Study on an estimation method for parameters of a dry laminated panel

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ABSTRACT

For improving sound insulation of a partition, it is often laminated with multi-plates. Although it is hard to solve theoretically a transmitting sound field through the laminated partition with its motion equation, the authors have shown that it can be easy to predict its transmission loss if the laminated panel is treated as a homogeneous panel. In this study, the author suggests an estimation method for parameters of the laminated panel as an application of the well-known Ross-Kerwin-Ungar model. The laminated panel spot screwed or spot gluing with two plates is modeled as three layers, which consist of two plates and a boundary layer between them. The boundary layer is assumed to have loss elastic modulus and very little thickness. The estimation method is verified and discussed by comparing estimated values with measured values of the parameters of the laminated panels.

INTRODUCTION

The sound insulation through a double leaf partition has investigated theoretically and experimentally by London [1]. He has obtained a theoretical solution of transmission sound for an oblique sound incident by using wave equations and acoustic impedance of each leaf. The stastical energy analysis has been used for sound transmission through the double leaf partition [2,3]. Although their predictions have shown good agreement with experiment results, the leaf has been a homogeneous panel. On the other hand, a sound transmission through a sandwich panel with face sheets and a core (e.g. isotropic, orthotropic and honeycomb) has been discussed [4], but it has treated with a single leaf partition. Moreover, a practical estimation of transmission loss for the double leaf partition consisted with gypsum boards have been suggested by considering a sound insulation theory and measured results for various partitions [5]. However, its laminated leaf varies just two kinds of panels and is restricted to leaves having same material and same thickness. These restrictions are unfit for variety of current building materials and constructions.

The author has theoretically analyzed a transmission loss through a double leaf dry partition [6]. The transmission loss was formulated by based on Helmholtz-Kirchhoff integrals and motion equations of each leaf. And estimation formulas for Young’s modulus and loss factor of a laminated leaf, which were made by a theoretical model and a measurement, were calculated by parameters of each panel. Finally, the author made a prediction formula for the transmission loss the double leaf partition with laminated leaves by combining above formulas, and discussed the predicted value with measured value of the transmission loss of the partitions in previous studies and catalogs. Its results suggested that the prediction formula can be applied to the double leaf partitions with various constructions and materials.

In this study, for accurately estimating the Young’s modulus and the loss factor of the laminate panels, the author suggests an estimation method for these parameters of the laminated panel as an application of the well-known Ross-Kerwin-Ungar model for viscoelastic laminae [7]. Its estimated values of the Young’s modulus and the loss factor are verified by their measured values for the laminated panel spot screwed or spot gluing with two laminae (e.g. gypsum board, plywood, metal plate and plastic plate).

ESTIMATION FORMULA OF PARAMETERS OF LAMINATED PLATE

Ross-Kerwin-Ungar model for a three layer plate

In Ross-Kerwin-Ungar(RKU) model, a three layer plate with damping material in flexure is consisted of a base elastic layer and two added layers, which are assumed to of this flexural motion but the middle layer experiences a superimposed shear motion, as shown in Figures 1 and 2.

"Damping of plate flexural vibrations by means of viscoelastic laminae," Structural Damping: (Ross, Ungar and Kerwin, 1959)

Figure 1. Element of a three layer plate in flexurral vibration.
“Damping of plate flexural vibrations by means of viscoelastic laminae,” Structural Damping: (Ross, Ungar and Kerwin, 1959)

**Figure 2.** Dimensions used in analysis of a three layer plate in flexural vibration.

A simple supported beam made of this three layer plate is assumed to be in the sinusoidal flexural vibration with a wave length, \( \lambda \). The net extensional force on each layer is obtained by integrating the stress over the layer, and sum of the force in the neutral plane of composite plate must vanish. Then, an expression for the flexural rigidity is obtained as below:

\[
 EI = \frac{E_1H_1^2}{12} + \frac{E_2H_2^2}{12} + \frac{E_3H_3^2}{12} - \frac{E_2H_2(H_{21} - D)}{2} \left( \frac{H_{31} - D}{1 + g^*} \right) + E_1H_1D^2 + E_2H_2(H_{21} - D)^2 + E_3H_3(H_{31} - D)^2 - \frac{E_2H_2(H_{21} - D)}{2} + E_1H_1(H_{31} - D) \left( \frac{H_{31} - D}{1 + g^*} \right),
\]

\[
 D = \frac{E_2H_2\left( H_{21} - \frac{H_{211}}{2} \right) + (E_2H_2H_{211} + E_3H_3H_{311})g^*}{E_1 + \frac{E_2H_2}{2} + g^*(E_1H_1 + E_2H_2 + E_3H_3)},
\]

\[
 H_{21} = \frac{H_1 + H_2}{2},
\]

\[
 H_{31} = \frac{H_2 + \frac{H_1 + H_3}{2}}{2},
\]

\[
 g^* = \frac{G_2(\lambda/2)}{E_3H_3H_3\pi^2}.
\]

In the equation 1, a first to third term are the flexural rigidity to the neutral plane of each layer, and a parameter \( D \) in fourth term or later is a position of the neutral plane of composite plate must vanish. Then, an expression for the flexural rigidity is obtained as below:

\[
 EI = \frac{E_1H_1^2}{12} + \frac{E_2H_2^2}{12} + \frac{E_3H_3^2}{12} - \frac{E_2H_2\left( H_{21} - D \right)}{2} \left( \frac{H_{31} - D}{1 + g^*} \right) + E_1H_1D^2 + E_2H_2\left( H_{21} - D \right)^2 + E_3H_3\left( H_{31} - D \right)^2 - \frac{E_2H_2\left( H_{21} - D \right)}{2} + E_1H_1\left( H_{31} - D \right) \left( \frac{H_{31} - D}{1 + g^*} \right),
\]

\[
 -E_3H_3\left( H_{31} - D \right) \left( \frac{H_{31} - D}{1 + g^*} \right),
\]

\[
 D = \frac{E_3H_3H_3g^*}{E_1H_1 + g^*(E_1H_1 + E_3H_3)},
\]

\[
 g^* = \frac{i\text{Im}(G_2)(\lambda/2)}{E_3H_3H_3\pi^2}.
\]

However, thickness, \( H_2 \), and loss elastic modulus, \( \text{Im}(G_2) \), of the boundary layer is verified by measuring the Young’s modulus and the loss factor of the laminated panel. Then, the Young’s modulus of the laminated panel is expressed as follow:

\[
 E = \frac{12EI}{(H_1 + H_3)^2}.
\]

And the loss factor of the laminated panel is expressed as follow:

\[
 \eta = \frac{\text{Im}(EI)}{\text{Re}(EI)}
\]

**MEASUREMENT OF A LOSS FACTOR OF THE LAMINATED PANEL**

**Test laminated beam**

Test laminated beams are made with two strips of gypsum board, plywood, metal plate and plastic plate. Three types of gypsum boards are selected as follows: Gypsum-Board Regular-type (GB-R), Fire-Resistant Gypsum-Board (GB-F) and Gypsum-board-Regular-Hard-Type (GB-R-H). Two types of metal plates are selected as follows: stainless and aluminium. Two types of plastic plates are selected as follows: acrylic plate and polyvinyl chloride (PVC) plate. The test laminated beams are combined with (1) two gypsum boards, (2) two plywood, (3) a plywood and a metal plate, and (4) a plywood and plastic plate. Table 1-4 show thickness, Young’s modulus and loss factor of these materials used in the measurement. And, thickness of metal plates and plastic plate are 0.5 mm, 1 mm, 2 mm and 4 mm.

The width of test laminated beam is 100 mm, and its length is 910 mm for gypsum board or 900 mm for plywood, metal plate and plastic plate. Test laminated beams are glued at three spots for gypsum boards, and screwed at three spots for other plate combination. These spots for screws and gluing are both ends and center of the strip. Test laminated beams are 82 samples.
Table 1. Parameters of gypsum board

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<td>$E$ (N/m$^2$)</td>
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Table 2. Parameters of plywood

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Table 3. Parameters of metal plate

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<td>$E$ (N/m$^2$)</td>
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<td>$\eta$</td>
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Table 4. Parameters of plastic plate

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<td>$\eta$</td>
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<td>$E$ (N/m$^2$)</td>
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<td>$\eta$</td>
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Procedure

Measurement of Young’s modulus and loss factor is made in an anechoic chamber as shown in Figure 4. Test laminated beam is hanged from a grating ceiling by two wires. An accelerometer is set on the median line and at 20 mm above the bottom. The impact hammer strikes on the median line and at 35 mm below the top. An averaged frequency characteristic of five impact responses is measured for each test laminated beam. Upper limit of measured frequency is 6.4 kHz and its resolution is 1 Hz.

The Young’s modulus and the loss factor of the test beam are obtained by substituting a resonance frequency $f_n$ and a half-value width $\Delta f_n$ of the response frequency characteristics to following equations:

$$E = \frac{48\pi^2 l^4 \rho \left( f_n^2 + \frac{1}{8} \Delta f_n^2 \right)}{h^2 \theta_n^4}, \quad (11)$$

$$\eta = \frac{\Delta f_n}{f_n}, \quad (12)$$

The Young’s modulus and the loss factor of the test beam are averaged with values of measured at resonance frequencies.

RESULTS AND DISCUSSION

The loss elastic modulus, $\text{Im}(G_2)$, for shear modulus of the boundary layer with various thickness, $H_2$, are obtained by the method of least square for best fitting estimated values to measured values for the Young’s modulus or the loss factor of test laminated beams. When the layer’s thickness varies from 10 nm to 0.1 mm every ten times, the loss elastic modulus, $\text{Im}(G_2)$, also increases every ten times. Then, ratio of the loss elastic modulus of the boundary layer to its thickness, $\text{Im}(G_2)/H_2$, is constant. Results in this section are discussed for 1 μm thickness of the boundary layer.

Figure 4 and 5 show relationship between estimated value and measured value of Young’s modulus or loss factor for all 82 test laminated beams. Figure 4 indicates that estimated Young’s modulus well fit measured Young’s modulus, and its correlation coefficient is 0.87. Although almost of samples are placed from $7\times10^8$ N/m$^2$ to $6\times10^9$ N/m$^2$, it equals or is smaller than the range of Young’s modulus of ordinary building materials for dry partitions, as shown in Table 1-4. And, loss elastic modulus, $\text{Im}(G_2)$, of the boundary layer is 79.62 N/m$^2$, and it is quite smaller than Young’s modulus of ordinary building materials (i.e. $10^8$ to $10^{11}$ N/m$^2$). Even if comparing complex elastic modulus of silicone gel (e.g. $2\times10^7$ N/m$^2$), it is as small as its loss factor is 0.01. This suggests that boundary layer does not have rigid connection between laminae and each lamina may freely vibrate by sliding at boundary layer. This shows that assumption as mentioned in estimation formula may be correct. To discuss spread of estimated values to measured values, shift of a coincidence frequency calculated by the estimated value to that by the measured value is obtained. It shows that 41.5% samples is in ± 1/6 octave range and 70.7% samples is in ± 1/2 octave range. This suggests that the estimation formula is roughly available for predicting Young’s modulus of laminated plate with ordinary building materials in a lump.

Figure 3. Procedure for measuring Young’s modulus and loss factor

Figure 4. Relationship between estimated values and measured values of Young’s modulus for all 82 samples
Figure 5 indicates that it is not correlative between estimate loss factor and measured loss factor, and its correlation coefficient is 0.24. Measured loss factors are placed from 0.02 to 0.18, and it equals or is larger than the range of loss factor of ordinary building materials for dry partitions, as shown in Table 1-4. This suggests that boundary layer may add energy loss to that of laminae themselves. To discuss spread of estimated values to measured value, ratio of error of estimated loss factor to measured loss factor is obtained. It shows that 8.5% samples in ±10% error to measured value, 25.6% samples in ±25% error and 56.1% samples in ±50% error. This suggests that the estimation formula is not available for predicting loss factor of laminated plate with ordinary building materials in a lump. In following section, Young’s modulus and loss factor of each combination of building materials, which are gypsum boards themselves, plywoods themselves, plywood and metal plate, and plywood and plastic plate, will be discussed in detail.

**Figure 5. Relationship between estimated values and measured values of loss factor for all 82 samples**

**Estimation of Young’s modulus for each combination of building materials**

In this section, estimation of Young’s modulus for each combination of building material is discussed in detail. For case of laminated beams consisted with two gypsum boards, it is not correlative between estimated values and measured values for 30 samples, and its correlation coefficient is 0.39. Therefore, discussion on combination of two gypsum boards is separated by combinations of same material plates (e.g. GB-R and GB-R) and combinations of different material plates (e.g. GB-R and GB-F).

Figure 6 shows relationship between estimated values and measured values of Young’s modulus for 12 samples of same material pairs of gypsum boards. It indicates that estimated values well fit measured values, and its correlation coefficient is 0.87. To discuss spread of estimated values to measured values, shift of a coincidence frequency calculated by the estimated value to that by the measured value is obtained. It shows that 58.3% samples is in ±1/6 octave range and 100% samples is in ±1/2 octave range. All samples are placed from 9x10⁹ to 2x10¹⁰, and are smaller than Young’s modulus of Gypsum boards used in this experiment, as shown in Table 1. And, loss elastic modulus, Im(G₂), of the boundary layer is 86.50 N/m². Even if comparing complex elastic modulus of silicone gel (e.g. 2x10⁴ N/m²), it is as small as its loss factor is 0.01. As mentioned above, this suggests that boundary layer does not have rigid connection between laminae and each lamina may freely vibrate by sliding at boundary layer.

On the other hand, Figure 7 shows relationship between estimated values and measured values of Young’s modulus for 18 samples of different material pairs of gypsum boards. It indicates that estimated values fit measured values, and its correlation coefficient is 0.75. To discuss spread of estimated values to measured values, shift of a coincidence frequency calculated by the estimated value to that by the measured value is obtained. It shows that 88.9% samples is in ±1/6 octave range and 100% samples is in ±1/2 octave range. All samples are placed from 1.5x10⁹ N/m² to 3x10¹⁰ N/m², and it roughly equals range of Young’s modulus of gypsum boards used in this experiment, as shown in Table 1. And, loss elastic modulus, Im(G₂), of the boundary layer is 340.4 N/m². Even if comparing complex elastic modulus of silicone gel (e.g. 2x10⁴ N/m²), it is as small as its loss factor is 0.05. This indicates that Young’s modulus of different material pair is rigidier than that of same material pair. It suggests that boundary layer of different material pair has larger resistance than that of same material pair.

**Figure 6. Relationship between estimated value and measured value of Young’s modulus for 12 samples of same material pairs of gypsum board**

**Figure 7. Relationship between estimated value and measured value of Young’s modulus for 18 samples of different material pairs of gypsum board**

Figure 8 shows relationship between estimated values and measured values of Young’s modulus for 15 samples of plywood. It indicates that estimated values well fit measured values, and its correlation coefficient is 0.88. To discuss spread of estimated values to measured values, shift of a coincidence frequency calculated by the estimated value to that by the measured value is obtained. It shows that 46.7% samples is in ±1/6 octave range and 66.7% samples is in ±1/2 octave range. All samples are placed from 7.1x10⁴ to 4.5x10⁵, and almost measured values are small than range of Young’s modulus of plywood used in this experiment, as shown in Table 2. And, loss elastic modulus, Im(G₂), of the boundary layer is 1.950 N/m² and it is smaller than that of gypsum board. Even if comparing complex elastic modulus of silicone gel (e.g. 2x10⁴ N/m²), it is as small as its loss factor is 0.0003. Although this indicates that boundary layer...
between laminae of plywood has less resistance than that of gypsum board, range of Young’s modulus of both is less different between them. Young’s modulus of almost samples is smaller than 2×10^9 N/m² and its range equals to that for same material pairs of gypsum board as shown Figure 6. This suggests that laminated panel with same material plates has smaller Young’s modulus than laminated panel with different plates, and it has smaller Young’s modulus than laminine.

Figure 8. Relationship between estimated value and measured value of Young’s modulus for 15 samples of plywood

Figure 9 shows relationship between estimated values and measured values of Young’s modulus for 19 samples of plywood and metal plate combination. It indicates that estimated values well fit measured values, and its correlation coefficient is 0.88. To discuss spread of estimated values to measured values, shift of a coincidence frequency calculated by the estimated value to that by the measured value is obtained. It shows that 55.6% samples is in ±1/6 octave range and 100% samples is in ±1/2 octave range. All samples are placed from 1×10^9 N/m² to 5×10^9 N/m², and it equals range of Young’s modulus of plywood and plastic plate used in this experiment, as shown in Table 2 and 3. And, loss elastic modulus, Im(G₂), of the boundary layer is 26.18 N/m² and it is almost same as that of same material pairs of gypsum boards. Even if comparing complex elastic modulus of silicone gel (e.g. 2×10^7 N/m²), it is as small as its loss factor is 0.003. Distribution of dots spread over the range from 1×10^7 N/m² to 5×10^9 N/m² as same as the case of two plywood combination shown in Figure 8, but these dots seems to shift to rigider than Figure 8. It assumes that laminated panel with different materials has larger Young’s modulus than that with same materials, as mentioned above.

Figure 10. Relationship between estimated value and measured value of Young’s modulus for 18 samples of plywood and plastic plate combination

Figure 10 shows relationship between estimated values and measured values of Young’s modulus for 18 samples of plywood and plastic plate combination. It indicates that estimated values well fit measured values, and its correlation coefficient is 0.88. To discuss spread of estimated values to measured values, shift of a coincidence frequency calculated by the estimated value to that by the measured value is obtained. It shows that 55.6% samples is in ±1/6 octave range and 100% samples is in ±1/2 octave range. All samples are placed from 1×10^9 N/m² to 5×10^9 N/m², and it equals range of Young’s modulus of plywood and plastic plate used in this experiment, as shown in Table 2 and 3. And, loss elastic modulus, Im(G₂), of the boundary layer is 26.18 N/m² and it is almost same as that of same material pairs of gypsum boards. Even if comparing complex elastic modulus of silicone gel (e.g. 2×10^7 N/m²), it is as small as its loss factor is 0.003. Distribution of dots spread over the range from 1×10^7 N/m² to 5×10^9 N/m² as same as the case of two plywood combination shown in Figure 8, but these dots seems to shift to rigider than Figure 8. It assumes that laminated panel with different materials has larger Young’s modulus than that with same materials, as mentioned above.

Estimation of loss factor for each combination of building materials

In this section, estimation of loss factor for each combination of building material is discussed in detail. Figure 11 shows relationship between estimated values and measured values of loss factor for 30 samples of gypsum boards. Distribution of dots concentrates around where the loss factor is 0.05 and it is not correlative between estimated values and measured values, and its correlation coefficient is 0.29. To discuss spread of estimation values to measured value, ratio of error of estimated loss factor to measured loss factor is obtained. It shows that 33.3% samples in ±10% error to measured value, 63.3% samples in ±25% error and 86.7% samples in ±50% error. This indicates that concentration of distribution of estimated and measured values gives low correlation between them but estimation formula of loss factor seems to be available when laminated panels are made of ordinary gypsum boards. And, loss elastic modulus, Im(G₂), of the boundary layer is 1.524 N/m² but it is quite smaller than that obtained by case of Young’s modulus. Even if comparing complex elastic modulus of silicone gel (e.g. 2×10^4 N/m²), it is as small as its loss factor is 0.0002.
For case of laminated beams consisted with two plywoods, it is not correlation between estimated values and measured values for 15 samples, and its correlation coefficient is 0.42. Therefore, discussion on combination of two plywoods is separated by combinations of same thickness plates and combinations of different thickness plates respectively.

Figure 12 shows relationship between estimated values and measured values of loss factor for 5 samples of same thickness pairs of plywoods. It indicates that estimated value well fit measured values, and its correlation coefficient is 0.88. To discuss spread of estimation values to measured value, ratio of error of estimated loss factor to measured loss factor is obtained. It shows that 20% samples in ±10% error to measured value, 80% samples in ±25% error and 80% samples in ±50% error. And, loss elastic modulus, \( \text{Im}(G_2) \), of the boundary layer is 0.453 N/m\(^2\) but it is smaller than that obtained by case of Young’s modulus. Even if comparing complex elastic modulus of silicone gel (e.g. \(2 \times 10^4\) N/m\(^2\)), it is as small as its loss factor is 0.00006.

Figure 13 shows relationship between estimated values and measured values of loss factor for 10 samples of different thickness pair of plywood.

The loss elastic modulus, \( \text{Im}(G_2) \), for the same thickness strips is about 8 times larger than that for the different thickness strips for the plywood. Because loss elastic modulus, \( \text{Im}(G_2) \), expresses the energy loss in the boundary layer, Figures 12 and 13 suggest that the laminated beams with different thickness strips have lager the equivalent loss factor than that with same thickness strips.

Figure 14 shows relationship between estimated values and measured values of loss factor for 19 samples of plywood and metal plate pairs. It seems that estimated value roughly fit measured values, and its correlation coefficient is 0.41. To discuss spread of estimation values to measured value, ratio of error of estimated loss factor to measured loss factor is obtained. It shows that 21.1% samples in ±10% error to measured value, 52.6% samples in ±25% error and 78.9% samples in ±50% error. And, loss elastic modulus, \( \text{Im}(G_2) \), of the boundary layer is 3698 N/m\(^2\) but it is quite larger than that obtained by case of Young’s modulus. Even if comparing complex elastic modulus of silicone gel (e.g. \(2 \times 10^4\) N/m\(^2\)), it is as large as its loss factor is 0.49.

The loss elastic modulus, \( \text{Im}(G_2) \), for a plywood-metal combination is about 1000 times larger than that for a different thickness plywood combination. This suggests that a combination of strips with quite different characteristics (typical parameters of plywood: \(\rho=500\) kg/m\(^3\), \(E=4\times10^4\) N/m\(^2\), \(\eta=0.02\)), stainless: \(\rho=8000\) kg/m\(^3\), \(E=2\times10^{11}\) N/m\(^2\), \(\eta=0.01\), and aluminium: \(\rho=2700\) kg/m\(^3\), \(E=2\times10^{10}\) N/m\(^2\), \(\eta=0.02\)) causes large additional energy loss by a friction at the boundary layer to the energy loss by the loss factor of both strips themselves.
Figure 15 shows relationship between estimated values and measured values of loss factor for 18 samples of plywood and plastic plate pairs. It seems that estimated value well fit measured values, and its correlation coefficient is 0.81. To discuss spread of estimation values to measured value, ratio of error of estimated loss factor to measured loss factor is obtained. It shows that 22.2% samples in ±10% error to measured value, 72.2% samples in ±25% error and 100% samples in ±50% error. And, loss elastic modulus, \( \text{Im}(G_j) \), of the boundary layer is 755.1 N/m² but it is larger than that obtained by case of Young’s modulus. Even if comparing complex elastic modulus of silicone gel (e.g. \( 2 \times 10^6 \) N/m²), it is as large as its loss factor is 0.1.

The loss elastic modulus, \( \text{Im}(G_j) \), for a plywood-plastic combination is about 200 times larger than that for a different thickness plywood combination. As mentioned above, this suggests that a combination of strips with different characteristics (typical parameters of acrylic plate: \( \rho=1200 \text{ kg/m}^3 \), \( E=10^9 \text{ N/m}^2 \), \( \eta=0.11 \), and PVC plate: \( \rho=1400 \text{ kg/m}^3 \), \( E=10^9 \text{ N/m}^2 \), \( \eta=0.04 \)) causes the additional energy loss by the friction at the boundary layer.

![Figure 15. Relationship between estimated value and measured value of loss factor for 18 samples of plywood and plastic plate combination](image)

**CONCLUDING REMARKS**

For accurately estimating the Young’s modulus and the loss factor of the laminate panels, this study suggested an estimation method for them of the laminated panel as an application of the well-known Ross-Kerwin-Ungar model for viscoelastic laminae [7]. The laminated panel was assumed as three layers consisted with two elastic laminae and a boundary layer between them which has only a frictional energy loss. Its estimated values of the Young’s modulus and the loss factor were verified by measured values for the laminated beams spot screwed or spot gluing with two laminae (e.g. plywood, gypsum board, metal plate and plastic plate).

Results of the estimation and the measurement suggest that this estimation formula for the Young’s modulus is roughly available for laminated panels with two various material laminae in a lump, but that this estimation formula for the loss factor is respectivelly available for laminated panels, if appropriate loss elastic modulus of the shear modulus for the boundary layer is chosen. Therefore, it is not convenient for predicting the loss factor of laminated panels.

The loss elastic modulus is different between estimation of the Young’s modulus and estimation of the loss factor. As the Ross-Kerwin-Ungar model is formulated for the complex flexural rigidity, it should be that the loss elastic modulus is given by one value for obtaining the Young’s modulus and the loss factor. In future, it needs to be verified that the boundary layer is assumed as a viscoelastic model but not as a pure viscous model.

And, in case of predicting individual laminated panel, the loss elastic modulus is larger for different thickness plate combinations than that for same thickness plate combinations. Moreover, it is much larger for a combination of plates with quite different characteristics than a combination of similar plates. However, it needs more considerations how the loss elastic modulus is decided for various material combinations.

**REFERENCES**