussian (note the logarithmic ordinate axis), while the non-turbulent function is significantly distorted at the tails.

The presence of these very large waves could be indicative of atomisation, where very large surface waves can pinch off, ejecting smaller droplets from the larger body of fluid. If we consider the system to consist of only the main body of fluid, the ejection of such droplets will remove energy from the system and therefore constitute an apparent damping. If the length scale at which the energy is removed (that is, the length scale of the unstable large wave) lies in the region of the turbulent cascade, this introduces a damping effect unaccounted for in the weak turbulence theory.

CONCLUSIONS

Using surface acoustic wave excitation and laser Doppler vibrometry, we have succeeded in observing wave turbulence at the micro-scale, over a wide band of frequencies up to 1 MHz. Our observations have opened a number of avenues for further investigation in this area: the process by which 19.5 MHz excitation energy excites a resonance on the order of 100 Hz, contrary to the predictions of Faraday wave theory, allowing energy to pass into higher frequencies and enter the cascade; the apparent decay in gradient at high excitation energy; and the appearance of very large waves at high excitation energy all demand further investigation.

The generation of the low-frequency resonance is of great importance not only due to its effect on the turbulent cascade, but also due to the many low-frequency phenomena associated with SAW excitation over which it undoubtedly has influence. As discussed above, the apparent decay in turbulence at large excitation levels, and the associated presence of very large waves, would seem to be related to the phenomenon of atomisation.

REFERENCES

[1] M. Y. Brazhnikov, G. V. Kolmakov, A. A. Levchenko, and L. P. Mezhov-Deglin. Observation of capillary turbulence on the water surface in a wide range of frequencies. *Europhysics Letters*, 58(4):510–516.

[2] E. Falcon, S. Fauve, and C. Laroche. Observation of intermittency in wave turbulence. *Physical Review Letters*, 98(15):154501, 2007.

[3] E. Falcon, C. Laroche, and S. Fauve. Observation of gravity-capillary wave turbulence. *Physical Review Letters*, 98(9):094503, 2007.

[4] M. Faraday. On a peculiar class of acoustical figures; and on certain forms assumed by groups of particles upon vibrating elastic surfaces. *Philosophical Transactions of the Royal Society of London*, 121:299–340, 1831.

[5] U. Frisch. Turbulence: The legacy of A. N. Kolmogorov. 1995.

[6] H. Li, J. R. Friend, and L. Y. Yeo. Surface acoustic wave concentration of particle and bioparticle suspensions. *Biomedical Microdevices*, 9:647, 2007.

[7] J. Miles. On Faraday waves. *Journal of Fluid Mechanics*, 248:671–683, 1993.

[8] A. Pushkarev and V. Zakharov. Turbulence of capillary waves - theory and numerical simulation. *Physica D*, 135 (1-2):98–116, 2000.

[9] A. Qi, L. Y. Yeo, and J. R. Friend. Interfacial destabilization and atomization driven by surface acoustic waves. *Physics of Fluids*, 20(7):074103, 2008.

[10] R. Shilton, M. K. Tan, L. Y. Yeo, and J. R. Friend. Particle concentration and mixing in microdrops driven by focused surface acoustic waves. *Journal of Applied Physics*, 104:014910, 2008.

[11] M. K. Tan, J. R. Friend, and L. Y. Yeo. Microparticle collection and concentration via a miniature surface acoustic wave device. *Lab on a Chip*, 7:618, 2007.

[12] W. B. Wright, R. Budakian, D. J. Pine, and S. J. Putterman. Imaging of intermittency in ripple-wave turbulence. *Science*, 278(5343):1609–1612, 1997.

[13] V. E. Zakharov and N. N. Filonenko. Weak turbulence of capillary waves. *Journal of Applied Mechanics and Technical Physics*, 8(5):37–40, 1967.