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Monte Carlo model of the uncertainty of SEA loss factors

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ABSTRACT

Finite Element Methods are widely used to model vibro-acoustic systems, but as the modal density becomes higher this type of model becomes inaccurate and impractical. This is why in the high modal density region the use of Statistical Energy Analysis (SEA) models has become increasingly popular. SEA has some obvious advantages such as its simple formal expression, being based on linear equation systems or the reduced number of variables involved. But SEA has drawbacks as well, such as the absence of local information or the necessity of frequency averaging. A key quantity in SEA models is the loss factor. This takes into account the energy dissipated within a given subsystem or when power flows from one subsystem to another. Even though analytical expressions exist for a number of subsystems of differing nature, the measurement of the loss factor is still advisable and a necessity for a large number of cases. The most commonly used method of measuring loss factors is the Power Injection Method. This method is based on the injection of power into every single subsystem in sequence while the energy in each subsystem is measured. In spite of its simplicity, there remain a number of problems where the accuracy of the results is influenced by various practical issues. In this paper, a Monte Carlo model is used to describe the uncertainty of a two subsystem-problem consisting of two planar elements connected along one side. The influence of the input variables is studied and the conditioning of the coefficient matrix that model the system is also taken into account.

INTRODUCTION

There are a great amount of studies that analyze advantages and drawbacks about the use of deterministic methods and non-deterministic methods based on statistical techniques to characterize dynamic systems. The conclusion can be stated as that both ways to solve the problem is efficient within its applicability range, what, in turn, will depend on the size of the system, the modal density of the whole system and its parts, the calculation load and the accuracy needed.

In this paper a non-deterministic method; the Statistical Energy Analysis, SEA, is used as a way to characterise the structural systems usual in fields apparently as far ones from each others, as aeronautic design or acoustical building elements analysis. In this method, the determination of the loss factors of the different parts of the system is of primary importance to specify the performances at high frequencies.

SEA was developed as an analysis method in the second half of the last century, and it can be very useful in the frequency range where deterministic methods become too much complex and unpractical. Lyon [1] was one of the first authors to give a general view of the method as a whole, including posterior updating revisions [2][3]. Later on Craik [4] contributed to generalize the application of SEA methods to the field of building acoustics showing its viability and utility.

One of the main advantages of SEA is that it can model complex vibro-acoustical systems using very few parameters (Input power, total energy of the system and loss factors, typically). On the other side, it cannot give local information about the system, or take into account little variations on the structure/material properties, including junctions between subsystems, for instance, unless they correspond to changes in their modal density characteristics.

To build up a SEA model, the structure must be split into subsystems. These subsystems are substructures with homogeneous properties, including its modal density that, in addition should have a high value. Once defined the subsystems, power flow among them must be determined, including dissipated power within them (see Fig. 1). Another important issue related with the applicability of these models is the coupling between systems, which is related with the amount of power flowing throughout all the structure and with the presence of global modes in it [3][4][5].

The principle SEA model is based on two main statements: On the stationary state power flow within the system, (including power going from one subsystem to another or power dissipated by any part of the structure) must be in equilibrium. The power flow is proportional to the total energy in a given subsystem (Fig. 1). Relationship between energy and power flow can be expressed as:

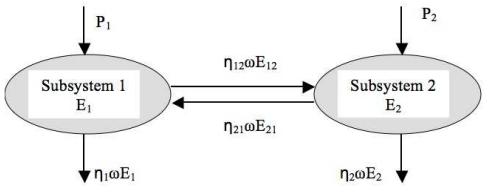


Figure 1. Power flow diagram of a two-subsystem SEA model

$$P_i = \omega E_i \sum_{j=1}^{N} \eta_{ij} - \omega \sum_{\substack{j=1 \\ j\neq i}}^{N} E_j \eta_{ji} \qquad i = 1,2,\mathrm{K}\ N \tag{1}$$

where P_i is the power input to the subsystem *i*, ω is the angular frequency, E_i the total energy of the *ith* subsystem, E_j the energy of the *jth* subsystem, η_i is the internal loss factor, *ILF*, and η_{ij} is the coupling loss factor, *CLF* between subsystems *i* and *j*. They take into account the energy dissipated within the subsystem and through the joint respectively.

To get loss factors, calculations based on the dynamics of the structure can be done. Besides they can be obtained from tests, as well. In this work an experimental method is used, the Power Injection Method, PIM, which consists on exiting consecutively each one of the subsystems the structure is compound of while the input power and every subsystem energies is measured [6][7]. In this way, the resulting equations system that describe the process is

$$\begin{bmatrix} E_{11} & 0 & E_{11} & -E_{21} \\ 0 & E_{21} & -E_{11} & E_{21} \\ E_{12} & 0 & E_{12} & -E_{22} \\ 0 & E_{22} & -E_{12} & E_{22} \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_{12} \\ \eta_{21} \end{bmatrix} = \begin{bmatrix} P_1/\omega \\ 0 \\ 0 \\ P_2/\omega \end{bmatrix} \tag{2}$$

where E_{ij} represents the energy in the *ith* subsystem when panel *j* is excited.

It is well known that in spite of the formal simplicity of the model some problems arise. One of these problems is the fact that energy matrix elements usually are of different magnitude orders giving rise to numerical problems that lead to ill conditioned matrices that, in turn, produce problems on the resolution process of the system. In this paper the conditioning of the matrix is taken into account to evaluate its influence on the results.

A way to minimize this problem was developed by Lalor [8], who separating *ILF* from *CLF* in two different equation systems. One for the *CLFs* and other for the *ILFs*:

$$\omega \begin{bmatrix} \frac{E_{11}}{E_{21}} - \frac{E_{12}}{E_{22}} & 0 \\ 0 & \frac{E_{22}}{E_{12}} - \frac{E_{21}}{E_{11}} \end{bmatrix} \begin{bmatrix} \eta_{12} \\ \eta_{21} \end{bmatrix} = \begin{bmatrix} \frac{P_2}{E_{22}} \\ \frac{P_1}{E_{11}} \end{bmatrix} \tag{3a}$$

$$\begin{bmatrix} E_{11} & E_{21} \\ E_{12} & E_{22} \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = \begin{bmatrix} \frac{P_1}{\omega} \\ \frac{P_2}{\omega} \end{bmatrix} \tag{3b}$$

For this simple case of two subsystems eqs (3a) and (3b) can be solved and the values of loss factors can be obtained:

$$\eta_{12} = \frac{P_2}{\omega E_{22} A}; \quad where \quad A = \frac{E_{11}}{E_{21}} - \frac{E_{12}}{E_{22}} \tag{4a}$$

$$\eta_{21} = \frac{P_1}{\omega E_{11} B}; \quad where \quad B = \frac{E_{22}}{E_{12}} - \frac{E_{21}}{E_{11}} \tag{4b}$$

$$\eta_1 = \frac{P_1}{\omega E_{11}} - \eta_2 E_{21} \tag{4c}$$

$$\eta_2 = \frac{P_2 - P_1 (E_{12}/E_{11})}{\omega (E_{22} - E_{21} E_{12})} \tag{4d}$$

In this paper, uncertainties of the loss factors obtained by the PIM are evaluated by the use of a Monte Carlo model[9]. The output variables of the model are the loss factors, *ILFs* and *CLFs*, while the input (statistical) variables are the energy in the subsystems and the input power to the subsystems, with Gaussian probability distribution densities. Both, input and output parameters are related by eqs. (4a) to (4d).

In addition the influence of the conditioning of the energy coefficients matrix is evaluated. To this end, singular values of the matrix are calculated and its minimum value is investigated.

MONTE CARLO MODEL

Structure description

The case studied consists of two plates joined by one side. Sizes of plate 1 and plate 2 are 1.60 x 1 x 0.003 m and 0.8 x1 x 0.003 m, respectively. They are assumed to be made of aluminun with c_L=5000 ms^{-1}, ρ = 2621.5 kgm^{-3}.

ILFs of both plates are assumed to be *0.01*, and plates junction is assumed to be in such a way that *CLFs* are

$$\eta_{ij} = \frac{\rho_o L_{ij} \tau_{ij}}{\pi^2 S_i \sqrt{f f_{ci}}} \tag{5}$$

being ρ_o the plate density, τ_{ij} the transmission coefficient (strength coupling will depend mainly on it [10][11][12]), L_{ij} the junction length, S_i the surface of the plate and f_c the critical frecuency, whose value is 4281 Hz. Both CLFs must satisfy

$$\eta_{ji} = \frac{n_i}{n_j} \eta_{ij} \tag{6}$$

where n_i stands for modal density of each plate

$$n_i(f) = \frac{\sqrt{3} S_i}{h_i c_{Li}} \quad modes/Hz \tag{7}$$

and h_i is the plate width. Values for plate *1* and *2* are:

$$n_1 = 0.1848 \ \ modes/Hz$$
$$n_2 = 0.0924 \ \ modes/Hz \tag{8}$$

Reference case definition

Once the system is completely defined, the response to a power input can be obtained by solving the direct SEA problem (material known and consequently, *LFs* known). The only non-intrinsic parameter involved is τ_{ij}. It describes the

vibratory power that goes into receiver subsystem relative to power incident to the subsystems boundary, so that strength of the coupling between the subsystems will depend on it. Two different values where here considered, *0.01* and *0.95*, weak and strong coupling respectively. Input power was assumed to be *1 W* for every frequency band, and energy of the vibratory state is obtained for both subsystems.

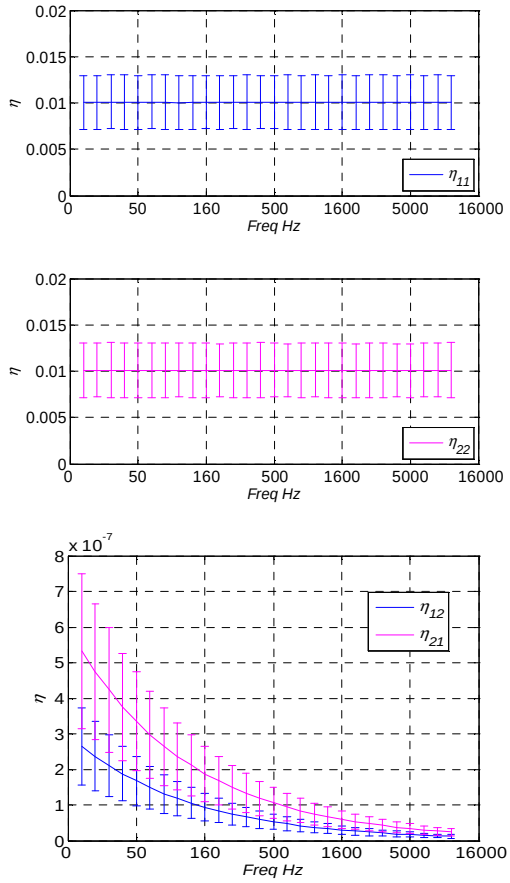


Figure 2. Mean loss factors values and error bars for a confidence level of 95 %. τ_{12} = *0.01*, $\sigma_{r\,P}$= *0.1.* and $\sigma_{r\,E}$=*0.1*

Test model definition

Mathematical test model is defined by eqs. (4), where input parameters are the input power to each subsystem and the whole set of energies every subsystem has on each case. Output parameters are the *CLFs* and *ILFs* of the system.

Input parameters are supposed to be random variables with a Gaussian probability distribution function, *pdf*, each one centred on the reference values obtained by the direct method, and standard deviations, σ_i, that in an actual case would depend on the uncertainty of the acquisition system and experimental setup, are chosen such that their value relative to the mean value ranges from 0 % to 20 % for the energy sets and from 0 % to 25 % for the input power, simulating different groups of tests sets.

Uncertainty estimation is based on the Monte Carlo method rather on propagating input uncertainties up to the output ones. Though, both could be used in this case, given the analytical expression of our test model, in real cases, where a much higher number of subsystems are involved, such relationships are difficult to obtain, if so. On the other hand, a Monte Carlo model in this more complicated scenario might reduce the mathematical effort, could reduce the number of hypothesis the uncertainty analysis is based of, and would allow the determination of the output parameters *pdfs*, and determine confidence interval when the output *pdfs* cannot be considered as Gaussians.

Then a Monte Carlo model was run for an ensemble of 5000 instances. All the *pdfs* were assumed to be Gaussian in this paper, and the standard uncertainty of the inputs is related to their variances.

RESULTS

Overview

Figure 2 shows the results for a single example. Mean values are so close to the reference ones that taking into account any bias correction is useless, and negligible compared with the uncertainty introduced by random variations.

Figure 3 shows an example of the behaviour of the uncertainties for a whole set of input variances. Results form a surface on the results space that can be different for each frequency band. Results show how the surface of the standard deviation of the *LFs* is monotone increasing with both σ_{rE} and $\sigma_{rP.}$

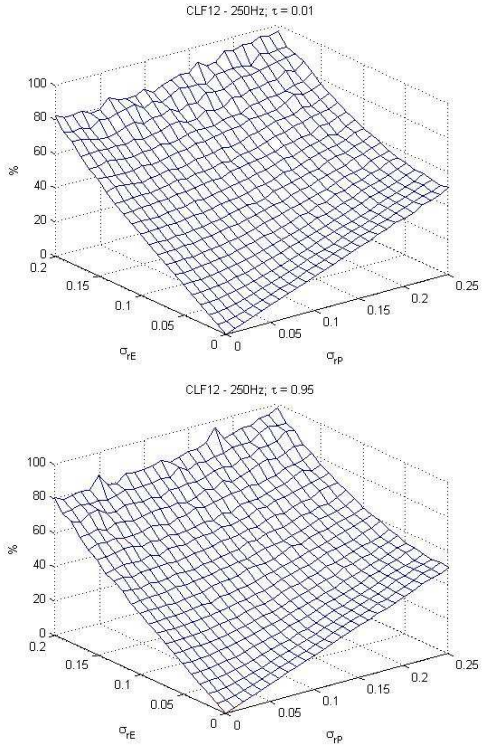


Figure 3. Relative uncertainty of the CLF values in the frequency band of 250Hz. Upper plot, τ_{12} = *0.01*; lower plot, τ_{12} = *0.95.* Confidence level: 95 %

Uncertainty model for the two-plate system

Given the nature of the results surface a bilinear model is defined for the uncertainties. We assume that the uncertainty of every *LF* follows the expression:

$$u(x,y) = f_p(x,y) \tag{9}$$

where *x*, and *y* are dummy variables that, in our case, will stand for the relative standard uncertainty of the energy and input power, and

$$f_p(x,y) = \xi_1 x^2 + \xi_2 y^2 + \xi_3 xy + \xi_4 x + \xi_5 y + \xi_6 \tag{10}$$

where coefficients ξ_i are determined by fitting *f(x,y)* to the calculated data by the Monte Carlo model, z_{ij}, that is, the function

$$F(z,x,y)=\sum_{j=1}^{m}\sum_{i=1}^{m}\left(z_{ij}-f_p(x_i,y_i)\right)^2 \qquad (11)$$

has to be minimized. What, in turn, leads to the equations system

$$\sum_{i,j}2\left(z_{ij}-f(x_i,y_i)\right)\frac{\partial f_p}{\partial \xi_p}(x_i,y_i)=0 \qquad \forall p=1,\ldots,n \qquad (12)$$

That can be expressed as:

$$\begin{bmatrix} \varepsilon(x_i^4) & \varepsilon(y_j^2x_i^2) & \varepsilon(y_jx_i^3) & \varepsilon(x_i^3) & \varepsilon(y_jx_i^2) & \varepsilon(x_i^2) \\ \varepsilon(y_j^2x_i^2) & \varepsilon(y_j^4) & \varepsilon(y_j^3x_i) & \varepsilon(x_iy_j^2) & \varepsilon(y_j^3) & \varepsilon(y_j^2) \\ \varepsilon(y_jx_i^3) & \varepsilon(y_j^3x_i) & \varepsilon(y_j^2x_i^2) & \varepsilon(y_jx_i^2) & \varepsilon(y_j^2x_i) & \varepsilon(y_jx_i) \\ \varepsilon(x_i^3) & \varepsilon(x_iy_j^2) & \varepsilon(y_j^2x_i^2) & \varepsilon(x_i^2) & \varepsilon(y_jx_i) & \varepsilon(x_i) \\ \varepsilon(y_jx_i^2) & \varepsilon(x_iy_j^2) & \varepsilon(y_j^2x_i^2) & \varepsilon(y_jx_i) & \varepsilon(y_j^2) & \varepsilon(y_j) \\ \varepsilon(y_jx_i^2) & \varepsilon(x_iy_j^2) & \varepsilon(y_jx_i) & \varepsilon(y_jx_i) & \varepsilon(y_j) & 1 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \\ \xi_5 \\ \xi_6 \end{bmatrix} = \begin{bmatrix} \varepsilon(z_{ij}x_i^2) \\ \varepsilon(z_{ij}y_j^2) \\ \varepsilon(z_{ij}x_iy_j) \\ \varepsilon(z_{ij}x_i) \\ \varepsilon(z_{ij}y_j) \\ \varepsilon(z_{ij}) \end{bmatrix} \qquad (13)$$

Where ε is the mathematical expectation of the respective variables.

This equation was solved for every frequency band. Figures 4 and 5 show the upper and lower quartile values and the median of the coefficient values obtained.

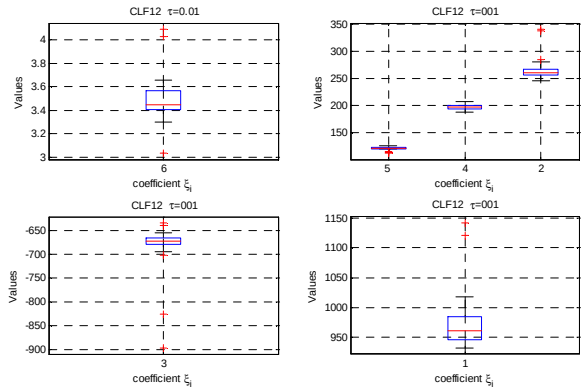


Figure 4. Coefficient values of the CLF uncertainty model. for $\tau_{12} = 0.01$

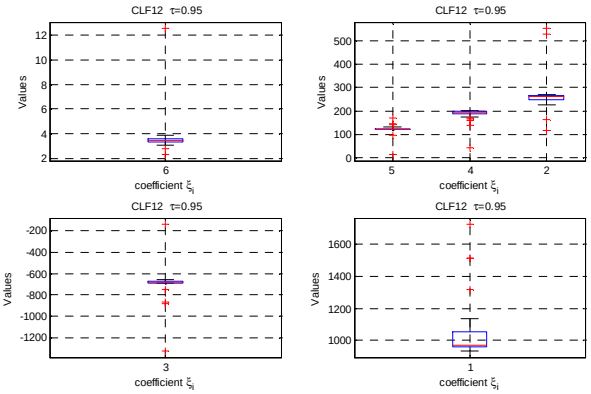


Figure 5. Coeficient values of the CLF uncertainty model. for $\tau_{12} = 0.95$.

Differences within a given coefficient for the whole frequency range are so small that the model can be simplified using the same expression for all the frequencies. These coefficients correspond to the median of the relative sigmas of the loss factor of all the frequencies for the different uncertainty values of energy and power. (see Table 1).

Table 1. Overall coefficients for each *LFs*

	ξ_1	ξ_2	ξ_3	ξ_4	ξ_5	ξ_6
η_{11}	583.58	249.45	-554.40	103.74	137.67	2.28
η_{12}	947.44	261.65	-672.00	199.57	121.82	3.47
η_{22}	584.72	252.63	-551.60	102.85	136.78	2.39
η_{21}	942.62	257.60	-676.28	201.56	123.02	3.30

Relative errors are obtained and show on in Table 2, which in the worse of cases rise up to 10%.

Table 2. Coefficients of adjustment Error for LFs

%	ξ_1	ξ_2	ξ_3	ξ_4	ξ_5	ξ_6
η_{11}	3.9	1.2	1.7	-1.6	2.4	2.2
η_{12}	5.8	2.8	2.3	-7.7	8.4	5.0
η_{22}	9.4	2.3	5.1	-9.8	8.2	9.2
η_{21}	5.6	2.7	2.7	-10.0	10.6	7.3

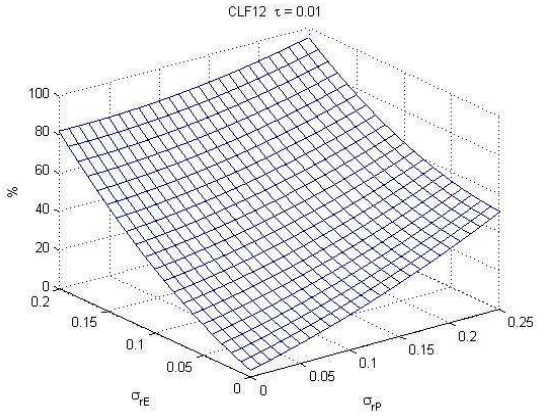
Thus the bilinear surface that describes the *LFs'* uncertainty of the systems as a function of the relative uncertainties in the energy and power is as follows:

$$u_\eta(\sigma_{rE},\sigma_{rP})=\xi_1\sigma_{rE}^{\ 2}+\xi_2\sigma_{rP}^{\ 2}+\xi_3\sigma_{rE}\sigma_{rP}+\xi_4\sigma_{rE}+\xi_5\sigma_{rP}+\xi_6 \qquad (14)$$

Where ξ_i are presented in Table 1

Therefore the uncertainty in the measurement of the *LFs* depends on the relative uncertainties of both the energy and the power according to Equation 14 for each of the *LFs*. According to this equation, Figure 5 presents the surfaces generated for one of the *CLFs* for both types of union studied.

Figure 5 shows the results for a given case. . It can be observed how the increase of either σ_{rE} or σ_{rP} causes an increase on the standard uncertainty of the corresponding *LF* uncertainty. It is noteworthy that the increase in the relative energy uncertainty produces higher values on the uncertainty than when the power uncertainty is increased.



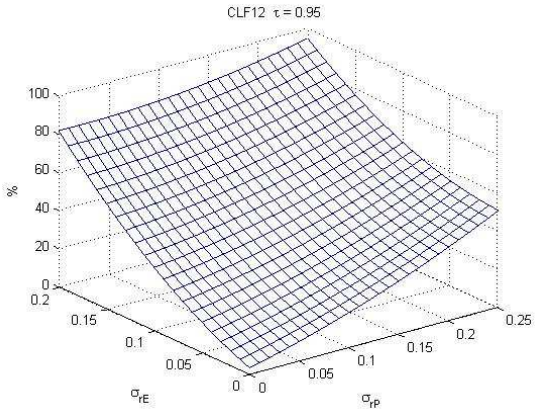


Figure 5. Model of the uncertainty for η_{12} Upper plot, τ_{12} = *0.01*; lower plot, τ_{12} = *0.95.*

In addition, results of the series of tests run shows how the model stays working even for different strengths of the junction.

INFLUENCE OF THE CONDITIONING OF THE PIM LINEAR EQUATION ON THE RESULTS

When usin PIM method to determine *LFs* negative values are usually obtained. The most ususal reason stated to explain this fact is that the linear system that relates the parameters of the PIM model is ill-conditioned. The development of calculation estrategies to avoid such a results is a very active line of work nowadays. In this section, the influence of the conditioning on the linear system of our case is investigated.

The calculation of the singular values of the energy matrix of Eq 2, results:

$$s^2 = \frac{(\gamma_1 + \gamma_2) \pm \sqrt{(\gamma_1 - \gamma_2)^2 + 4\gamma_3}}{2} \tag{15}$$

Where:

$$\begin{aligned} \gamma_1 &= (E_{11} + E_{12})^2 + (E_{12})^2 \\ \gamma_2 &= (E_{21} + E_2)^2 + (E_2)^2 \\ \gamma_3 &= (E_1 + E_{12})(E_{21} + E_2) + E_2 E_{12} \end{aligned} \tag{16}$$

There are two singular values of double multiplicity, where the minimun value corresponds to the negative sign of the eq.15. In our case this value is very small in the high frequencies area.

The condition number is defined as

$$\kappa(E) = \frac{s_{\max}(E)}{s_{\min}(E)} \tag{17}$$

where s_{max} and s_{min} stands for maximum and minimum singular values of the Energy matrix. This is a very simple case where κ will be the ratio between the two corresponding singular values (eq 15).

$$\kappa(E) = \left(\frac{\mu^2 + \mu\sqrt{\mu^2 - 4}}{2} - 1 \right)^{1/2} \tag{18}$$

Where:

$$\mu = \frac{(E_{11} + E_{12})^2 + E_{12}^2 + (E_{21} + E_{22})^2 + E_{22}^2}{(E_{11}E_{22} - E_{12}E_{21})} \tag{19}$$

The condition number values obtained in this work for the various cases analysed are between $\kappa = 2$ and $\kappa = 7$.For the latter, negative loss factors can be presented.

Actually present case is quite small so it is to be expected that no big problems arise from the matrix conditioning as it is happening. Nevertheless, the model can be extended to higher dimensional problems where conditioning of the system matrix plays a role.

CONCLUSIONS

An uncertainty model of the *LFs* has been developed for systems consisting of two plane plates, which generates a quadratic surface whose coefficients can be easily determined.

Based on the previous results, it can be said that the *LF* uncertainty depends on the energy and power uncertainties Playing a greater role the one coming from the subsystems energy.

It should be noted that in spite of the dependence of the *LFs* with frequency, the uncertainties model can be defined the same for all frequencies if based on relative uncertainties.

It has been shown that for the case under study matrix conditioning does not influence much the validity of the results. Though, this conclusion cannot be extended to higher dimensional systems that must be studied in the future.

Finally, another important point that emerges from this study is the independence of the uncertainty surface with the junction strength between subsystems.

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