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Short-Interval Period Distributions for Structural Health Monitoring

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ABSTRACT

A structure, for example a building, a bridge etc. has degraded since it was built due to a physical damage, for example an earthquake or time-passing by. Detecting such degrading is much important to predict an accident from security point of view and to maintain a structure from economical point of view. Degrading is detectable as changes in dynamic properties of a structure. It is practical to detect changes in frequency responses of a structure with non-stationary vibrations, e.g. natural force of winds. This issue is considered to be identification of transfer function with a non-stationary input signal. The discrete Fourier transform (DFT) is well used to obtain the spectrum of a response of a structure. The amplitude spectrum by DFT is subject to temporal changes in the non-stationary input signal. On this issue, we have a statistical approach in this article. We introduce short-interval period (SIP) to detect the spectrum of the signal. The SIP is defined as distribution of statistical frequency of dominant frequencies of fractions of measured data. Therefore, the SIP is independent from the magnitude of the unknown signal. This paper shows a theory of the SIP and an application to a scale model experiment. In the experiment, the SIP distribution resulted in stable spectrum independent from a sequence of random numbers as a non-stationary input. The SIP is available for the estimation of the dynamic properties for many types of structures.

INTRODUCUCTION

Detecting degradation of structure, for example a building, a bridge etc. is important for living safety and comfort. In many cases, such degrading does not appear in its appearance. Degradation of buildings due to a physical damage is detectable as changes in dynamic properties of the structure. The properties are evaluated by a transfer function of the structure and those in the properties are estimated by changes in the function. It is, however, impractical to measure the function intentionally.

A structure trembles due to winds, ground motions etc. The tremble data of the structure is considered to be convolution of a transfer function of the structure and vibration sources, which are wind and ground motion etc. If we could extract the transfer function from the tremble data, it's very much practical. However, even if the transfer function of the structure varies much slowly and is assumed to be stable for a short period, tremble data for the structure is also non-stationary because the vibration sources are non-stationary. Therefore, extracting the transfer function from the non-stationary data is the issue.

On this issue, discrete Fourier transform, DFT, is a well-known method for estimating a transfer function. However, a transfer function estimated by DFT is subject to non-stationary of the data to be analyzed. Hirata, one of the authors, proposed a method, that is short-interval period, SIP, for estimating a frequency characteristic from a non-stationary data. This paper shows that SIP is applicable to the issue by a scale-model experiment.

SHORT-INTERVAL PERIOD

Short-Interval Period, SIP is a new method for estimating a frequency characteristic of a transfer function with a non-stationary input. SIP is given as statistical frequency of a dominant frequency for short-interval data. SIP requires a dominant frequency for a signal of short-length. Discrete Fourier Transform, DFT, is a well-known method for analyzing a signal in the frequency domain. But, we do not use the DFT for detecting a short-interval period because of its inherent limitation of frequency resolution(Kay and Marple Jr. 1981).

The dominant frequency or period of a short-interval seqnuence $W(n)(n=0,1,\cdots,M)$ can be given by the non-harmonic Fourier analysis(Hirata 2005,Hirata 2008). In the process of the analysis, we put $W(x)=W(n)$ where $x=n-M/2$, and obtain Fourier coefficients $a(f)$ and $b(f)$ for an arbitrary frequency such that

$$a(f)=\frac{\sum_{x=-\frac{M}{2}}^{\frac{M}{2}} W(x)\sin(2\pi fx)}{\sum_{x=-\frac{M}{2}}^{\frac{M}{2}} \sin^2(2\pi fx)}, \tag{1}$$

$$b(f)=\frac{\sum_{x=-\frac{M}{2}}^{\frac{M}{2}} W(x)\cos(2\pi fx)}{\sum_{x=-\frac{M}{2}}^{\frac{M}{2}} \cos^2(2\pi fx)}. \tag{2}$$

Hence, if we put

$$y(x,f)=a(f)\sin(2\pi fx)+b(f)\cos(2\pi fx) \tag{3}$$

and

$$Y(f)=\sum_{x=-\frac{M}{2}}^{\frac{M}{2}} y^2(x,f), \tag{4}$$

we have the dominant frequency f_p which satisfies

$$Y(f_p) = \text{the maximum of } Y(f). \tag{5}$$

It should be noted that we attain a least-squares fit of $W(x)$ to a sinusoid by $y(x, f_p)$.

The SIP distribution is given by a number of dominant frequencies (or periods) of short sequences which are fractions of measured data. Thus, the normalized SIP distribution $D(f_p)$ gives the probability of the dominant frequency f_p found in measured data.

If we assume that a structure is excited by the force of random noise and assign the frequency f in Eqs.(1) and (2) such that

$$f = f_n = n\Delta f; \quad n = 1, 2, \cdots N \tag{6}$$

where $\Delta f < 1/M$, we have, from the reference(Hirata 2005),

$$D(f_j) - D(f_i) = Q_{ij}(r)\frac{S(f_j) - S(f_i)}{S(f_j) + S(f_i)} \tag{7}$$

where $0 < Q_{ij}(r) < 1$ and $S(f_j)$ is the power frequency response of a structure at a frequency f_j and so on.

Hence, we get an approximation:

$$S(f_n) \approx kD(f_n) \tag{8}$$

where k is an appropriate constant. It should be mentioned that the spectral resolution given by $D(f_n)$ depends little on the length of the short sequence when $Mf_n > 1$.

SCALE-MODEL EXPERIMENT

Scale-model

The scale-model is a framework model of a building of three-storied. Its dimensions are 18 cm (W) * 21 cm (D) * 38 cm (H). The scale-model has four pillars and each pillar is comprised of thin square lumbers. Figure 1 shows that images of the scale-model degrading by reducing the lumbers. Frequency characteristic of the impulse responses at Fs of 6 (kHz) before degrading (condition A) and after degrading (condition B) are shown in Fig. 2. These two responses are unknown in the real world.

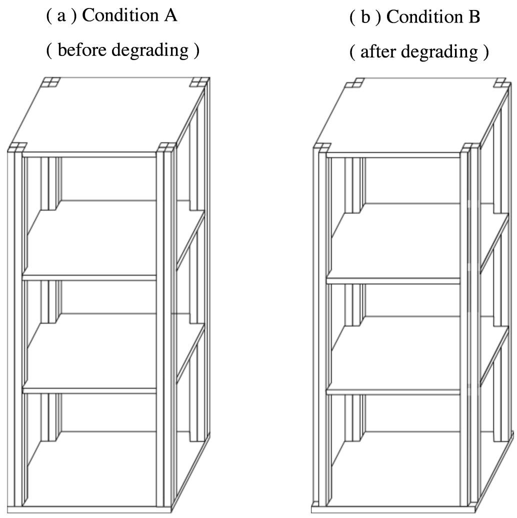

Figure 1: Structure conditions (before degrading and after degrading); Condition A (a) and Condition B (b)

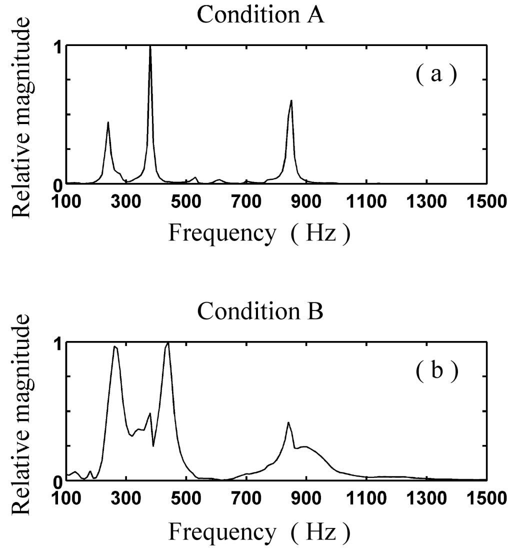

Figure 2: Power spectrum of structure; condition A (a) and condition B (b).

FREQUENCY CHARACTERISTICS ESTIMATION USING STATIONARY VIBRATION

First of all, DFT and SIP analysis were compared by stationary noise vibration for structure frequency charracteristics estimation. Noise signals as stationary vibration were prepared using Matlab function ("randn"). Responses of the scale-model for condition A and condition B, vibrated by the noise signal are simulated by convolution of the two impulse responses and the noise signals, respectively. The two simulated signals of 144,000-sample long are used for frequency chracteristics estimation.

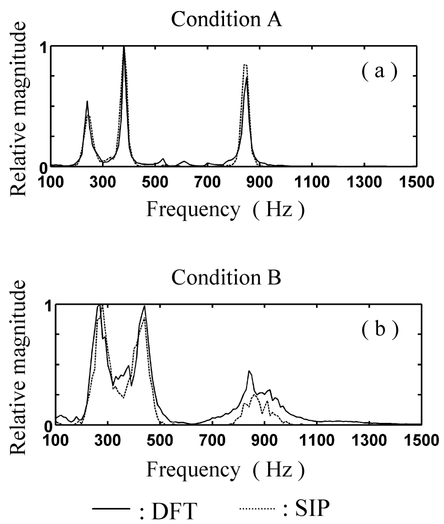

Figure 3: Frequency characteristics estimation using stationary noise vibration; condition A (a) and condition B (b).

DFT analysis

The two simulated signals for condition A and condition B are, respectively, divided into 240 sections. Each is 600-sample long. For each section, DFT over 600 samples are computed. Therefore, frequency resolution is 10 (Hz). Furthermore, frequency characteristic over 240 sections are averaged.

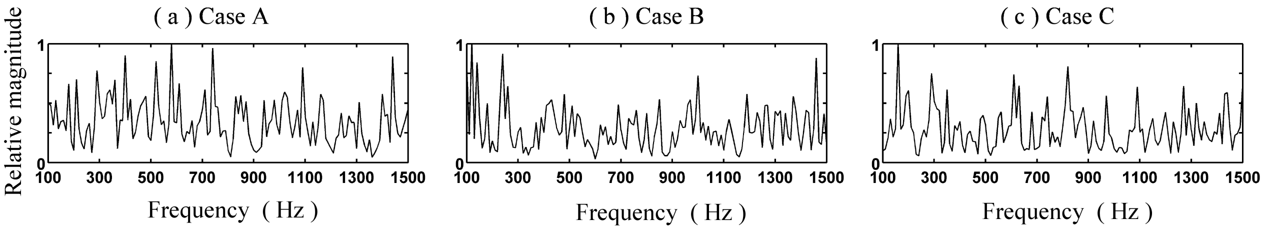

Figure 4: Averaged power spectrum of non-stationary vibrations; case A (a), case B (b) and case C (c).

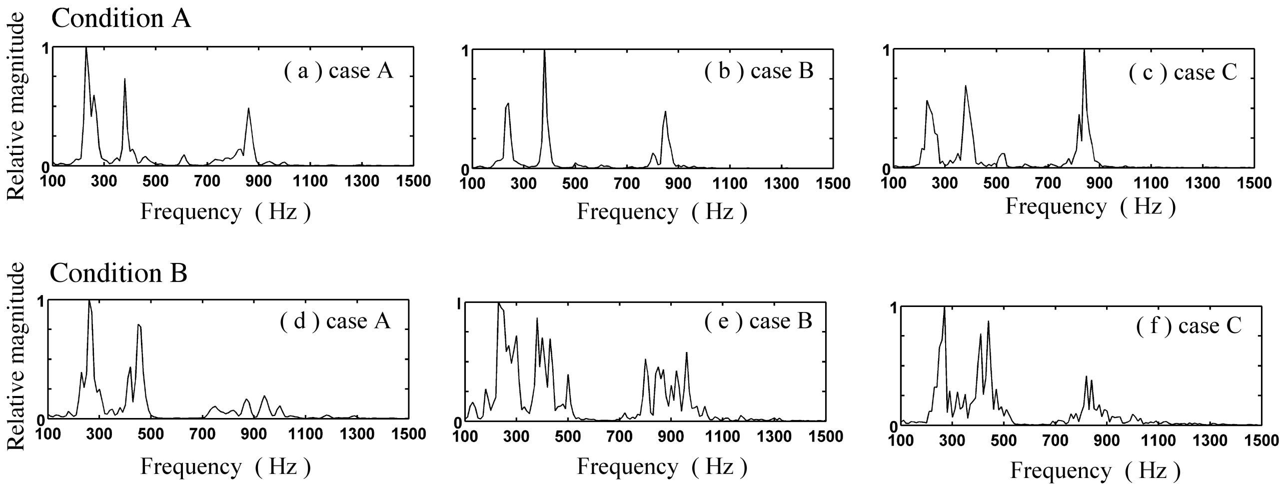

Figure 5: Averaged power spectra of structure condition A and B under non-stationary vaibration culculated by DFT. Condition A excited by non-stationary noise case A (a), case B (b) and case C (c). Condition B excited by non-stationary noise case A (d), case B (e) and case C (f).

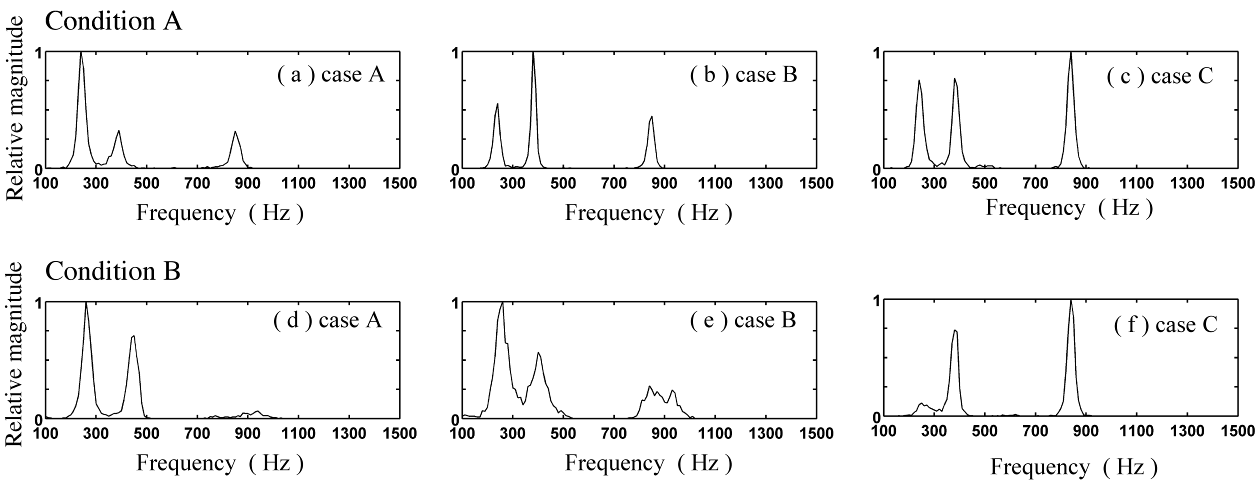

Figure 6: SIP distributions of structure condition A and B under non-stationary vaibration. Condition A excited by non-stationary noise case A (a), case B (b) and case C (c). Condition B excited by non-stationary noise case A (d), case B (e), case C (f).

SIP analysis

The two responses simulated are, respectively, divided into 2,400 sections. Each section is 60-sample long. For each section, a dominant frequency is extracted with frequency resolution of 10 (Hz), by the non-harmonic Fourier analysis. And SIP is given as statistical frequency of the dominant frequencies.

Figure 3 shows that estimated frequency characteristics of the two impulse responses by DFT and SIP analysis under stationry noise vibration. The horizontal axis shows frequency and the vertical axis denotes relative magnitue. Relative magnitude was defined normalized power of each frequency for DFT and normalized distribution for SIP analysis. In condition A, similar frequency characteristics are appeared in DFT and SIP analysis both. These charactercs are also similar to original frequency characteristics of impulse response in Fig. 2. In condition B, DFT has details of impulse frequency characteristic compared with SIP analysis. And, SIP distriburion shows peaks of frequecy characteristics explicitly. In any case, similar frequency characteristics are obtained from stationary noise vibration by DFT and SIP analysis both.

IMPULSE RESPONSE ESTIMATION USING NON-STATIONARY VIBRATION

Noise signals of 144,000-sample long as non-stationary vibration were prepared for natural vibration simulator as natural force of winds, seismic wave and etc. Figure 4 shows that the power spectra of three noise signals (case A, B, C) averaged over 600-samples long 240 windows. Responses of the scale-model for condition A and condition B, vibrated by the noise signal are simulated by convolution of the two impulse responses and the noise signals, respectively. And, DFT and SIP analysis were computed as same as previous section.

Evaluation

Figure 5 shows power spectra of different structure conditions A and B which were excited by three different noise signals. Upper three panels (a-c) are for condition A and lower ones (d-f) are for condition B. Frequency characteristics of the under condition A has three peaks which are similar to impulse response condition A. And, condition B have different shapes from the measured impulse response. According to this results, DFT can estimate frequency characteristics under non-stationary vibration, if impulse response has strong peaks such a condition A.

Figure 6 shows SIP distributions as well as Fig.5. For SIP distributions, condition A has three peaks explicitly. For condition B, SIP distributions have simple peaks compare with power spectra using DFT. It denotes that SIP distribution can estimate peaks of frequency characteristics, if peaks of orignal frequency characteristics are not sharp. This results suggests that SIP distribution is effective to estimate peak frequencies as resonance frequency. Consequently, SIP analysis is applicable for health monitoring under non-stationary vibration.

SUMMARY

Hirata proposed Short-Interval Period, SIP, that is a method for estimating a transfer function from non-stationary data. This paper reports on a scale-model experiment, which shows that SIP detects changes in a frequency characteristic from non-stationary data and is applicable for estimation of a structure.

ACKNOWLEDGMENT

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