Effects of correlation property of spread-spectrum signal on BER of M-ary underwater acoustic communication system

Jun Wang, Haibin Wang and Lixin Wu
State Key Lab. Of Acoustics, Institute of Acoustics, Chinese Academy of Sciences, Beijing, PR CHINA

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ABSTRACT

M-ary mode is widely used to enhance underwater communication data rate. However, in M-ary underwater acoustic communication system, it is infeasible to generate theoretically orthogonal spread-spectrum signals using sequences with finite length, and thus inter-code interference is inevitable. In order to predict the BER and provide theoretical criterion for the design of communication system parameters, theoretical formula was derived to describe the variation of BER versus several variables, such as SNR, parameters of signal and size of spread-spectrum signal set. Both simulations and theoretical analysis indicate that correlation property of spread-spectrum signal has a great effect on the BER. For a given SNR and parameters of signal, the size of spread-spectrum signal set must be limited to obtain expected BER. For a given BER, in order to obtain higher data rate, either SNR must be enhanced or parameters of signal must be adjusted.

INTRODUCTION

Compared with a binary system, M-ary mode has a higher communication data rate, which is increased by $\log_2 M$ multiple, where M is size of the spread-spectrum signal set. But in practice, lengths of spread-spectrum sequences need to be finite, which causes the generated signals to be not ideally orthogonal. Thus BER performance of the M-ary communication system will degrade considerably with the increasement of the signal set size value M. Therefore this increasement of M is not unbounded. In order to analyse the above interference quantificationally and predict the BER, the paper derives theoretical formula to describe the variation of BER versus several variables, such as SNR, parameters of signal and size of spread-spectrum signal set.

THEORY ANALYSIS

M-ary mode signal model

Assume that each signal in the spread-spectrum signal set is $s_i(t)$, where $i = 1,2.....M$ (M is the signal set size) denotes different signals. Here $s_i(t)$ should be signal with good correlation property, like M-sequence phase-modulation signal, chaotic signal, and pseudo-random signal etc. Without loss of generality, the received signal $y(t)$ can be given by:

$$y(t) = s_1(t) + n(t) \tag{1}$$

where $s_1(t)$ is the transmitted signal selected from the signal set, and $n(t)$ is Gaussian noise whose power spectrum density is $m_0/2$. The receiver detects $s_1(t)$ from $y(t)$ by making correlation between $y(t)$ and each spread-spectrum signal in the signal set and choosing the one that owns the maximum output value. Correlation output of signal $s_1(t)$ is given by

$$E_1 = \int_0^T y(t)s_1(t)dt \tag{2}$$

where T is time length of spread-spectrum signals, and correlation output of other copy signals in the signal set is given by

$$E_2 = \int_0^T y(t)s_q(t)dt, q = 2.....M \tag{3}$$

Correlation property of the spread-spectrum signal

In this paper, cross correlation coefficient of above spread-spectrum signals is considered to be a random variable that varies with different spread-spectrum signals. Thus mean value and second order moment of the cross correlation coefficient are separately defined as

$$E(\int_0^T s_m(t)s_n(t)dt) = bE_s, E(\int_0^T s_m(t)s_n(t)dt)^2 = aE_s^2 \tag{4}$$

where $m \neq n, m, n = 1.....M$, $E_s = \int_0^T s_m(t)s_m(t)dt$.

For convenience, following assumption is taken.

Assumption: The cross correlation coefficient accords with a Gaussian distribution with mean value b and variance $a - b^2$. And the mean value b can be considered little enough to be neglected.
For most kinds of spread-spectrum signals, above assumptions are reasonable. As shown in figure 1, simulation result of a chaotic signal set is in agreement with a theoretical Gaussian distribution.

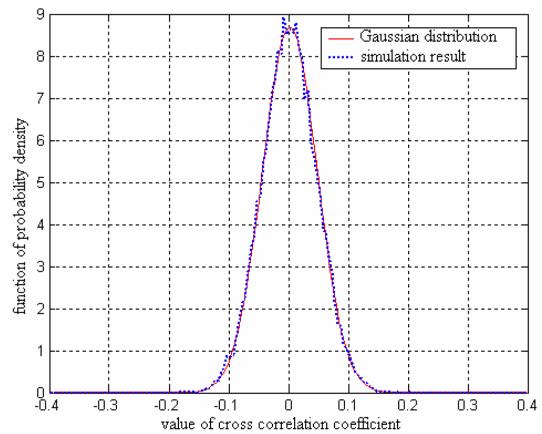

Figure 1. Comparison of theoretical Gaussian distribution and simulation distribution

Furthermore the variance value a is dependent on parameters of spread-spectrum signals, such as time length T, bandwidth B and code length N. To take the above-mentioned chaotic signal as an example, figure 2 shows the theoretical result of its variance a by different signal parameters.

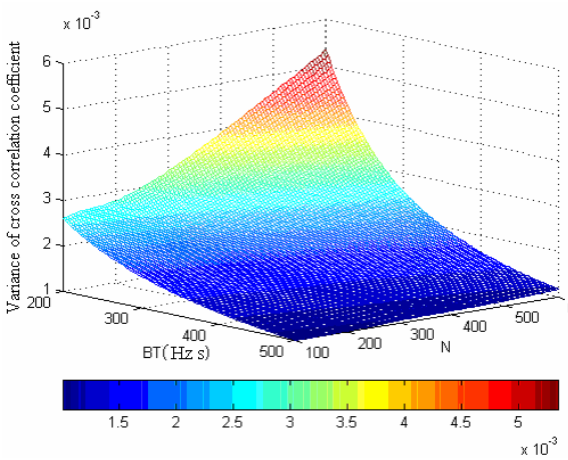

Figure 2. Variance versus signal parameters

Derivation of BER formula

From above assumption and analysis, it is easy to get that E_1 and E_2 are both in accordance with Gaussian distribution. And their mean value and variance are derived and given by

$$E(E_1) = E_s \text{,} \quad E(E_2) = 0 \qquad (5)$$

$$Var(E_1) = \frac{m_0}{2} E_s \text{,} \quad Var(E_2) = aE_s^2 + \frac{m_0}{2} E_s \qquad (6)$$

Then, when signal $s_1(t)$ is transmitted, probability density functions of E_1 and E_2 can be expressed as

$$f(E_1 \mid s_1) = \sqrt{\frac{1}{2pVar(E_1)}} \exp(-\frac{(E_1 - E_s)^2}{2Var(E_1)}) \qquad (7)$$

$$f(E_2 \mid s_1) = \sqrt{\frac{1}{2pVar(E_2)}} \exp(-\frac{E_2^{\ 2}}{2Var(E_2)}) \qquad (8)$$

In order to overcome phase ambiguity at the receiving side, we adopt absolute value of the correlator output. For any possible value x of E_1, the probability of correct decision is equal to the union probability of $|x|$ being greater than absolute outputs of all other $M-1$ correlators, and that's

$$\begin{aligned} P_c &= P(\text{correct} | E_1 = x, s_1) \\ &= P(\left|\int_0^T y(t)s_q(t)dt\right| < |x|, (q = 2..M) \, | E_1 = x, s_1) \end{aligned} \qquad (9)$$

Assume that the above outputs are independent and with identical distribution, then equation (9) can be expressed as

$$P_c = \left[P(\left|\int_0^T y(t)s_q(t)dt\right| < |x| \, | E_1 = x, s_1) \right]^{M-1}, q \neq 1 \qquad (10)$$

According to equation (8),

$$P_c = \left[\int_{-x}^{x} \sqrt{\frac{1}{2pVar(E_2)}} \exp(-\frac{E_2^{\ 2}}{2Var(E_2)}) dE_2 \right]^{M-1} \qquad (11)$$

So that, according to (5), (6) and (7),

$$P_c = \int_{-\infty}^{+\infty} \frac{\exp(-\frac{z^2}{2})}{\sqrt{2p}} \left[\int_{-\left|\frac{\sqrt{S}+z}{\sqrt{aS+1}}\right|}^{\left|\frac{\sqrt{S}+z}{\sqrt{aS+1}}\right|} \frac{\exp(-\frac{u^2}{2})}{\sqrt{2p}} du \right]^{M-1} dz \qquad (12)$$

where $S = \frac{2E_s}{m_0}$ is output SNR of the correlator.

Since signals in the set are supposed to be selected and transmitted equiprobably, incorrect decision probability P_e of the M-ary mode is given by

$$P_e = 1 - \int_{-\infty}^{+\infty} \frac{\exp(-\frac{z^2}{2})}{\sqrt{2p}} \left[\int_{-\left|\frac{\sqrt{S}+z}{\sqrt{aS+1}}\right|}^{\left|\frac{\sqrt{S}+z}{\sqrt{aS+1}}\right|} \frac{\exp(-\frac{u^2}{2})}{\sqrt{2p}} du \right]^{M-1} dz \qquad (13)$$

As to the M-ary mode, one transmitted signal corresponds to $n = \log_2 M$ bits, and every incorrect decision may cause $n2^{n-1}/(2^n - 1)$ error bits on average. Thus BER of the M-ary mode can be expressed as

$$P_{eb} = \frac{1}{n}\frac{n2^{n-1}}{2^n-1}P_e \approx \frac{1}{2}P_e$$
$$= \frac{1}{2}\left(1 - \int_{-\infty}^{+\infty}\frac{\exp(-\frac{z^2}{2})}{\sqrt{2p}}\left[\int_{-\left|\frac{\sqrt{S}+z}{\sqrt{aS+1}}\right|}^{\left|\frac{\sqrt{S}+z}{\sqrt{aS+1}}\right|}\frac{\exp(-\frac{u^2}{2})}{\sqrt{2p}}du\right]^{M-1}dz\right) \quad (14)$$

SIMULATION RESULT

Figure 3 illustrates BER comparison of ideally orthogonal signal set and actual signal set with $M = 1024$, and shows that correlation property of spread-spectrum signal has a considerable effect on the BER.

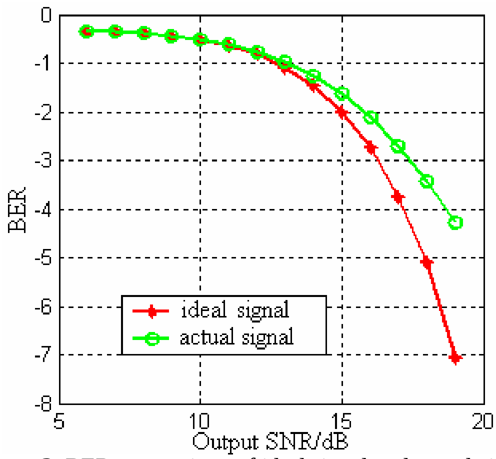

Figure 3. BER comparison of ideal signal and actual signal

Figure 4 shows BER versus SNR and signal set size M with $BT = 250 Hz \cdot s$. For a given SNR, BER increases with increment of signal set size M.

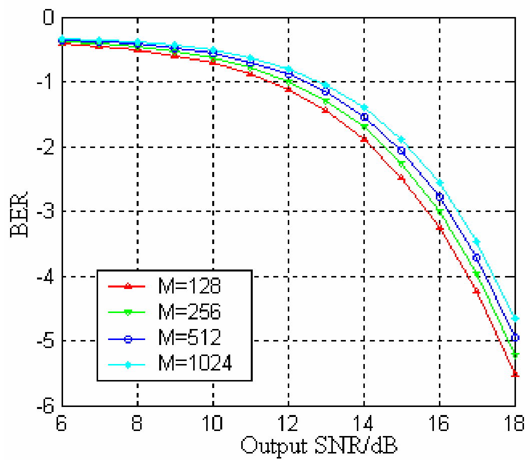

Figure 4. BER versus SNR and signal set size M

Figure 5 shows BER versus SNR and signal parameters with $M = 256$. For a given SNR, BER increases with decrement of product of signal bandwidth and time length BT.

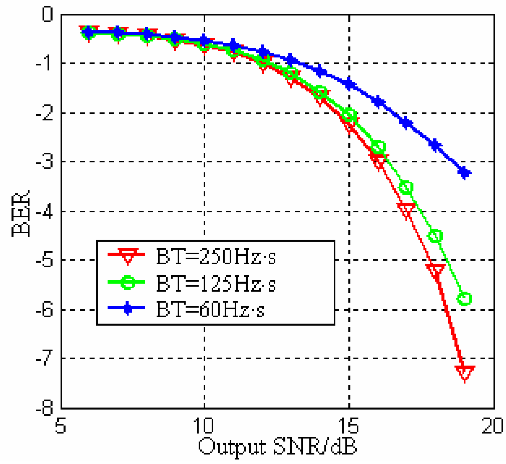

Figure 5. BER versus SNR and signal parameters

CONCLUSION

This paper derives theoretical formula to describe the variation of BER versus several variables, such as SNR, parameters of signal and size of spread-spectrum signal set. And simulation result indicates that correlation property of spread-spectrum signal has a considerable effect on the BER.
For a given SNR and parameters of signal, the size of spread-spectrum signal set must be limited to obtain expected BER.
For a given BER, in order to obtain higher data rate, either SNR must be enhanced or parameters of signal must be adjusted.

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