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Ranking energy paths in a SEA model

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ABSTRACT

A link between graph theory and statistical energy analysis (SEA) has been recently established. This allows resorting to the former to solve many issues related to energy transmission paths in SEA models. In this work, we benefit from this connection to implement an algorithm for ranking the set of K maximum energy transmission paths from a source subsystem to a target subsystem, in a SEA model. Problems arising if the stochastic nature of loss factors was to be incorporated in the computation of paths are also outlined. The algorithm can prove very useful for the noise control engineer. For instance, knowing whether energy transmission between sources and targets in a SEA system is drawn by a limited set of paths or not, can be helpful to determine noise control treatments. Moreover, it is at the core of the existence of transmission loss regulations between dwellings.

INTRODUCTION TO SEA GRAPHS

Traditionally, a statistical energy analysis (SEA) model of a physical system has been represented by means of a set of boxes standing for subsystems, which are linked by arrows accounting for power interchange. Additional arrows are used to represent external power input and subsystem internal dissipated power [1]. For the example in Figure 1a, which shows a given configuration of connected plates, the corresponding SEA model would look like the block diagram in Figure1b.

On the other hand, a graph consists of a pair $\mathscr{G} = (\mathscr{U}, \mathscr{E})$ with $\mathscr{U} = \{u_1, u_2, \ldots, u_N\}$ standing for the set of *nodes* and E for the set of edges $(u_i, u_j) \in \mathscr{E} \subset \mathscr{U} \times \mathscr{U}$. We consider ordered edges $(u_i, u_j) \neq (u_j, u_i)$ in which case $\mathscr{G}$ is known as a directed graph or *digraph*. A graph can be weighted or labelled with a *path algebra* [2], which basically consists in assigning a given number or symbol w_{ij} to each edge (u_i, u_j) and defining two sets of binary operations between them. Having a look at the plate model in Figure 1a, one may think of assigning a weighted graph to it that it could look somewhat as that in Figure 2. Comparison of the SEA block diagram in Figure 1b with the graph in Figure 2b naturally accounts for the following questions:

- Is it possible to define a SEA graph? Which weights should be assigned to it?
- If this was the case, could SEA graphs be useful for noise control engineering purposes? That is to say, is it possible to benefit from developments in graph theory to help the acoustic engineer?

In [3] a positive answer was given to these questions. It was shown that a natural and convenient way to build a SEA graph was by identifying its adjacency matrix with the generating matrix of the SEA system series solution. We remind that the weighted adjacency matrix of a graph has entries

$$\mathscr{A}(i,j) = \begin{cases} w_{ij} & \text{if} \quad (u_i, u_j) \in \mathscr{E} \\ \emptyset & \text{if} \quad (u_i, u_j) \notin \mathscr{E}, \end{cases} \tag{1}$$

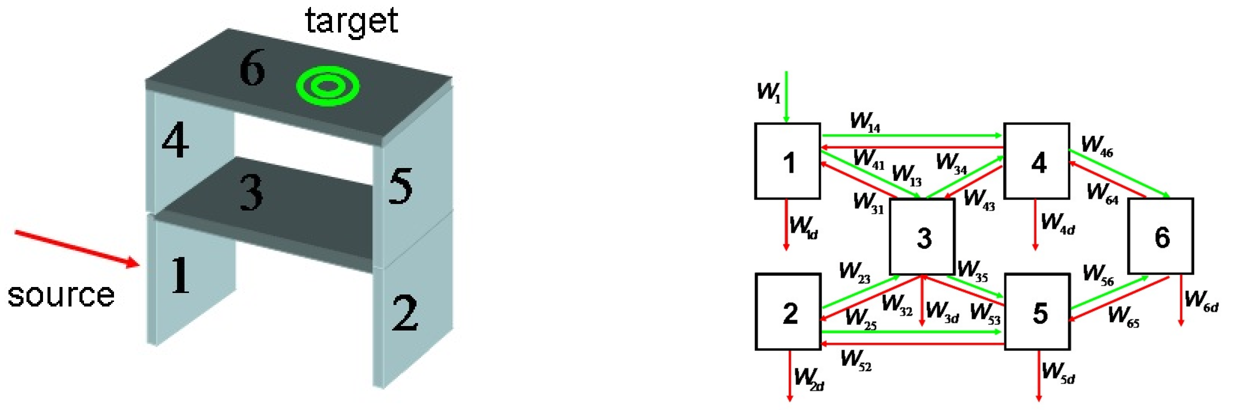
which allows to represent any weighted graph by means of a squared matrix. For a SEA system with loss factor matrix $\mathscr{H}$, the subsystem energy vector $\boldsymbol{E}$ for a given external energy vector input $\boldsymbol{E}_0$ can be found as $\boldsymbol{E} = \mathscr{H}^{-1}\boldsymbol{E}_0$. $\mathscr{H}^{-1}$ can be computed from the Neumann series $\mathscr{H}^{-1} = (I - S)^{-1} = \sum_{k=0}^{\infty} S^k$, where S stands for the generating matrix of the series, having entries $S(i,j) = \eta_{ji}/\eta_i(1 - \delta_{ij})$ [4]. As usual, η_{ij} stand for the coupling loss factors, whereas η_i stands for the total loss factor of subsystem i ($\eta_i = \eta_{id} + \sum_j \eta_{ij}$, η_{id} being the dissipation loss factor).

It is then possible to define a SEA graph $G_{\text{SEA}} = (U_{\text{SEA}}, E_{\text{SEA}})$ with adjacency matrix given by

$$\mathscr{A}_S(i,j) = \begin{cases} \eta_{ji}/\eta_i & \text{if} \quad (u_i, u_j) \in E_{\text{SEA}} \\ 0 & \text{if} \quad (u_i, u_j) \notin E_{\text{SEA}}. \end{cases} \tag{2}$$

The powers of $\mathscr{A}_S \equiv S$ provide very useful information on the SEA system. For example, the element $S^k(i,j)$ gives the energy contribution of all paths of order k linking subsystems j and i [4, 5]. If instead of the "natural" SEA graph induced by S, one considers different algebras and weights but keeping the same graph connectivity, further results can be obtained on the existence and number of paths of a given order. Extremal (maximum and minimum) kth order paths can also be identified in this way [3]. Moreover, once the link between graph theory and SEA has been established, many possibilities arise. For instance, it was proposed in [3], and further extended in [6], to make use of graph cut algorithms to diminish the energy transmitted from a set of source subsystems to a set of target subsystems in a SEA model, with the sole modification of a few number of system loss factors. This constitutes an alternative and/or complement to more standard approaches to solve the problem based on optimization plus sensitivity analysis techniques [7, 8].

In this work we will focus on another SEA path problem pointed out in [9], where the necessity to find efficient algorithms for identifying dominant energy transmission paths between sources and targets in a SEA model was expressed. It has been recently shown in [10] that it is possible to tackle this problem by resorting again to graph theory. The idea is to perform appropriate modifications to an algorithm originally intended to solve the so called *standard K shortest path problem* in graphs. In particular, the MPS algorithm [11] that relies on the computation



(a) System of connected plates (b) SEA model for connected plates

Figure 1: Physical system and corresponding SEA model

of the so called *deviation paths* has been used to rank the set of K dominant energy transmission paths between a source and a receiver in a SEA model. In this paper, we will review the main results in [10], and comment on the possibilities and difficulties that will emerge if the random nature of graph weights in $\mathscr{A}_S$ is to be taken into account. Some rather straightforward options such as making use of the Hurwicz principle or computing path uncertainties may suffice for engineering purposes. However, if the random nature of graph weights is to be incorporated in the computation path algorithm, the main difficulty will come from the fact that Bellman's principle of optimality will no longer be satisfied.

The paper is organized as follows. In section 2 the problem to be solved will be stated. The solution proposed in [10] will be reviewed in section 3 and applied to the SEA system in Figure 1. In section 4 some strategies and problems arising when dealing with stochastic weights will be pointed out. A more realistic example consisting of vibroacoustic transmission between adjacent dwellings in a building will be considered to illustrate some situations. Conclusions close the paper in section 5.

STATEMENT OF THE PROBLEM

Let us remember that a pth order path between a source node s and a target node t in a graph consists in the edge sequence $p_{st} := \left\{ \left(s, u_{h_1}\right), \left(u_{h_1}, u_{h_2}\right), \dots \left(u_{h_{p-1}}, t\right) \right\}$. If G_{SEA} is weighted with the path algebra $\mathfrak{P}_2 \equiv \left(\mathbb{R}_0^+, \max, \cdot\right)$ and has the adjacency matrix $\mathscr{A}_S$ in (2), it follows that the weight of a path will be given by

$$w\left(p_{st}\right) = \prod_{m=0}^{p-1} w_{h_m h_{m+1}} = \prod_{m=0}^{p-1} \frac{\eta_{h_m h_{m+1}}}{\eta_{h_{m+1}}} \equiv \prod_{p} w_{st}, \tag{3}$$

where the equivalence has been introduced to ease the notation. Note that (3) is nothing but the standard Craik definition of an

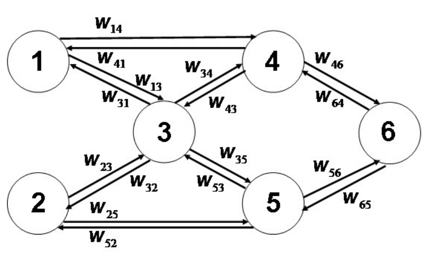


Figure 2: SEA graph for connected plate system?

energy transmission path [5]. The weight of two concatenated paths p_{sj} and q_{jt} will be given by

$$w\left(p_{sj} \circ q_{jt}\right) = w\left(p_{sj}\right) w\left(q_{jt}\right) = \prod_{p} w_{sj} \prod_{q} w_{jt} = \prod_{p+q} w_{st}, \tag{4}$$

and the join operation of the algebra, $\vee \equiv \max$, for a pair of paths returns that with maximum weight,

$$w\left(p_{st}\right) \vee w\left(q_{st}\right) = \max\left\{w\left(p_{st}\right), w\left(q_{st}\right)\right\}. \tag{5}$$

The above results can be generalized for any paths in the graphs although they have been particularized here for paths linking the source and the target. Note also that a path in G_{SEA} will obviously contain loops.

Let us denote by $\mathscr{P}_{st}$ the set of all paths joining s and t. This set will never be empty given that a SEA graph is strongly connected provided no coupling loss factor is neglected in the model. Then, the problem of ranking the set of, say, K main energy transmission paths in a SEA model can be stated as follow:

Problem 1 *Given a source subsystem s and a target subsystem t in the SEA graph G_{SEA}, we aim at finding the set of K paths $\mathscr{P}_{st,K} = \left\{p_{st,1}, p_{st,2} \dots p_{st,K}\right\} \subseteq \mathscr{P}_{st}$ such that*

- $w(p_{st,k}) \geq w(p_{st,k+1})\ \forall k \in \{1, \dots K-1\}$,
- $w(p_{st,K}) \geq w(q_{st})\ \forall q_{st} \in \mathscr{P}_{st} - \mathscr{P}_{st,K}$,
- *$p_{st,k}$ is found just before $p_{st,k+1} \forall k \in \{1, \dots K-1\}$.*

The following results guarantee the well-posedness of Problem 1, in the sense that a finite solution can be found for it.

Theorem 1 *Consider a SEA digraph $G_{\text{SEA}} = (U_{\text{SEA}}, E_{\text{SEA}})$ with adjacency matrix given by the generating matrix $\mathscr{A}_S$ of the Neumann series solution of the SEA system. There exists a finite and maximum energy transmission path from the source s to the target t in G_{SEA}.*

Proof. The proof is given in [10].

Corollary 1 *The problem of finding the $K \in \mathbb{Z}^+$ dominant paths in the SEA graph G_{SEA} is finite.*

Proof. It is a direct consequence of Theorem 1.

RANKING THE SET OF K MAIN ENERGY TRANSMISSION PATHS

Trying to solve Problem 1 making use of the series development for the subsystem energy vector, i.e., using the kth powers of the SEA graph adjacency matrix is clearly inefficient. If the path algebra $\mathfrak{P}_2 \equiv \left(\mathbb{R}_0^+, \max, \cdot\right)$ is used, the powers of the adjacency matrix $\mathscr{A}_S^k$ will provide the maximum transmission paths of order k, linking any pair of subsystems in the SEA model. That is to say the entry $A_S^k(i,j)$ will contain the k-th order dominant path from j to i. However, it is clear that if one proceeds accordingly, several important paths will be missing. For instance, it could easily happen that the third dominant energy transmission path of second order in a SEA model transfers more energy than the main transmission path of third order. Consequently, to avoid disregarding any path one should compute all paths up to a large enough k and then sort them. This is a clearly inadequate way to proceed, as already quoted in [9], given that for large k the number of path can become prohibitive.

It has been recently proposed in [10] that Problem 1 can be efficiently solved by introducing some modifications to the so called MPS algorithm [11]. This algorithm makes use of the concept of deviation paths, originally introduced in [12]. Let us next give a general overview on how the algorithm performs (the reader is referred to [10] and references therein for full technical details).

The adapted MPS algorithm works on the tree $\mathscr{T}_t^* = (U_{\text{SEA}}, E_t^*)$ rooted at t that contains all maximum transmission paths from the source subsystem to the target subsystem. This tree can be easily computed by means of slight modifications to well-known algorithms, such as the celebrated Dijkstra algorithm [13]. The maximum energy transmission path from any node i to the target t will be designated by p_{it}^* and its weight by $\pi_i \equiv w(p_{it}^*)$. The algorithm starts choosing the maximum energy path from the source to the target $p_1 \equiv p_{st}^* \in \mathscr{T}_t^*$, and proceeds to build a set $\mathscr{X}$ of possible candidates to be the second dominant transmission path p_2. These candidates are computed from deviation paths of p_1. Given the finite pth order path from source to target $p_{st} := \left\{ \left(s, u_{h_1}\right), \left(u_{h_1}, u_{h_2}\right), \dots \left(u_{h_{p-1}}, t\right) \right\}$, the qth order path $q_{st} := \left\{ \left(s, v_{h_1}\right), \left(v_{h_1}, v_{h_2}\right), \dots \left(v_{h_{q-1}}, t\right) \right\}$ is termed a deviation path from p_{st} if the following fulfils

- the subpath $q_{st} \supset q_{sh_j} = p_{sh_j} \subset p_{st}$ for $j < p,\ j < q$,
- the first arc of the subpath $q_{h_j t}$ is different from the first arc in the subpath $p_{h_j t}$, i.e., $(u_{h_j}, u_{h_{j+1}}) \neq (v_{h_j}, v_{h_{j+1}})$,
- the subpath $q_{h_{j+1}t} = q_{h_{j+1}t}^* \in \mathscr{T}_t^*$, i.e., $q_{h_{j+1}t}$ is the maximum energy transmission path from the node $v_{h_{j+1}}$ to the target t.

The node $u_{h_j} = v_{h_j}$ is known as the deviation node of q_{st} and $(v_{h_j}, v_{h_{j+1}})$ as the deviation arc of q_{st} from p_{st}.

In order to build the deviation paths from p_1, the source node s is first considered as a deviation node. Then, the weights of all deviation paths $(u_s, u_{h_j}) \circ p_{h_j t}^* \ \forall \ (u_s, u_{h_j}) \in \mathscr{E}_s^-$ are computed and the one with maximum weight is stored in $\mathscr{X}$ ($\mathscr{E}_s^-$ stands for the adjacent nodes of s whose edges are such that s is their tail node). The subsequent node in p_1 is next considered as a deviation node. The weights of all paths $p_{sh_1} \circ (u_{h_1}, u_{h_j}) \circ p_{h_j t}^* \ \forall \ (u_{h_1}, u_{h_j}) \in \mathscr{E}_{h_1}^-$ are computed and the one having maximum weight is also saved in $\mathscr{X}$. The process continues until all nodes in p_1 have been analyzed. Then, the maximum path in $\mathscr{X}$ is selected to be the second dominant path in the SEA graph, p_2, and becomes removed from $\mathscr{X}$. To find the third maximum transmission path in the SEA model we proceed computing all deviation paths from p_2. The third dominant energy transmission path p_3 is selected as the maximum path in $\mathscr{X}$, which now contains all previously stored deviation paths from p_1 and all deviation paths from p_2. The process is iterated until the K maximum energy transmission paths have been obtained. The MPS algorithm turns to be very efficient and has a theoretical computational cost $\mathscr{O}(|E| \log N + KN)$, with N denoting the number of subsystems in the SEA graph and $|E|$ the number of edges [14].

The time expended in the comparison of deviation paths can be strongly decreased with the use of edge *reduced weights* and by sorting the graph edge set E_{SEA} in the *forward star form* [11]. For the SEA problem under consideration, the reduced weight of an edge $\bar{w}_{ij}$ can be defined as

$$\bar{w}_{ij} := \frac{\pi_j}{\pi_i} w_{ij}. \quad (6)$$

$\bar{w}_{ij}$ could be understood as the additional energy attenuation that a path from u_i to t will experience if $u_j \notin p_{it}^*$. This is so given that if $u_j \in p_{it}^*$, then $\pi_i = w_{ij}\pi_j$ and $\bar{w}_{ij} = 1$.

Reduced weights can be shown to satisfy the following interesting relations (see [10]):

$$\bar{w}_{ij} = 1 \ \forall \ (u_i, u_j) \in E_{\text{SEA}} \cap E_t^*, \quad (7)$$

$$\bar{w}_{ij} \leq 1 \ \forall \ (u_i, u_j) \in E_{\text{SEA}}, \quad (8)$$

$$w(p_{st}) \geq w(q_{st}) \text{ iff } \bar{w}(p_{st}) \geq \bar{w}(q_{st}). \quad (9)$$

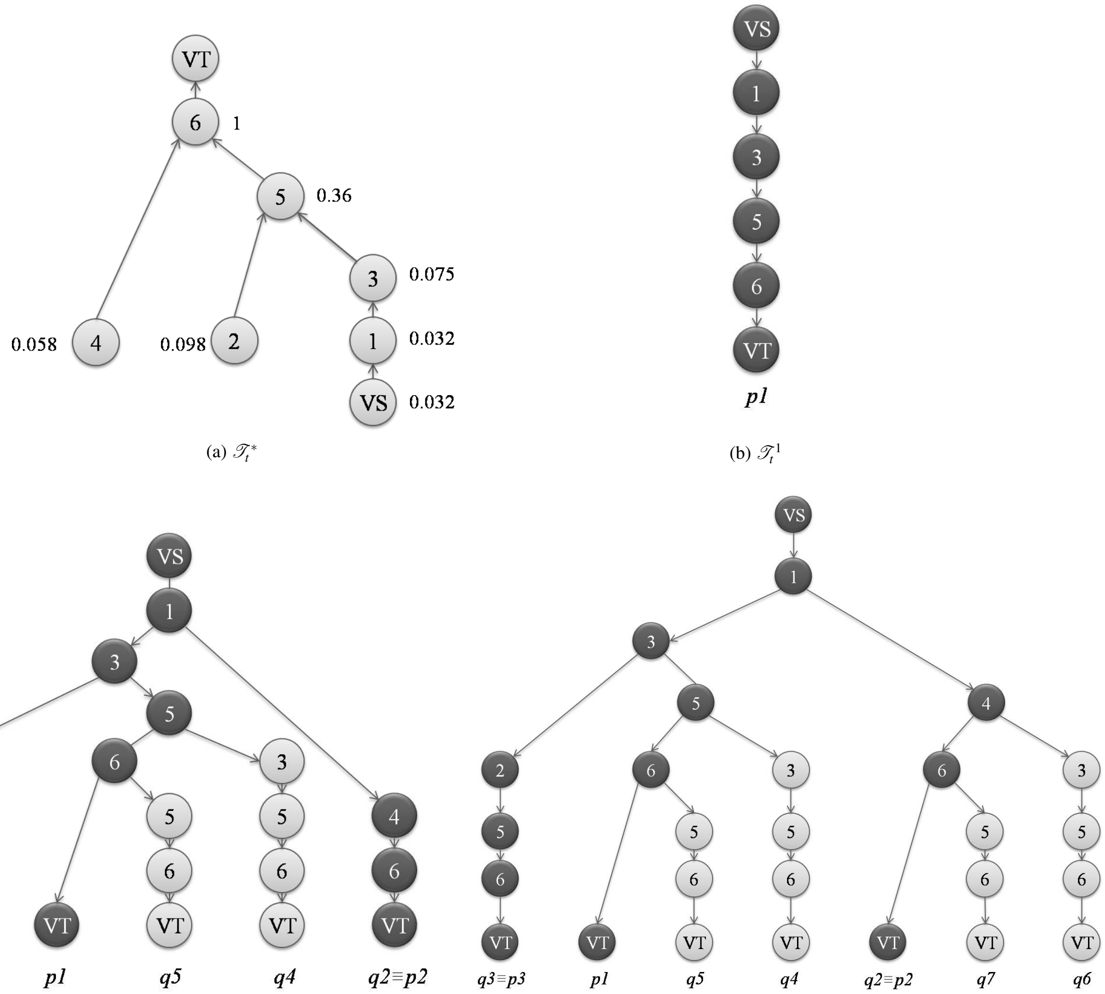
Note that (9) expresses the fact that if a path p_{st} with weight $w(p_{st})$ transmits more energy than a path q_{st} with weight $w(q_{st})$ (i.e., $w(p_{st}) \geq w(q_{st})$) then it is also true that $\bar{w}(p_{st}) \geq \bar{w}(q_{st})$. So one can sort deviation paths either by considering their weights or their reduced weights. However, the latter is more advantageous given that it is quite straightforward to see that the reduced weight of a deviation path $\bar{w}[p_{sh_{j-1}} \circ (u_{h_{j-1}}, u_{h_j}) \circ p_{h_j t}^*] = \bar{w}(p_{st})\bar{w}[(u_{h_{j-1}}, u_{h_j})]$ (see [10]). Consequently, it will suffice to compare the deviation arc reduced weights $\bar{w}[(u_{h_{j-1}}, u_{h_j})]$ to obtain the maximum deviation path of a given deviation node. Moreover, only one deviation arc per deviation node needs to be considered if the graph edge set is sorted in the forward star form. This basically consists in factorising E_{SEA} as $E_{\text{SEA}} = \cup_{i=1}^N \mathscr{E}_i^-$, with the tail node u_i being smaller than u_{i+1}. The edges in every $\mathscr{E}_i^-$ are then rearranged, the edge having maximum reduced weight being positioned the first in the list. This edge will be the only one considered from all u_i deviation arcs, so there is no need to evaluate any further edge in $\mathscr{E}_i^-$.

Table 1: Loss factors and modal densities used for the SEA graph in Figure 2.

η_{13}	0.086	η_{31}	0.081	η_{1d}	0.057	n_1/n_1	1
η_{14}	0.045	η_{41}	0.051	η_{2d}	0.057	n_2/n_1	1
η_{23}	0.085	η_{32}	0.080	η_{3d}	0.055	n_3/n_1	1.06
η_{25}	0.045	η_{52}	0.051	η_{4d}	0.045	n_4/n_1	0.88
η_{34}	0.064	η_{43}	0.077	η_{5d}	0.026	n_5/n_1	0.88
η_{35}	0.065	η_{53}	0.079	η_{6d}	0.055	n_6/n_1	1.06
η_{46}	0.011	η_{64}	0.009				
η_{56}	0.094	η_{65}	0.076				

The whole process can be illustrated building a pseudo tree $\mathscr{T}_t^k$ of deviation paths for each kth iteration. The first pseudo tree $\mathscr{T}_t^1$ will only contain the maximum energy transmission path p_1 from the s to t. The second pseudo tree $\mathscr{T}_t^2$ will represent p_1 plus all its maximum deviation paths, from which p_2 will be selected. $\mathscr{T}_t^3$ will show p_1, plus its deviation paths, plus the deviation paths from p_2, and so on.

As an example, we have considered the SEA graph in Figure 2 (associated to the connected plate model in Figure 1a) with



(a) $\mathscr{T}_t^*$ (b) $\mathscr{T}_t^1$ (c) $\mathscr{T}_t^2$ (d) $\mathscr{T}_t^3$

Figure 3: Tree $\mathscr{T}_t^*$ of maximum energy transmission paths to the target and pseudo trees $\mathscr{T}_t^k$, $k = 2...3$.

arbitrary loss factors and normalized modal densities in Table 1. In Figure 3a, the tree $\mathscr{T}_t^*$ of maximum energy transmission paths from the source subsystem $s \equiv u_1$ (plate 1) to the target subsystem $t \equiv u_6$ (plate 6) is plotted. Numerical values for the weights of the maximum paths connecting each node with the target are given besides each vertex. The maximum energy path from $\mathscr{T}_t^*$ is chosen as the first pseudo tree, $\mathscr{T}_t^1$ (see Figure 3b). Note that this path is the third order transmission path $p_1 = \{(s,u_3),(u_3,u_5),(u_5,t)\}$ and not the second order path passing through plate 4. In fact, this is the second dominant path $p_2 = \{(s,u_4),(u_5,t)\}$ as observed from the second pseudo tree $\mathscr{T}_t^2$ in Figure 3c. In Figure 3d we have plotted the third pseudo tree $\mathscr{T}_t^3$, from which the third dominant path p_3 is selected. This is the fourth order path $p_3 = \{(s,u_3),(u_3,u_2),(u_2,u_5),(u_5,t)\}$. Note that paths highlighted in dark grey in Figure 3 belong to the set of dominant paths. In particular, $\mathscr{T}_t^3$ contains p_1, p_2 and p_3. The weights of these paths, which correspond to the energy ratios $(E_6/E_1)_{p_i}$ (i.e., energy ratio corresponding to the energy transmitted through p_i) have values $(E_6/E_1)_{p_1} = 0.032$, $(E_6/E_1)_{p_2} = 0.016$ and $(E_6/E_1)_{p_3} = 0.009$.

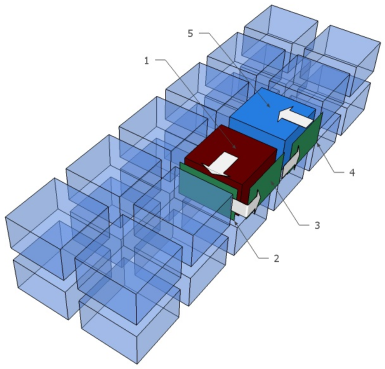
Finally, observe that virtual source VS and target VT nodes have been added to the pseudo trees and maximum transmission tree in Figure 3. Virtual nodes have no influence in the final results because they are linked to s and t through edges of unit weight, but they are necessary for the good performance of the algorithm [10, 11].

SEA GRAPH WITH STOCHASTIC WEIGHTS

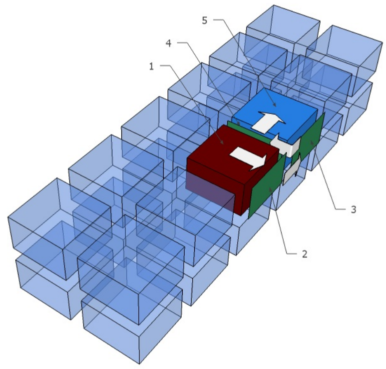
The algorithm described in the previous section considers deterministic weights, i.e, each edge has been assigned a constant value, $w_{ij} = \eta_{ji}/\eta_i$, and it is assumed that w_{ij} exactly represents the connection between subsystems j and i. In graph theory, finding extremal paths in the deterministic case is sometimes referred to as solving the *standard* extremal path problem, to discriminate from w_{ij} being a random variable described by means of a probability density function (pdf). Given that SEA is a statistical approach, the loss factors in the SEA model will correspond to averaged values according to the randomness of the parameters involved in their computation (see [1] for details on the sources of uncertainty in SEA). As known, the subsystem energy vector $\boldsymbol{E} = \mathscr{H}^{-1}\boldsymbol{E}_0$ corresponds to a mean energy value and much efforts have been placed to compute its variance (see e.g., [15] and references therein for recent results on the subject).

Consequently, it is licit to wonder about the reliability of the ranking of paths output by the MPS algorithm, given that the SEA graph weights do not actually correspond to deterministic values, but to stochastic ones. Several strategies of increasing complexity can be followed to address this topic (see e.g., [16] in the general context of graph theory).

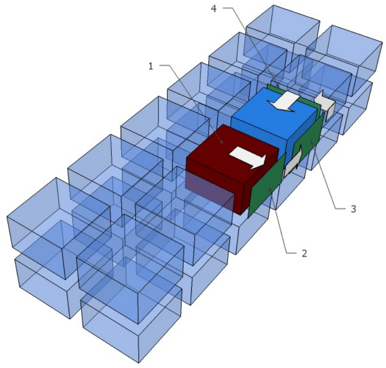
First of all, let us focus on the meaning of the MPS path ranking



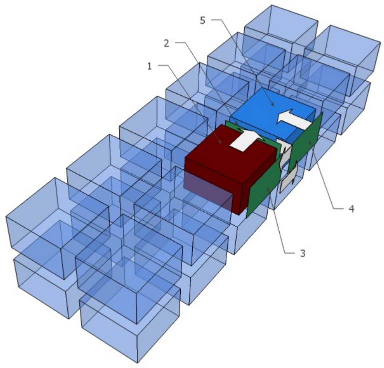
(a) Path I



(b) Path J



(c) Path K



(d) Path L

Figure 4: Some energy transmission paths between source room (red) and target room (blue).

results when random weights are considered. Assume that each weight w_{ij} of an edge in a path is described by a pdf $f_{ij}(w_{ij})$ and that all w_{ij} are independent random variables. Let us denote by $\langle w_{ij} \rangle$ the expected value of w_{ij}. A pth order path p_{st} linking the source and the target will be described by the joint pdf $f_{p_{st}} = \prod_p f_{ij}(w_{ij})$ and its weight expected value will be given by

$$\langle w(p_{st}) \rangle := \prod_p \langle w_{st} \rangle . \quad (10)$$

Therefore, under the assumption of independent random variables for the SEA graph weights, the outcome of the MPS algorithm is to find the set of K paths with largest expected value but without taking into account any deviation (i.e., neglecting the stochastic nature of weights when concatenating paths).

One option to gain some knowledge on the path ranking reliability may consist in applying Hurwicz's principle (see e.g., [16]). If we have an idea of the limiting values for the loss factors, we can compute the edge weights corresponding to the most favorable (less energy transmission) and (maximum energy transmission) worst cases, w_{ij}^{up} and w_{ij}^{low}. This can be done by respectively diminishing the coupling loss factors to their minimum possible values, as well as increasing the dissipation loss factors to their maximum possible values, and viceversa [6]. The following weight is assigned to each edge in E_{SEA}

$$\alpha w_{ij}^{low} + (1-\alpha) w_{ij}^{up}, \quad \alpha \in [0,1] . \quad (11)$$

Then, the problem to be solved is that of first finding the path such that

$$p_1^{\alpha} = \arg \max_{p_{st} \in \mathscr{P}_{st}} \left[\alpha w^{low}(p_{st}) + (1-\alpha) w^{up}(p_{st}) \right] , \quad (12)$$

and then proceed analogously for the remaining $K-1$ ones. Note that $\alpha = 1$ corresponds to the best case and $\alpha = 0$ to the worst one. Consequently, in practice on could apply the MPS algorithm three times to account for the expected value (EV) and the $\alpha = 0, 1$ cases.

The latter has been done for the more or less realistic benchmark case tested in [10]. This consisted in computing energy transmission paths between two adjacent dwellings in a building. The building was made of 24 identical rooms distributed between two floors. The reader is referred to [10] for modeling details such as geometric and material characteristics. An overall of $K = 500$ paths were computed showing that the first 20 dominant paths already transmitted 90% of the energy in the reception room (hence justifying the existence of noise regulations based on direct anf first order flanking paths between adjacent dwellings).

The first 15 energy transmission paths have been computed according to (12) for the EV and $\alpha = 0, 1$ cases. The paths have been labelled from A to O according to their ranking in the EV results. In Figure 4, we have plotted the paths I, J, K and L from the source room (red colour) to the receiever room (blue color) (i.e., the paths corresponding from the ninth to the twelfth dominant paths). In Figure 5 the ranking of paths is shown for the three cases being analyzed. The center column contains the classification for the EV case, whereas the left column corresponds to the best situation ($\alpha = 1$), and the right column to the worst one ($\alpha = 0$). As observed, the first eight dominant paths remain identical for the three cases. However, for $\alpha = 1$ paths L and K become interchanged with respect to EV, while for $\alpha = 0$ the interchanged paths are I and J. Moreover, two new paths (P and Q) appear and path M becomes relegated to the 15

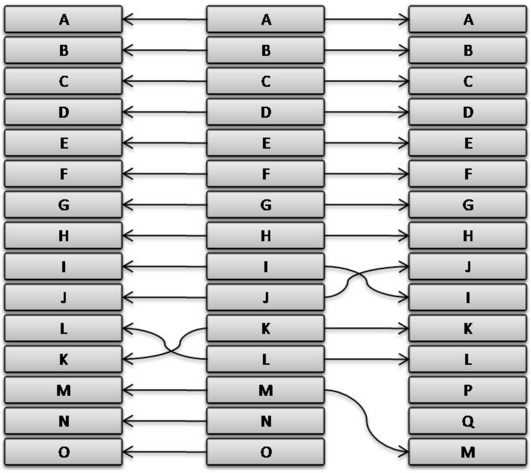


Figure 5: Ranking of energy transmission paths. (left column) Best case $\alpha = 1$ (center column) Expected value (right column) Worst case $\alpha = 0$

position. The weights for $\alpha = 0,1$ cases have been computed assuming variations of $\pm$2dB in the involved loss factors.

Another way to check the reliability of the ranked paths is that of evaluating the uncertainty of their weights. In [9], the variance of a path was expressed in terms of the variance of its coupling and total loss factors. Consider the expression in (3) for the weight of a path p_{st} from source to receiver. Given that the path can have loops, let us rewrite its weight as

$$w(p_{st}) = \prod_{p!} w_{h_m h_{m+1}}^{\alpha_{m+1}} = \prod_{p!} \left(\frac{\eta_{h_m h_{m+1}}}{\eta_{h_{m+1}}} \right)^{\alpha_{m+1}}, \quad (13)$$

where α_{m+1} stands for the multiplicity of each edge in the path and $p!$ is used to indicate that no terms can be repeated in the product (these are accounted for with the exponents α_{m+1}). From standard uncertainty propagation theory, it follows that the variance σ_g^2 of a function $g(x_1, x_2, \ldots, x_n)$ depending on n variables x_i, $i = 1, n$ is given by $\sigma_g^2 = \sum_n \sigma_i^2 (\partial_i g)^2$. Then it can easily be checked that the variance of the weight of p_{st} fulfills

$$\left(\frac{\sigma_{w(p_{st})}}{w(p_{st})} \right)^2 = \sum_{p!} \alpha_{m+1}^2 \left(\frac{\sigma_{w_{h_m h_{m+1}}}}{w_{h_m h_{m+1}}} \right)^2. \quad (14)$$

Taking into account that $w_{h_m h_{m+1}} = \eta_{h_m h_{m+1}} / \eta_{h_{m+1}}$ we can readily relate the variance of the path weight with the variance of the coupling loss factors and total loss factors. This yields [9]

$$\left(\frac{\sigma_{w(p_{st})}}{w(p_{st})} \right)^2 = \sum \alpha_m^2 \left(\frac{\sigma_{\eta_{h_m}}}{\eta_{h_m}} \right)^2 + \sum \alpha_{m+1}^2 \left(\frac{\sigma_{\eta_{h_m h_{m+1}}}}{\eta_{h_m h_{m+1}}} \right)^2, \quad (15)$$

which expresses the variance of the path as a summation of the total loss factors variances plus the coupling loss factors variances. Notwithstanding, it should be remarked that the total loss factor variances depend in turn on the internal and coupling loss factors, and these on the statistics of the physical parameters involved in their computation. However, should we have some knowledge on the variances of the SEA total and coupling loss factors, we could evaluate the variance of each of the ranked K paths. If the energy of, say, the $K-3$ path, was in the range of the energy of the $K-4$ path minus a certain factor multiplying its variance, then we could only assume a certain degree of validity for their mutual ranking, depending on the pdfs being involved. It is also worthwhile to note that the uncertainty will generally be higher for the longest paths, as they involve a larger number of subsystems.

Although resorting to the Hurwicz principle or computing uncertainties might suffice for noise control engineering purposes, the possibility of directly including randomness in the computation of energy paths could be considered. In graph theory much work has been devoted to this subject since it finds applications in many areas of operational research, from transportation and routing problems to robotics (among many others). The main difficulty of the stochastic extremal path problem stems from the fact that in general, the optimality principle no longer holds. In other words, the optimal path between two nodes is not made of suboptimal paths, which makes most strategies developed for the standard critical path problems useless. Let us illustrate this with a simple toy example.

Consider the graph in Figure 6 with edge weights described by independent random variables. Let us assume that we know the mean value and variance of every edge weight, so that for any path p_{ij} we can compute its mean m_{ij} and variance σ_{ij}^2 (from repeated application of the standard expressions for the product of two independent random variables). Namely, for any two paths p_{ij} and p_{jk}, the mean and variance of the concatenated path $p_{ik} = p_{ij} \circ p_{jk}$ are given by

$$m_{ik} = m_{ij} m_{jk}, \quad (16)$$

$$\sigma_{ik}^2 = \sigma_{ij}^2 \sigma_{jk}^2 + \sigma_{ij}^2 m_{jk}^2 + m_{ij}^2 \sigma_{jk}^2. \quad (17)$$

Next, suppose that we aim at finding a path from $s \equiv u_1$ to $t \equiv u_4$ that maximizes the objective function

$$u(p_{st}) = m_{st} - c\sigma_{st}, \quad (18)$$

with c standing for a constant to be tuned according to the range of intended deviation (we take $c = 1/2$ for simplicity in this example). Maximizing (18) could correspond to asking for the weakest transmission paths resulting from variations of the maximum energy transmission path, to be as large as possible. If use is made of the values in Table 2, the following results are obtained.

Table 2: Mean values and variances for edges in Figure 6.

m_{12}	3.55	σ_{12}^2	0.8
m_{13}	2.1	σ_{13}^2	0.75
m_{32}	2.1	σ_{32}^2	0.75
m_{24}	1	σ_{24}^2	0.2
m_{34}	0.5	σ_{34}^2	0.5

The path that maximizes the objective function (18) results from the concatenation $p_1 \equiv p_{st}^1 = p_{13} \circ p_{32} \circ p_{24}$ that has value $u(p_{st}^1) = 2.642$, to be compared with $u(p_{st}^2) = 2.617$ from path $p_{st}^2 = p_{12} \circ p_{24}$. However, the optimal path from $s \equiv u_1$ to u_2 is $p_{12} \notin p_{st}^1$ with cost $u(p_{12}) = 3.103$, instead of $p_{13} \circ p_{32} \in$

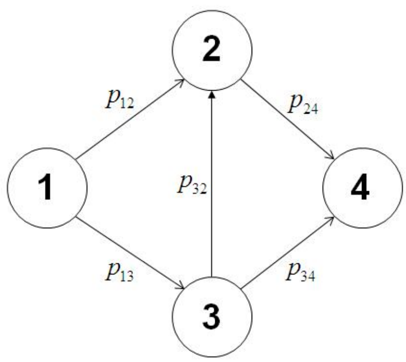


Figure 6: Benchmark graph to show failure of the optimality principle

p^1_{st} with value $u(p_{13} \circ p_{32}) = 3.07$. Consequently, the path p^1_{st} that maximizes the objective function is not made of optimal subpaths.

The objective function (18) has been only intended to illustrate the failure of optimality but, a large variety of alternative mathematical models exist. Probably, the most extended approach to solve stochastic path problems is that of making use of the expected utility criterion of Von Neumann and Morgensten, i.e., the optimal path is the one maximizing the expected value of a given utility function. When dealing with a summation of random variables, which would correspond to dealing with the logarithms of weights in our SEA case (probably at the price of having to deal with positive and negative values), it follows that the resulting problem still satisfies the optimality principle whenever the utility function is either affine linear or exponential. Moreover, for quadratic utilities and/or exponential pdfs, the stochastic problem can be turned into a deterministic, but multidimensional, critical path problem (i.e., vectors are now assigned to graph weights instead of scalars) [16]. Dynamic programming strategies can then be still applied to find the solution by resorting to the concepts of dominance and efficient paths. However, in more general situations this is not possible and many other strategies have been attempted such as extended dominance, convex and quasi-convex optimization, hybrid approaches that combine stochastic and genetic algorithms, etc (see e.g., [17–19] and references therein).

Finally it is worthwhile to mention that whatever strategy could be followed, its success will also rely on the knowledge of the involved SEA statistics. This is not an easy point, given that some simplifying hypothesis which allow to make use e.g., of the very practical Central Limit theorem [20], do not always apply in SEA [21].

CONCLUSIONS

In this work, we have benefit from the connection between SEA and graph theory to efficiently rank the set of dominant energy transmission paths between a source subsystem and a target subsystem, in a SEA model. On the one hand, we have reviewed the basis of a recently developed algorithm to do so, which relies on the notion of deviation paths. On the other hand, several options to take into account the stochastic nature of SEA graph weights in the ranking process have been outlined. These can be used to check the reliability of the ranked paths, and constitute the basis for future work that will directly incorporate the random nature of weights in the ranking process.

Ranking paths in a SEA model is certainly a problem of interest for noise control engineering. For example, if a set of paths can be shown to transmit negligible energy when compared to others in a SEA model, the latter could be simplified in future computations. Moreover, path ranking can be very useful to help determining noise abatement treatments. Besides, many situations exist in which a large percentage of energy is carried out by a finite and rather small number of paths. For instance, this is at the core of transmission loss regulations between adjacent dwellings. Ranking paths algorithms can easily be used to analyze those cases in which these regulations are known to fail.

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