Proceedings of 20th International Congress on Acoustics, ICA 2010

23-27 August 2010, Sydney, Australia

Time-domain computation of broadband noise due to interaction of a rectilinear cascade of flat plates with inflow turbulence

Daehawn Kim (1), Cheolung Cheong (1)

(1) School of Mechanical engineering, Pusan National University, Busan, Korea

PACS: 43.28.Ra

ABSTRACT

In this paper, time-domain computational aeroacoustic (CAA) methods are developed to predict broadband noise generation and propagation by a rectilinear cascade of flat plates interacting with ingesting turbulent gust. Utilizing three-dimensional time-domain CAA model, the three-dimensional characteristics of inflow turbulence noise due to the turbulence-cascade interaction are investigated with an emphasis placed on the effects of wavenumber components of ingesting turbulence in the span-wise direction. Through the comparison of three-dimensional results with two-dimensional ones, the characteristics of sound pressure spectrums obtained from the three-dimensional results are revealed, which is mainly due to the different dispersion relations of acoustic waves in two- and three-dimensions. These differences between two- and three-dimensional results become more significant in the lower frequency components. In addition, some preliminary numerical results on the effects on the broadband noise of a rectilinear cascade of leaned or swept flat plates are illustrated, which can provide basic principle for the low-noise design of the stator in turbo-fan engines.

INTRODUCTION

Manufacturers of aero-engines have made efforts to reduce noise from aero-engines due to more and more stringent community noise regulations. A significant contributor to overall aero-engine noise is fan noise which radiates tones at harmonics of blade-passing frequency superimposed on a broadband noise. Fan broadband noise is now dominant since longer blades and slower their rotating velocity have reduced the tonal noise [1]. The fan broadband noise is divided into self-noise and inflow turbulence noise. The model problems considered in this paper is related to inflow turbulence noise of fans.

Until now, most researches on the inflow turbulence noise have been conducted based on two-dimensional linear theory in frequency-domain for the cascade of flat plates. The recent works for inflow turbulence noise have been carried out by Gliebe [1], Hanson [2] and Cheong et al. [3–5]. However, the methods which these researches were based on cannot consider effects on the broadband noise of configuration of airfoils, non-uniform flow field and nonlinear interaction involved in its generation and propagation. In present paper, we predict broadband inflow turbulence noise in time domain by utilizing Computational AeroAcoustic (CAA) techniques to overcome limitations on frequency-domain approaches. Kim et al. [6] predicted broadband noise due to interaction between ingesting turbulent gust and a cascade of flat plates in two-dimensional time-domain. Their results show good agreements with results from Linsub[4] program in frequency-domain. The approach used in the two-dimensional prediction, however, is to start with a three-dimensional turbulence spectrum, integrate over all span-wise wavenumbers, and then apply this reduced spectrum at zero span-wise wavenumber. i.e., all input gusts are in phase along the full vane span. Therefore, two-dimensional approaches cannot considers variations of incident turbulent gust along the span-wise direction and thus contribution of span-wise turbulent components to the broadband inflow turbulence noise.

We developed three-dimensional time-domain program utilizing CAA techniques, which can compute effects of the span-wise turbulent components on three-dimensional broadband inflow turbulence noise. After computing sound pressure spectrums, we compared two- and three-dimensional results. Through this comparison, it is revealed that different dispersion relation between two- and three-dimensional acoustic waves causes main differences between two in two- and three-dimensional results. In lower frequency range, these differences between two results are more significant. In addition, utilizing the developed three-dimensional program, the effects on the pressure spectra of a rectilinear cascade of leaned or swept flat plates are quantitatively analysed, which can provide basic principle for the low-noise design of the stator in turbo-fan engines.

NUMERICAL METHODS

Governing Eqs. With BCs and configuration of a cascade

To compute the generation and propagation of the broadband noise, we employed the linearized Euler equation as governing equations in Eq. (1) and configuration of rectilinear cascade of flat plates is shown in Figure 1.

$$\frac{\partial \rho}{\partial t}+U_j\frac{\partial \rho}{\partial x_j}+\rho_0\frac{\partial u_j}{\partial x_j}=0 \qquad (1)$$
$$\frac{\partial u_i}{\partial t}+U_j\frac{\partial u_i}{\partial x_j}+\frac{1}{\rho_0}\frac{\partial p}{\partial x_i}=0$$

where, U_j and u_j (j = 1, 2, 3) denote mean and perturbation velocities, respectively. U_2 and U_3 are set to be zero since the angle of incidence is zero and the slip boundary conditions must be satisfied on the two-sided walls. ρ_0 is mean density, and ρ and p are perturbation quantities of density and pressure, respectively. The fluctuating pressure can be calculated by using the isentropic relation, $(dp/d\rho)=a^2$. Here a is the speed of sound.

The 7-point stencil Dispersion-Relation-Preserving(DRP) scheme [7] is utilized to compute the spatial derivatives and optimized Adams-Bashford method[7] is used as the time integration method. To eliminate reflecting waves into computational domain at inlet/outlet boundaries, Perfectly-Matched Layer technique [8] is implemented as non-reflecting boundary conditions. At inlet boundary, specified inflow turbulent gust should be well generated while outgoing waves are simultaneously absorbed with reflections. This inflow boundary condition of turbulent gust is described in the following section. In Figure 1, slip condition is applied on the planes of $x_3 = 0$ and $x_3 = R$ and the surface of blades. Periodic boundary condition is implemented on the planes of $x_2 = 0$ and $x_2 = 4$.

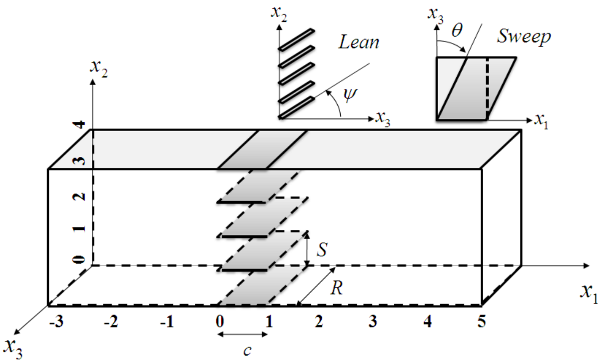


Figure 1. Configuration of rectilinear cascade of flat plates and definition of lean/sweep angles

Inflow turbulence modelling

For the specification of inflow turbulence, the normal component of the turbulent velocity to the chord of the rectilinear cascade can be defined as

$$w(x_1,x_2,x_3,t)=\iiint_{-\infty}^{\infty}\hat{\hat{w}}(k_1,k_2,k_3)e^{-i[k_1(x_1-Ut)+k_2x_2+k_3x_3]}dk_1dk_2dk_3 \qquad (2)$$

where k_1, k_2 and k_3 are the wave numbers in the axial, circumferential and span-wise directions, respectively. The wave number spectrum, $\hat{\hat{w}}(k_1,k_2,k_3)$, of the turbulent velocity can be evaluated by recognizing the turbulent fluctuation as a stochastic process. The turbulent velocities are periodic in the x_2-direction (circumferential direction) and satisfy the duct modal solutions in the x_3-direction. As a result the wave numbers in the x_2 and x_3 directions must satisfy the following relations

$$k_{2,m}=\frac{2\pi}{Bs}m,\ k_{3,l}=\frac{\pi}{R}l, \qquad (3)$$

where m and l denote mode number in circumferential and span-wise direction, respectively. B is the number of blade and s is distance between two adjacent blades. The Fourier integrals over the k_2 and k_3 are therefore converted to Fourier series as follows,

$$\iint dk_2\,dk_3 \rightarrow \sum_{m=-M}^{M}\sum_{l=-L}^{L}\frac{2\pi}{Bs}\frac{\pi}{R}. \qquad (4)$$

Combining Eq.(4) with Eq.(2) leads to

$$w(x_1,x_2,x_3,t)=\frac{2\pi}{Bs}\frac{\pi}{R}\sum_{m=-M}^{m=M}\sum_{l=-L}^{l=L}\int_{-\infty}^{\infty}\hat{\hat{w}}(k_1,k_2,k_3)e^{-[k_1(x_1-U_1t)+k_{2,m}x_2+k_{3,l}x_3]}dk_1 \qquad (5)$$

The integration over the wavenumber, k_1, in the axial direction is numerically made. Consequently, In Eq. (5), $\hat{\hat{w}}(k_1,k_2,k_3)$ may be rewritten by using the wave number power spectra density of turbulent gust, $\Phi(k_1,k_2,k_3)$. Assuming that the incoming turbulence is homogenous and isentropic, its power spectral density may be modelled by using the Liepmann spectrum defined in the form,

$$\Phi(k_1,k_2,k_3)=\frac{2\overline{w}^2\Lambda^3}{\pi^2}\frac{\Lambda^2\left(k_1^2+k_3^2\right)}{\left[1+\Lambda^2\left(k_1^2+k_2^2+k_3^2\right)\right]^3}, \qquad (6)$$

where $\overline{w}^2$ is the mean square value of turbulence velocity in the direction normal to the chord and Λ is the turbulence integral length scale.

NUMERICAL RESULTS

Prediction of three-dimensional broadband inflow turbulence noise

Three-dimensional broadband noise due to interaction between turbulence and the rectilinear cascade is predicted by utilizing CAA techniques described in previous section. Length, velocity and pressure are scaled by chord length(c=1m), mean velocity(U_1=170m/s) and $\rho_0 a^2$, respectively. $\overline{w}^2$ is set to be $1\times10^{-4}U_1^2$ and Λ is 0.1c. In Figure 1, the geometric parameters are set to be R/c=0.8 and s/c=1. The maximum wavenumber, $k_{1,N}$, of axial direction in turbulent velocity is set to be 6π. The maximum mode numbers N, M and L in each direction are set to be 15,7and 3, respectively.

Results from time-domain analysis can be represented in frequency-domain using joint spatial-temporal transform defined in the form

$$p(x,n,m,l)=\frac{2}{N_{\Delta x_2}N_{\Delta x_3}N_{\Delta t}}\sum_{j=0}^{N_{\Delta x2}-1}\sum_{k=0}^{N_{\Delta x3}-1}\sum_{r=0}^{N_{\Delta t}-1}p(x_1,x_{2,j},x_{3,k},t_r)e^{-2\pi i\left(\frac{mj}{N_{\Delta x2}}+\frac{lk}{N_{\Delta x3}}+\frac{nr}{N_{\Delta t}}\right)} \qquad (7)$$

where $N_{\Delta t}$ is sampling number over the time period and Δt is sampling time interval and $n=0,1,2,\cdots,(N_{\Delta t}-1)/2$ is the harmonic number. $N_{\Delta x2}$ is the number of points and $-N_{\Delta x2}/2<m<N_{\Delta x2}/2-1$ is the mode number in the x_2-directino. $N_{\Delta x3}$ is the number of points and $-N_{\Delta x3}/2<l<N_{\Delta x3}/2-1$ is the mode number in span-wise direction.

The modal amplitudes, $|p(x,n,m,l)|$, are calculated at the planes of x = –2 and 4 using Eq.(7). The computed modal amplitudes are shown in Figure 2. In Figure 2, the black curved line represents the cut-off line. The cut-off line can be computed by using the dispersion relation of the acoustic wave solutions to Eq. (1) in the form,

$$(\omega + U_1 k_1 + U_2 k_2 + U_3 k_3)^2 - a^2\left(k_1^2 + k_2^2 + k_3^2\right) = 0 \tag{8}$$

When U_2 = U_3 =0, Eq.(8) can be rearrange for the axial wavenumber k_1 as follows

$$k_1 = \frac{U_1\omega \pm a\sqrt{\omega^2 - \left(a^2 - U_1^2\right)\left(k_2^2 + k_3^2\right)}}{a^2 - U_1^2} \tag{9}$$

From Eq.(9), cut-on ratio, Θ, is defined as

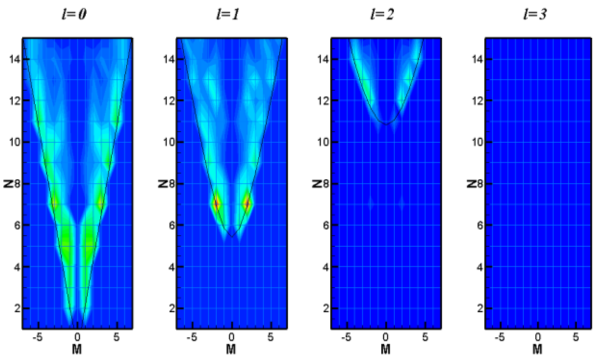
$$\Theta = \sqrt{\frac{\left(a^2 - U^2\right)\left(k_2^2 + k_3^2\right)}{\omega^2}} \tag{10}$$

As shown in Figure 2, modal amplitudes inside the cut-off lines have significant magnitudes since the acoustic wave can propagate without attenuation in the case of $|\Theta|<1$.

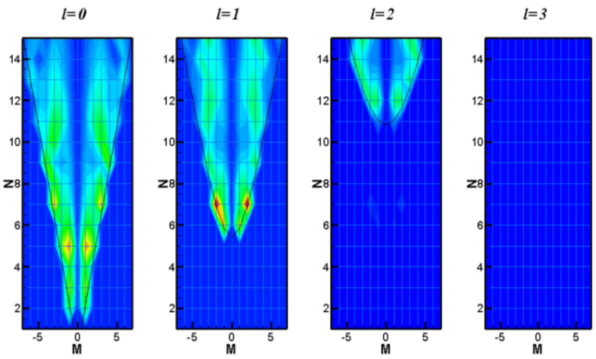
As the span-wise mode number, *l*, increases, the number of the cut-on frequency modes decreases. All the modes in the frequency range considered in this study are cut-off at *l*=3. Therefore there are no significantly responding acoustic pressure modes. Similar results can be shown in sound pressure spectrums which are calculated by

$$p_n = \sum_{m=-M/2}^{M/2-1} \left| p(x,n,m,l) \right|^2 \tag{11}$$

The computed sound pressure spectrums, p_n, are shown in Figure 3.

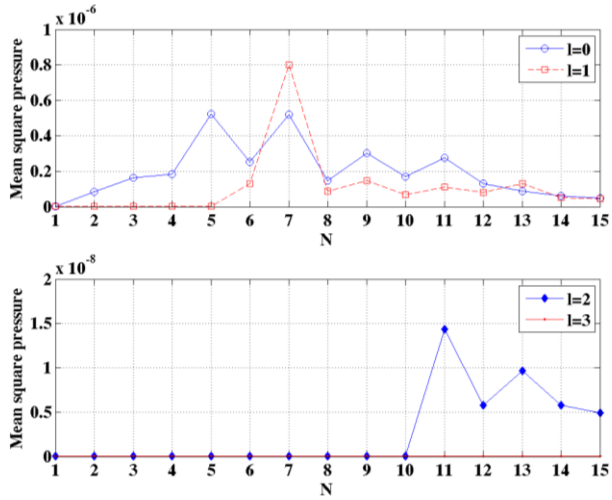


(a) *x* = –2

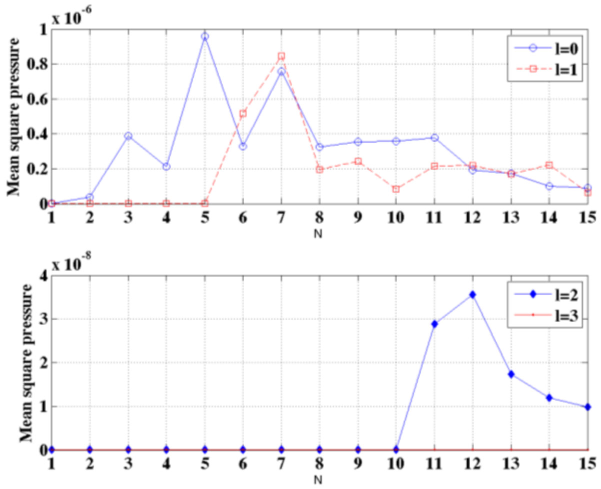


(b) *x* = 4

Figure 2. Mode amplitude of joint spatial-temporal transformed pressure, $|p(x,n,m,l)|$



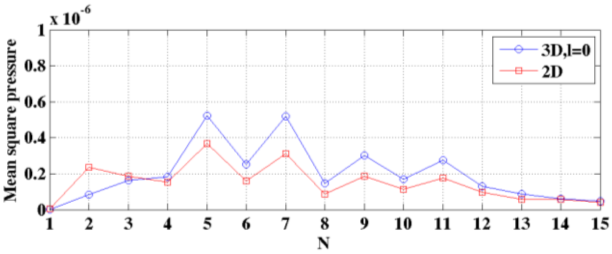
(a) *x* = –2



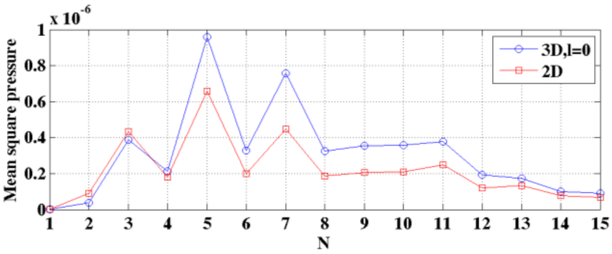
(b) *x* = 4

Figure 3. Sound pressure spectrum

All of the modes in the frequency range from *n*=1 to 15 are generated by the inflow turbulence gust of *l*=0. However, the cut-off frequency mode numbers, *n*, equal 6 in the case of *l* = 1, and 11 in the case of *l* = 2. At *l* = 3, all the frequency components are cut-off. These characteristics of sound pressure spectrum are main differences compared with two-dimensional ones. That is, generated broadband noise due to turbulence-cascade interaction in the two-dimension matches the case that span-wise mode number *l* equals zero in the three-dimensional ones. However, the three-dimensional computations generate the sound field depending on the variation of the turbulences in the span-wise direction. In this respect, it can be inferred that the differences in the sound pressure spectra computed in the two- and three-dimension originate from different dispersion relations of acoustic waves in the two- and three-dimensions. Comparison of sound pressure spectra predictions from two- and three-dimensional computations are shown in Figure 4. Overall patterns of sound pressure spectrum show similarity in the two computation. Discrepancies in amplitude of spectrums are due to different turbulent spectrum, Φ: two-dimensional turbulent spectrum is integrated over all span-wise wavenumbers.



(a) $x = -2$

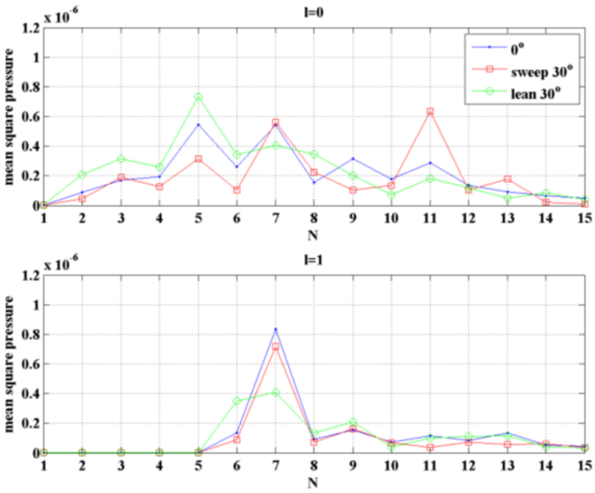


(b) $x = 4$

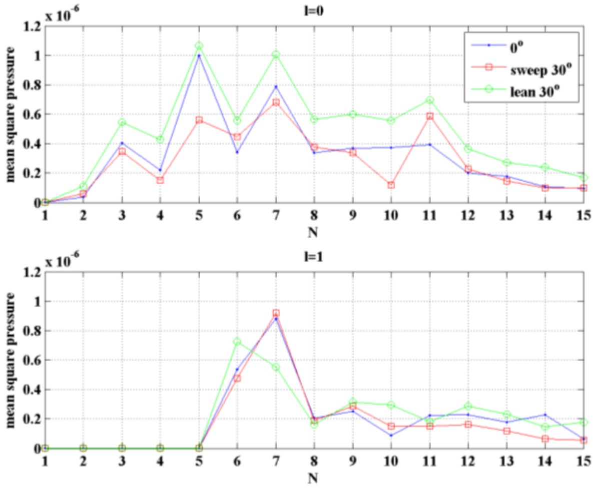
Figure 4. Comparison of sound pressure spectrums from two- and three-dimensional computation

Effects of leaned/swept flat plates on three-dimensional broadband inflow turbulence noise

In this section, we examined the effects of the leaned and swept flat plates on the broadband inflow turbulence noise utilizing the developed three-dimensional time-domain program. The definition of angles of lean and sweep is given in Figure 1. The angle of lean/sweep is set to be $\psi = \theta = 30^o$. The sound pressure spectrums for the leaned/swept blades are shown in Figure 5. As the angle of sweep (θ) increases, Mach number(U_1/a) normal to leading edge decreases as $(U_1/a)\cos\theta$. Consequently, amplitude of sound pressure spectrum for the swept blades decreases as shown in Figure 5. However, for the leaned blades, the reduction of sound pressure is small compared with the case of the swept flat plates, as shown in Figure 5. The swept blades seem to be more effective as a noise reduction measure for the inflow broadband noise than the leaned blades. Note that the dimensionalized maximum frequency corresponding to N = 15 is approximately 1000 Hz. To make clear the effects of the swept/leaned blades in the audible frequency range, it is needed to extend the computed frequency range.



(a) $x = -2$



(b) $x = 4$

Figure 5. Sound pressure spectrums predicted for the leaned/swept rectilinear cascade

CONCLUSIONS

Three-dimensional broadband inflow turbulence noise due to turbulent-cascade interaction was predicted. From comparison between two- and three-dimensional sound pressure spectrums, the effects of turbulent component of span-wise direction on the broadband noise were estimated. The different characteristic of three-dimensional broadband sound fields, compared with two-dimensional one, originates from the three-dimensional dispersion relation of acoustic waves in association with span-wise wavenumber components. Furthermore, the reduction of broadband noise due to leaned and swept rectilinear cascade were quantitatively analysed, which reveals that the swept cascade more effectively reduces the broadband noise than the leaned cascade, in the given frequency range. Our future research is to extend linear based program to the nonlinear and to develop time-domain program for the cascade consisting of real airfoils.

ACKNOWLEDGMENT

This work was supported by National Research Foundation of Korea (NRF) grant funded by Korea government (MEST) (No. 2009-0052961). This research was also supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (No. 2010-0009221).

REFERENCES

1 P. R. Gliebe, “Fan broadband noise - the floor to high bypass engine noise reduction” *Noise-Con96*, Seattle, Washington, 133–138, (1996)
2 D. B. Hanson, “Theory for broadband noise of rotor and stator cascades with inhomogeneous inflow turbulence including effects of lean and sweep” *NASA CR-2001-210762*, (2001)
3 Cheong, C., Phillip, J. and Lee, S., “Computation of broadband noise of a 2-D flat airfoil cascade subjected to ingested turbulence” T*ransaction of the Korean Society for Noise and Vibration Engineering* **15**, (2005)

4 Cheong, C., Phillip, J. and Lee. S., "High-frequency Approximate formulation for the acoustic power spectrum due to cascade-turbulence interaction" *J. Acoust. Soc. Am*. **119**, 108–122 (2006)
5 Cheong, C., Vincent J., Phillip, J. and Lee, S., "Decomposition of model acoustic power due to cascade-turbulence interaction" *J. Sound and Vib.* **324** 57-73(2009)
6 Kim, S., Cheong, C., "Mode-decomposition of the broadbande noise due to inflow turbulence cascade interaction by using time-domain computational aeroacoustic thechnique" *16th International Congress on Sound Vib. Krakow,Poland* (2009)
7 Tam, C. K. W., and Webb, J. C., "Dispersion-Relation-preserving finite difference scheme for computational aeroacoustics" *J. Sound Vib*. **107** (1993)
8 Hu, F.Q., "A stable, perfectly matched layer for linearized Eualer equations in unsplit physical variables" *J. Sound Vib.* **173** (2001)